SPECTRAL GEOMETRY CONFERENCE MAY 16-20 2016

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Registration 9:50 Welcoming	L. Hermi	J. Rossi	P. T. Nam	R. Laugesen
10:00 -11:00	C. Gordon	D. Webb	M. Chasman	M. van den Berg	R. Mahadevan
	Coffee	Coffee	Coffee	Coffee	Coffee
11:20-12:20	M. Loss	Free	E. Stockmeyer	E. Harrell	B. Siudeja
	Lunch		Lunch	Lunch	Lunch
13:45-14:45	C. Jerez		C. Feuillade	M. Courdurier	C. Sing-Long
14:45-15:45	M. Dambrine		M. Pereira	H. van den Bosch	T. Weidl
	Coffee		Coffee	Coffee	Coffee
16:15-17:15	M. Ashbaugh		A. Osses	G. Csato	

Mark S. Ashbaugh

Department of Mathematics
University of Missouri, Columbia, MO

A Sharp Lower Bound for the First Eigenvalue of the Vibrating Clamped Plate under Compression

Abstract:

We give a sharp lower bound to the fundamental frequency of a vibrating clamped plate under compression in the context of plates of different shapes of fixed area. Mathematically, the problem is that of bounding the first eigenvalue of a certain 4th-order partial differential operator with leading term the bi-Laplacian from below by a positive constant over the square of the domain's area. We give a Rayleigh-Faber-Krahn-type result for this problem for small compressions. Thus, our lower bound is saturated for a disk, and the constant appearing in our inequality is that for the disk under the appropriate compression. (This is joint work with R. Benguria and R. Mahadevan.)

Merc Chasman

Division of Math & Science University of Minnesota Morris

Low Eigenvalues of Plates

Abstract:

Many of the same spectral questions we consider for drums and membranes can be asked of plates. However, with the fourth-order bi-Laplacian replacing the usual Laplacian in our eigenvalue equations, fewer of these problems have been solved. We will survey several results involving low eigenvalues of plates and the approaches used to achieve them, and take a brief look at some open problems.

Matías Courdurier

Departamento de Matemáticas
Pontificia Universidad Católica de Chile

Construction of some soliton solutions for a Schrödinger equation with space-time dependent non–linearity.

Abstract:

The Gross-Pitaevskii equation

$$i\frac{\partial\psi}{\partial t} = -\Delta\psi - |\psi|^2\psi,$$

represents a one body approximation of an N–body system of linear Schrödinger equations. The non–linear term arises as a mean-field approximation when the interactions between particles in the original system are of the form $\delta(x_i - x_j)$. In this talk we will look at the equation

$$i\frac{\partial\psi}{\partial t} = -\Delta\psi + \overline{V}(x,t)|\psi|^2\psi, \quad (1)$$

where the role of $V(x,t)$ in front of the mean-field term is to possibly consider different spatial-temporal dependencies in the interaction between particles of the original N—body system. For some specific $V(x,t)$ in equation (1) we will present the construction of a family of soliton solutions, by finding "eigen-functions" to the associated reduced equation. This is a joint work with O. Bourget and C. Fernandez.

Marc Dambrine

Université de Pau, France, and Universidad de Chile

An extremal eigenvalue problem for the Wentzell–Laplace operator

Abstract:

We consider the question of giving an upper bound for the first nontrivial eigenvalue of the Wentzell-Laplace operator of a domain Ω, involving only geometrical informations. We provide such an upper bound, by generalizing Brock's inequality concerning Steklov eigenvalues, and we conjecture that balls maximize the Wentzell eigenvalue, in a suitable class of domains, which would improve our bound. To support this conjecture, we prove that balls are critical domains for the Wentzell eigenvalue, in any dimension, and that they are local maximizers in dimension 2 and 3, using an order two sensitivity analysis. We also provide some numerical evidence.

