Non-geodesic motion with and without mass

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in colaboraion with

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Afunalhue
January 2017

Outline

- Motivation
- Introduction
- Collisions of STOPs with mass
- STOP without mass
- Conclusion?

Based on:

*C. Armaza, M. Banados, B.K. Class.Quant.Grav. 33 (2016) no.10, 105014

**C.Armaza, S.Hojman, B.K., N. Zalaquett, Class.Quant.Grav. 33 (2016) no.14, 145011

Motivation

Big quest:
Unify gravity with particle physics!

Simple question first:
What about particles in curved space-time?

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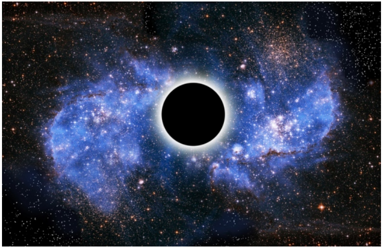
Motivation

Phenomenological question:

QG messengers have spin and travel in curved space-time



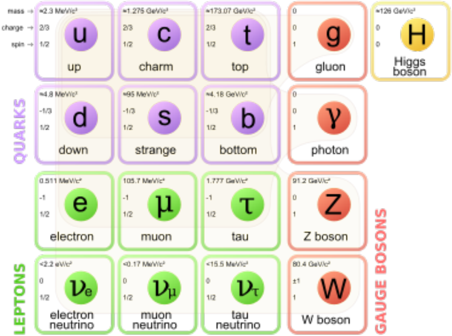
Up to now, all conclusions drawn by using geodesics

$$\frac{dp^\mu}{d\lambda} = 0$$

How wrong is this?

Motivation

Formal question:
Almost all SM particles have spin



Exact motion in curved background?

Introduction

Introduction

Three different approaches

- Using properties of $T^{\mu\nu}$
  Mathisson (1937), Papapetrou (1951), Dixon (1970)
- Using Lagrangian formulation
  Hanson-Regge (1974), Hojman (1975)
- Solutions of and limits of fields in curved space-time
  ... Hojman (2016)

One common answer!

Three different approaches:

- Using properties of $T^{\mu\nu}$
  Mathisson (1937), Papapetrou (1951), Dixon (1970)
- Using **Lagrangian formulation**
  Hanson-Regge (1974), Hojman (1975)
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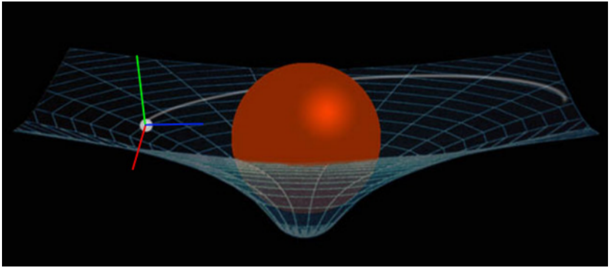
Variables:

- Position $x^\mu(\lambda)$
- Internal orientation $e_a^\mu(\lambda)$ (with $e_b^\mu e_a^\nu g_{\mu\nu} = \eta_{ab}$)



Variables:

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Velocities:

- Position $u^\mu = \frac{dx^\mu}{d\lambda}$
- Angular $\sigma^{\mu\nu} = \eta^{ab} e_a^\mu \frac{De_b^\nu}{D\lambda} = -\sigma^{\nu\mu}$ (with $\frac{De_b^\nu}{D\lambda} = \frac{de_b^\nu}{d\lambda} + \Gamma^\nu_{\rho\tau} e_b^\rho u^\tau$)

Lagrangian: Function of invariants:

$$L = L(a_1, a_2, a_3, a_4) \tag{2}$$

- $a_1 = u^\mu u_\mu$
- $a_2 = \sigma^{\mu\nu}\sigma_{\mu\nu}$
- $a_3 = u^\alpha \sigma_{\alpha\beta}\sigma^{\beta\gamma}u_\gamma$
- $a_4 = \sigma_{\alpha\beta}\sigma^{\beta\gamma}\sigma_{\gamma\delta}\sigma^{\delta\alpha}$

with canonical momenta

$$P^\mu = -\frac{\partial L}{\partial u_\mu} \tag{3}$$

and

$$S^{\mu\nu} = -\frac{\partial L}{\partial \sigma_{\mu\nu}} \tag{4}$$

Lagrangian: Function of invariants:

$$L = L(a_1, a_2, a_3, a_4) \tag{5}$$

Variational calculus ...
EOM δx^μ:

$$\frac{DP^\mu}{D\lambda} = -\frac{1}{2} R^\mu_{\nu\alpha\beta} u^\nu S^{\alpha\beta} \tag{6}$$

EOM $\delta\theta^{\mu\nu}$ (parte independiente de e^μ_a):

$$\frac{DS^{\mu\nu}}{D\lambda} = P^\mu u^\nu - P^\nu u^\mu \tag{7}$$

EOMs:

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$$\frac{DS^{\mu\nu}}{D\lambda} = P^\mu u^\nu - P^\nu u^\mu$$

Note:

- Same EOMs found in many different ways
- Degrees of freedom do not match (coupled equations)
- Three rotations in rest frame not 6
- $\Rightarrow$ need functional form of $L(a_i)$
- $\Rightarrow$ need constraints!

