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Computational Methods for Unbiased Risk Estimation .

Abstract:

In many engineering applications, one seeks to estimate, recover or reconstruct an unknown object of interest from an incomplete set of linear measurements. Mathematically, the unknown object can be represented as the solution to an underdetermined system of linear equations. In recent years it has been shown that it is possible to recover the true object by exploiting a priori information about its structure, such as sparsity in compressed sensing or low-rank in matrix completion. However, in practice the measurements are corrupted by noise and exact recovery is not possible.

A popular approach to address this issue is to solve an unconstrained convex optimization problem to obtain an estimate that both explains the measurements and resembles the known structural characteristics of the true object. The objective function quantifies the trade-off between data fidelity and structural fidelity, which is usually controlled by a single regularization parameter. One possible criterion for selecting the value of this parameter is to minimize an unbiased estimate for the prediction error as a surrogate for the true prediction risk. Unfortunately, evaluating this estimate requires an expression for the weak divergence of the predicted observations. Therefore, it is necessary to characterize the regularity of the solution to the convex optimization problem with respect to the measurements.

In this talk I will present a conceptual and practical framework to study the regularity of the solution to a popular class of such optimization problems. The approach consists of using an auxiliary optimization problem that characterizes the smoothness of the predicted observations. In particular, we can relate the analytic singularities of the predicted observations with the geometric singularities of the feasible set to this problem. I will then present a disciplined approach for obtaining closed-form expressions for the derivatives of the predicted measurements that are amenable to computation. Finally, I will explain how the expressions establish a connection between the geometry of the convex optimization problem and the unbiased estimate for the prediction risk.