Christopher Feuillade

Instituto de Física, Pontificia Universidad Católica de Chile

Using acoustics to count fish in the sea

Abstract:

Acoustical scattering from individual and ensembles of swim bladder fish in the sea is important for both military and commercial reasons. At low frequencies (250 Hz?10 kHz), scattering from these fish is dominated by the bladder resonance response. Dense schools of bladder fish frequently consist of individuals of similar size, arranging themselves about one fish length apart. At near–resonance frequencies, acoustical interactions between fish can cause the ensemble scattering to become highly complex. Additionally, since the wavelength at resonance is generally many times the fish spacing, the scattered fields interfere strongly. Both features must be incorporated to realistically describe scattering from fish schools. An effective methodology is available through the application of self-consistent multiple scattering techniques, coupled with the solution of sets of coupled differential equations, and incorporates a verified swim bladder scattering "kernel" for an individual fish. All orders of multiple scattering interactions between the fish are included, and the aggregate scattering field calculated by coherent summation. The resulting mathematical model leads to two important developments. First, the ?forward? problem of predicting the expected strength of SONAR echoes from schools of fish, for varying oceanic conditions, and different species of fish, is facilitated. Second, the inverse problem of estimating the number of fish in a school, using measurements of both the scattered field of the school, and the transmitted field, which is attenuated as it passes through the school, becomes tractable. This second application has potential importance as a method for monitoring fisheries and fish stocks.

Evans M. Harrell

School of Mathematics, Georgia Institute of Tecnology

What does the average eigenvalue know about a graph?

Abstract:

The average of the first k eigenvalues of the Laplacian (or similar operator) on a discrete or quantum graphs satisfies various inequalities related to the structure of the graph. I will present some methods for obtaining inequalities for averages of eigenvalues and will use them to obtain sharp bounds in terms of the structure of the graph. This work is joint with J. Stubbe and J. Dever.

Carolyn Gordon

Department of Mathematics, Darmouth College

Isospectrality: a partial survey

Abstract: We will address constructions of isospectral operators in a variety of settings such as the Laplace spectrum on compact Riemannian manifolds and scattering on noncompact Riemannian manifolds. The primary techniques are Sunada?s technique and its many generalizations, which typically result in metrics with the same local but different global geometry, and the torus action method, which results in metrics with different local geometry. We will also identify geometric invariants that are not spectrally determined.

Lotfi Hermi

Department of Mathematics, University of Arizona

Isoperimetric Inequalities for Convex Cones and Wedge-Like Domains

By introducing geometric factors and physical parameters which lend themselves to the Payne interpretation in Weinstein fractional space, we prove new isoperimetric inequalities for the fundamental eigenvalue of the Dirichlet problem for wedge-like membranes in two dimensions, and convex cones in higher dimensions. These inequalities generalize and improve sharp inequalities that date back to the works of Payne-Weinberger (1960), Payne-Rayner (1973), Crooke-Sperb (1978), and Chiti (1982). We also motivate, conjecture, and prove an isoperimetric inequality relating the fundamental eigenvalue of a wedge-like membrane to its "relative torsional rigidity". This new sharp inequality beats both Faber-Krahn and Payne-Weinberger at the isoperimetric game. The central tool of this recent development is the use of a new weighted form of the Kohler-Jobin symmetrization method, which we introduce. (Joint work with A. Hasnaoui)

Carlos Jerez

Departamento de Ingeniería Matemática
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Optimal Preconditioning for the Hypersingular Operators on Screens

Abstract:

We propose a new Calderón–type preconditioner for the hypersingular integral operator for the Laplacian on screens in $\mathbb{R}^3$. We introduce a modified weakly singular operator, which is the exact inverse of the hypersingular operator on the unit disk. It forms the foundation for dual–mesh based operator preconditioning. Applied to low–order boundary element Galerkin discretizations, it achieves h-uniformly bounded condition numbers. Heuristic extensions to general screens even with non-smooth boundaries are discussed. Their good performance is confirmed by numerical tests.