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- $\Rightarrow$ need functional form of $L(a_i)$
- $\Rightarrow$ **need constraints!**

Constraints:
Different approaches have different constraints (same EOMs).

- Dixon: $S^{\mu\nu} u_\nu = 0$
- Tulczyjew: $S^{\mu\nu} P_\nu = 0$
- Others ...

Constraints:
Different approaches have different constraints (same EOMs).

- Dixon: $S^{\mu\nu} u_\nu = 0$
- **Tulczyjew:** $S^{\mu\nu} P_\nu = 0$
- **Others ...(invent for** $m = 0$**)**

Finally:

EOMs:

$$\frac{DP^\mu}{D\lambda} = -\frac{1}{2} R^\mu_{\nu\alpha\beta} u^\nu S^{\alpha\beta} \tag{8}$$

$$\frac{DS^{\mu\nu}}{D\lambda} = P^\mu u^\nu - P^\nu u^\mu \tag{9}$$

Constraints:

$$S^{\mu\nu} P_\nu = 0 \tag{10}$$

(for $m \neq 0$)

Collisions of STOPs

Collisions of STOPs in Schwarzschild background

Astrophysical background:

Collisions of STOPs

Astrophysical background:
(geodesics)

- Black holes can in principle produce $E_{CM} \to \infty$, but one neds
- Extremely rotating black hole
- Collision at the horizon
- Angular momentum l: critical

$\Rightarrow$ Unlikely, hard to observe

Collisions of STOPs

Idea:

Let the particle rotate and the black hole be spherical

- Can one produce $E_{CM} \to \infty$?
  If yes:
- Has the collision to be at the horizon?
- Has the angular momentum l: to be critical?
- Is there a notion of extremely rotating particle?

$\Rightarrow$ Solve equations (8–10) for

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2\, d\Omega^2, \tag{11}$$

and see ...

Solution in equatorial plane (draw...):

$$\frac{P^t}{m} = \left(1 - \frac{2M}{r}\right)^{-1} \frac{e - Mjs/r^3}{1 - Ms^2/r^3}, \tag{12}$$

$$\frac{P^\phi}{m} = \frac{1}{r^2}\left(\frac{j - es}{1 - Ms^2/r^3}\right), \tag{13}$$

$$\left(\frac{P^r}{m}\right)^2 = \left(\frac{e - Mjs/r^3}{1 - Ms^2/r^3}\right)^2 - \left(1 - \frac{2M}{r}\right)\left[1 + \frac{1}{r^2}\left(\frac{j - es}{1 - Ms^2/r^3}\right)^2\right], \tag{14}$$

with velocities

$$\frac{dr}{dt} \equiv \frac{u^r}{u^t} = \frac{P^r}{P^t}, \tag{15}$$

$$\frac{d\phi}{dt} \equiv \frac{u^\phi}{u^t} = \left(\frac{1 + 2Ms^2/r^3}{1 - Ms^2/r^3}\right)\frac{P^\phi}{P^t}. \tag{16}$$

Collisions in equatorial plane (draw ...):
Collision energy:

$$E_{\rm cm}^2 = -(\vec{P}_1 + \vec{P}_2)^2 = m_1^2 + m_2^2 - 2\,\vec{P}_1 \cdot \vec{P}_2. \tag{17}$$

gives

$$E_{\rm cm}^2 = \frac{2m^2}{\Delta_1\Delta_2\Delta}\Bigg\{ r(r^3 - Mj_1 s_1)(r^3 - Mj_2 s_2) + \Delta\Big[\Delta_1\Delta_2 - r^4(j_1 - s_1)(j_2 - s_2)\Big] - \sqrt{r(r^3 - Mj_1 s_1)^2 - \Delta[\Delta_1^2 + r^4(j_1 - s_1)^2]}\sqrt{r(r^3 - Mj_2 s_2)^2 - \Delta[\Delta_2^2 + r^4(j_2 - s_2)^2]}\Bigg\}, \tag{18}$$

where $\Delta \equiv r - 2M$ and $\Delta_i \equiv r^3 - Ms_i^2$, $i = 1, 2$.

$\Rightarrow$ Poles! Trajectories reach poles?

