NONLOCAL PERIMETER, CURVATURE AND MINIMAL SURFACES FOR MEASURABLE SETS

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ABSTRACT. We study the nonlocal perimeter associated with a nonnegative radial kernel $J : \mathbb{R}^N \to \mathbb{R}$, compactly supported, verifying $\int_{\mathbb{R}^N} J(z)dz = 1$. The nonlocal perimeter studied here is given by the interactions (measured in terms of the kernel J) of particles from the outside of a measurable set E with particles from the inside, that is,

$$P_J(E) := \int_E \left(\int_{\mathbb{R}^N \setminus E} J(x-y)dy \right) dx.$$

We prove that an isoperimetric inequality holds and that, when the kernel J is appropriately rescaled, the nonlocal perimeter converges to the classical local perimeter. Associated with the kernel J and the previous definition of perimeter we can consider minimal surfaces. In connexion with minimal surfaces we introduce the concept of J-mean curvature at a point x, and we show that again under rescaling we can recover the usual notion of mean curvature. In addition, we study the analogous to a Cheeger set in this nonlocal context and show that a set Ω is J-calibrable (Ω is a J-Cheeger set of itself) if and only if there exists τ such that $\tau(x) = 1$ if $x \in \Omega$ satisfying $-\lambda_\Omega^J \tau \in \Delta_1^J \chi_\Omega$, here λ_Ω^J is the J-Cheeger constant $\lambda_\Omega^J = \frac{P_J(\Omega)}{|\Omega|}$ and, Δ_1^J is given, formally, by

$$\Delta_1^J u(x) = \int_{\mathbb{R}^N} J(x-y) \frac{u(y)-u(x)}{|u(y)-u(x)|} dy.$$

Moreover, we also provide a result on J-calibrable sets and the nonlocal J-mean curvature that says that a J-calibrable set can not include points with large curvature. Concerning examples, we show that balls are J-calibrable for kernels J that are radially nonincresing, while stadiums are J-calibrable when they are small but they are not when they are large.

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