Richard Laugesen

Department of Mathematics, University of Illinois, USA

Optimal shapes for lattice point counting

Abstract:

What shape of domain minimizes the n—th eigenvalue of the Dirichlet Laplacian, for large n? (Here we normalize the area to equal 1.) Does the minimizer approach a disk as n tends to infinity? Supporting this idea is the discovery by Antunes and Freitas that among rectangles, the minimizer approaches a square in the limit. Their result for rectangles relies on lattice point counting in ellipses. In joint work with Shiya Liu (University of Illinois), we extend to more general lattice counting problems, proving again that the "most balanced" situation is optimal in the limit. Our work in progress aims to connect these insights back to spectral problems.

Michael Loss

School of Mathematics, Georgia Tech

60 years “Foundations of Kinetic Theory”

Abstract: In 1956 Mark Kac published his celebrated article “Foundations of Kinetic Theory” in which he laid out a program for studying systems of colliding particles by means of probabilistic methods. He proposed a master equation to describe a one dimensional gas in a spatially homogeneous situation and defined what we nowadays call “propagation of chaos”. Further he proved propagation of chaos and gave a satisfactory derivation of the associated non-linear Boltzmann-Kac equation. In the past 15 years this program received renewed attention and this talk is a summary of some of the results in this area.

An eigenvalue optimization problem for the p-Laplacian

ANISA M.H. CHORWADWALA[1], R. MAHADEVAN[2],

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keywords : shape optimization; Dirichlet p-Laplacian; shape derivative analysis; moving plane method; comparison principles

Abstract

It has been shown by Ashbaugh and Chatelain (personal communication), Harrell et. al. (SIAM J. Math. Analysis 33 (2001), 240-259), Kesavan (Proc. R. Soc. Edinb. 133A (2003), 617-624) among others that the first eigenvalue for the Dirichlet Laplacian in a punctured ball, with the puncture having the shape of a ball, is maximum if and only if the balls are concentric. Recently, Emamizadeh and Zivari-Rezapour (Proc. Am. Math. Soc. 136 (2007), 1325-1331) have tried to generalize this result to the case of the p-Laplacian but could succeed only in proving a domain monotonicity result for a weighted eigenvalue problem in which the weights need to satisfy some restrictive assumptions which excludes the constant weight. In this talk, we show how to obtain the result for the original eigenvalue problem for the Dirichlet p-Laplacian ($1 < p < \infty$) with constant weight. The uniqueness of the maximizing domain in the nonlinear case is still an open question.

Acknowledgments

The communicating author thanks CONICYT for the financement received through the project FONDECYT 1130595.

References

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Phan Thanh Nam

Institute for Science and Technology, Austria

Nonexistence of minimizers in the liquid drop model and TFDW theory

Abstract:

It is a conjecture that the energy in the liquid drop model is minimized by a ball when the mass is small, and the minimizer does not exist when the mass is large. The nonexistence part has been proved by Knuepfer-Muratov and Lu-Otto, but their methods are rather involved. We will provide a new, simpler proof for the nonexistence with an explicit bound. Our techniques can be developed to prove the nonexistence highly negative ions in the Thomas-Fermi-DiracWeizscker theory, which has been open for a long time. The talk is based on joint works with Rupert L. Frank, Rowan Killip and Hanne Van Den Bosch.

Axel Osses

Departmento de Ingeniería Matemática
Universidad de Chile

The making of a video about "Can one hear the shape of a drum?"

Abstract:

We describe the main challenges encountered during the making of a video for public scientific dissemination of math concepts, where "Can one hear the shape of a drum?" was one of them. From the difficulty of making a simple, brief script for general public to the required software and models to realistically animate the membrane and sound of drums in matlab for the film. In particular, the numerical verification of isospectrality for polygonal drums was a real and almost impossible challenge. We revisit some works in this direction and we show the final video obtained that can be useful for similar future educational or dissemination films.