Radial turning points:

$$\left(\frac{P^r}{m}\right)^2 = a\left(1-\frac{Ms^2}{r^3}\right)^{-2}[e-V_+(r)][e-V_-(r)], \qquad (19)$$

where the effective potential is given by

$$V_\pm(r) = \frac{b \pm \Sigma^{1/2}}{a} \qquad (20)$$

with

$$a = 1-\left(1-\frac{2M}{r}\right)\frac{s^2}{r^2}, \qquad b = -\frac{js}{r^2}\left(1-\frac{3M}{r}\right), \qquad (21)$$

and

$$\Sigma = \left(1-\frac{2M}{r}\right)\left(1-\frac{Ms^2}{r^3}\right)^2\left[1+\frac{j^2}{r^2}-\left(1-\frac{2M}{r}\right)\frac{s^2}{r^2}\right]. \qquad (22)$$

Collisions of STOPs

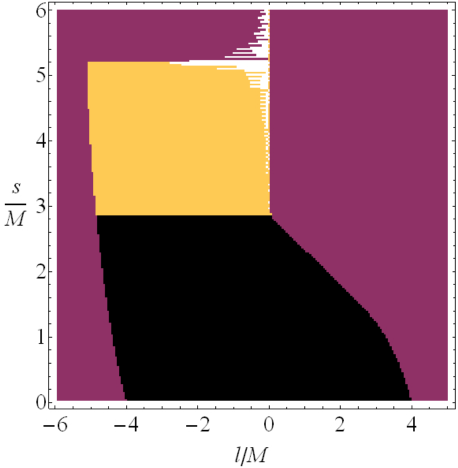
In analytic analysis one finds:

- For retrograde ($l < 0$) trajectories
- Spin: $8M^2 \leq s^2 \leq 27M^2$

Divergence can be reached and lie outside of BH!

Collisions of STOPs

In numerical analysis one finds:



E_{CM} divergent for yellow region

Massless STOPs

Massless STOPs

Literature considers

- $S^{\mu\nu} P_\mu = 0$
- $S^{\mu\nu} U_\mu = 0$
- $S^{\mu\nu} U_\mu = a U^\nu$ and $P^\mu U_\mu = \frac{da}{d\tau}$

Always:

Massless STOPs travel on simple null geodesics

$$\frac{dP^\mu}{d\tau} = 0 \tag{23}$$

Nothing else?

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Massless STOPs

Nothing else? Actually many possibilities ...
For simplicity define

$$V^\mu = S^{\mu\nu} P_\nu \tag{24}$$

$$W^\mu = S^{*\mu\nu} P_\nu \tag{25}$$

$$J^2 = \frac{1}{2} S^{\mu\nu} S_{\mu\nu} \tag{26}$$

For example

$$W^\mu = \lambda P^\mu|_{\lambda\neq 0}, \text{ with } V^\mu = \alpha P^\mu \tag{27}$$

(studied 21 cases and combinations)

Massless STOPs

$$W^\mu = \lambda P^\mu|_{\lambda\neq 0}, \text{ with } V^\mu = \alpha P^\mu \tag{28}$$

One finds algebraically for $\alpha \neq 0$:

- $P^2 = W^2 = V^2 = 0$ (indeed massless)
- $S^* S = \alpha\lambda$
- $J^2 = \alpha^2 - \lambda^2$

Always nice to have non-trivial algebraic relations

Massless STOPs

What does that mean for trajectories?

$\Rightarrow$ Not necessarily $dP^\mu/d\tau = 0$

What does that mean: "Not necessarily $dP^\mu/d\tau = 0$"

In principle $\neq 0$,

but in some symmetric cases and initial conditions still might be ...

example

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Massless STOPs

Example:
Massless radial Schwarzschild (draw)

$$S^{tr} = -\frac{\pm\alpha}{c}, \quad (29)$$

$$S^{\theta\phi} = \frac{\pm\lambda}{r^2}, \quad (30)$$

$$S^{r\theta} = -\frac{C_4}{r}, \quad (31)$$

$$S^{t\theta} = -\frac{\pm c C_4}{gr}, \quad (32)$$

$$S^{t\phi} = -\frac{\pm cj}{gr}, \quad (33)$$

$$S^{r\phi} = -\frac{j}{r}, \quad (34)$$

$$\frac{\dot{r}}{\dot{t}} = \frac{\pm g}{c}, \quad (35)$$

$$P^t = \frac{2cE - \pm\alpha g'}{2cg}, \quad (36)$$

$$P^r = \pm\frac{2cE - \pm\alpha g'}{2c^2}. \quad (37)$$

Massless STOPs

Example:
Massless radial Schwarzschild
Obviously still radial like null-geodesics but ...

$$\Delta E = \frac{\alpha}{c}\frac{1}{2}g'|_{r^+} \tag{38}$$

Hawking relation!

Massless STOPs

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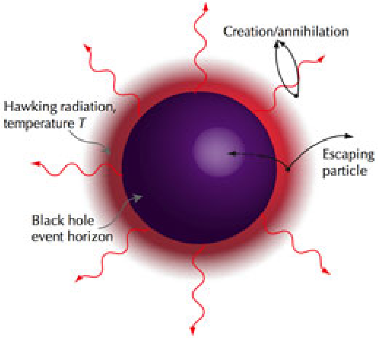

Without QFT in curved space-time

Conclusions

Non-geodesic motion of STOPs

- Window of visible effects (collisions)
- Window to QFT-QG link (massless)

Thank you

Thank you !