THE NEUMANN PROBLEM FOR THE LAPLACIAN OPERATOR IN OSCILLATING THIN DOMAINS

MARCONE CORRÊA PEREIRA

Abstract

In this talk we discuss some results from [1, 2, 3] concerning to the asymptotic behavior of the solutions of a homogeneous Neumann problem for the Laplacian operator posed in thin domains with locally periodic structure on the boundary. Using Multiple Scale Method and Oscillating Test Functions from Homogenization Theory we obtain the homogenized equation proving weak convergence in H^1 Sobolev spaces. Next we introduce a notion of convergence in order to investigate the convergence of the resolvent operators defined by these problems.

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Pesquisa parcialmente financiada por CNPq 302960/2014-7 and 471210/2013-7, FAPESP 2013/22275-1 and 2015/17702-3.

NONLOCAL PERIMETER, CURVATURE AND MINIMAL SURFACES FOR MEASURABLE SETS

JOSÉ M. MAZÓN, JULIO D. ROSSI AND JULIÁN TOLEDO

Abstract. We study the nonlocal perimeter associated with a nonnegative radial kernel $J : \mathbb{R}^N \to \mathbb{R}$, compactly supported, verifying $\int_{\mathbb{R}^N} J(z)dz = 1$. The nonlocal perimeter studied here is given by the interactions (measured in terms of the kernel J) of particles from the outside of a measurable set E with particles from the inside, that is,

$$P_J(E) := \int_E \left(\int_{\mathbb{R}^N \setminus E} J(x-y)dy \right) dx.$$

We prove that an isoperimetric inequality holds and that, when the kernel J is appropriately rescaled, the nonlocal perimeter converges to the classical local perimeter. Associated with the kernel J and the previous definition of perimeter we can consider minimal surfaces. In connexion with minimal surfaces we introduce the concept of J-mean curvature at a point x, and we show that again under rescaling we can recover the usual notion of mean curvature. In addition, we study the analogous to a Cheeger set in this nonlocal context and show that a set Ω is J-calibrable (Ω is a J-Cheeger set of itself) if and only if there exists τ such that $\tau(x) = 1$ if $x \in \Omega$ satisfying $-\lambda_\Omega^J \tau \in \Delta_1^J \chi_\Omega$, here λ_Ω^J is the J-Cheeger constant $\lambda_\Omega^J = \frac{P_J(\Omega)}{|\Omega|}$ and, Δ_1^J is given, formally, by

$$\Delta_1^J u(x) = \int_{\mathbb{R}^N} J(x-y) \frac{u(y)-u(x)}{|u(y)-u(x)|} dy.$$

Moreover, we also provide a result on J-calibrable sets and the nonlocal J-mean curvature that says that a J-calibrable set can not include points with large curvature. Concerning examples, we show that balls are J-calibrable for kernels J that are radially nonincresing, while stadiums are J-calibrable when they are small but they are not when they are large.

References

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Key words and phrases. Sets of finite perimeter, nonlocal operators, Cheeger sets, Calibrable sets.

2010 *Mathematics Subject Classification:* 45C99, 28A75, 49Q05.

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Computational Methods for Unbiased Risk Estimation .

Abstract:

In many engineering applications, one seeks to estimate, recover or reconstruct an unknown object of interest from an incomplete set of linear measurements. Mathematically, the unknown object can be represented as the solution to an underdetermined system of linear equations. In recent years it has been shown that it is possible to recover the true object by exploiting a priori information about its structure, such as sparsity in compressed sensing or low-rank in matrix completion. However, in practice the measurements are corrupted by noise and exact recovery is not possible.

A popular approach to address this issue is to solve an unconstrained convex optimization problem to obtain an estimate that both explains the measurements and resembles the known structural characteristics of the true object. The objective function quantifies the trade-off between data fidelity and structural fidelity, which is usually controlled by a single regularization parameter. One possible criterion for selecting the value of this parameter is to minimize an unbiased estimate for the prediction error as a surrogate for the true prediction risk. Unfortunately, evaluating this estimate requires an expression for the weak divergence of the predicted observations. Therefore, it is necessary to characterize the regularity of the solution to the convex optimization problem with respect to the measurements.

In this talk I will present a conceptual and practical framework to study the regularity of the solution to a popular class of such optimization problems. The approach consists of using an auxiliary optimization problem that characterizes the smoothness of the predicted observations. In particular, we can relate the analytic singularities of the predicted observations with the geometric singularities of the feasible set to this problem. I will then present a disciplined approach for obtaining closed-form expressions for the derivatives of the predicted measurements that are amenable to computation. Finally, I will explain how the expressions establish a connection between the geometry of the convex optimization problem and the unbiased estimate for the prediction risk.

Bartłomiej Siudeja

Department of Mathematics, University of Oregon

On mixed Dirichlet-Neumann eigenvalues of triangles

Abstract:

We consider a fixed triangular domain and various mixed Dirichlet–Neumann eigenvalue problems on that domain. We are interested in the dependence of the smallest eigenvalue of the problem on the choice of the sides for the Dirichlet boundary. It turns out that the longer the Dirichlet side the higher the eigenvalue. Similarly with two Dirichlet sides.

Edgardo Stockmeyer

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Infinite mass boundary conditions for Dirac operators.

Abstract:

We study a self–adjoint realization of a massless Dirac operator on a bounded connected domain $\Omega \subset \mathbb{R}^2$ which is frequently used to model graphene. In particular, we show that this operator is the limit, as $M \to \infty$, of a Dirac operator defined on the whole plane, with a mass term of size M supported outside Ω.

Michiel van den Berg

Bristol University, UK

On the heat content of a polygon.

Abstract:

Let D be a bounded, connected, open set in the plane with polygonal boundary. Suppose D has initial temperature 1 and the complement of D has initial temperature 0. We obtain the asymptotic behaviour of the heat content of D as time t goes to 0. We then apply this result to compute the heat content of a particular fractal polyhedron in $\mathbb{R}^3$ for small t.

Hanne Van Den Bosch

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Spectral gaps of Dirac operators describing graphene quantum dots

Abstract:

Low energy electronic excitations in graphene, a two-dimensional lattice of carbon atoms, are described effectively by a two–dimensional Dirac operator. For a bounded flake of graphene (a quantum dot), the choice of boundary conditions determines various properties of the spectrum. These properties, in turn, influence the transport of electrons through the dot, so it is interesting to study them. For a simply connected flake and a family of local boundary conditions, we obtain an explicit lower bound on the spectral gap around zero. This is joint work with Rafael Benguria, Søren Fournais and Edgardo Stockmeyer.

David L. Webb

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Transplantation: Some easy applications

Abstract:

The notion of “ransplantation”, introduced independently by P. Bérard, P. Buser, and S. Zelditch and developed and generalized by many others, has been a useful tool in generating many interesting examples of Riemannian manifolds, quantum graphs, and other geometric objects that are in some sense isospectral. We will review the algebraic background of transplantation, which is surprisingly simple, and we will illustrate the technique by means of a few examples. These will include a retrospective understanding of some older results, as well as some more recent work of Peter Herbrich and others.

Timo Weidl

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Recent developments in sharp semiclassical bounds

Abstract:

As Mark Kac pointed out, according to Weyl's law one can hear the area and the perimeter in the high tones of the drum. Moreover, the Berezin and the Li-Yau inequalities use the area via the first term in Weyl's asymptotic formula to state uniform spectral bounds on partial eigenvalue sums. I will report on various attempts to improve these bound by taking even terms of lower order into account. In particular, I will sketch some recent results on the magnetic Laplacian and on the Heisenberg Laplacian, respectively.

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