



Vacuum energy, Casimir effect, and Newton's non-constant

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Content

- What we would like to know
- Vacuum energy density in the lab
- Vacuum energy density in the Universe
- Vacuum energy density combined
- Scale-dependent framework
- Towards experiment
- Discussion and Conclusion

Vacuum energy

What we know so far?

rimordial / uctuations

smic microwave background





Quantum vacuum:

 ρ_Q

accelerates plates

accoloratos I Inizoro

Cosmological vacuum:

accelerates Universe

 ρ_{Λ}

Reionization

Reionized universe

*https://physicsworld.com/a/the-casimir-effect-a-force-from-nothing/

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Vacuum energy

How are they related?



In the lab

Casimir effect

Predicted 1948 ref [10] Observed 1997



$$\rho_{C} = -\frac{\hbar\pi^{2}}{720a^{4}}$$

$$\Rightarrow \quad \frac{F_{Q}}{A} \approx \rho_{Q} \cdot a$$



Albert Einstein

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

 \Rightarrow Friedman eq.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \equiv 0$$



S. Perlmutter, A. Riess, B. Schmidt, & others





 $\dot{a} \neq 0$

 $\ddot{a} > 0 \Rightarrow \Lambda > 0$

Λ as an energy density $\rho_{\Lambda,0} = \frac{\Lambda_0 c^4}{8\pi G_0} = 5.35 \times 10^{-10} J/m^3$

Quantum origin?

Yakov Zeldovich, 1967

Steven Weinberg, 1998



ref [4] Big theoretical puzzle

Big theoretical puzzle

In short QFT with cutoff $\rho_Q \sim c\kappa_0^4/\hbar^3$ As ratio

$$\Upsilon_0 \equiv \frac{\rho_{\Lambda_0}}{\rho_{Q,0}(\kappa)} = \frac{\Lambda_0 c^3 \hbar^3}{8\pi G_0 \kappa_0^4} = \begin{cases} 10^{-121} & \text{for } \kappa_0 = c \sqrt{\frac{c\hbar}{G_0}} \\ 10^{-55} & \text{for } \kappa_0 = cm_Z. \end{cases}$$



Big theoretical puzzleexperimental input neededuse Casimir experiment to gain insightChange ρ_Q See whether ρ_Λ



changes





Scale Dependent Framework

Gravity as effective QFT









variable couplings

Weinberg, Wetterich, Donoghue,

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scale-dependent (SD) couplings $\Lambda = \Lambda(k)$, G = G(k), two main consequences:

• Modified Einstein equations $G_{\mu\nu} = 8\pi G(k)T_{\mu\nu} - \Lambda(k)g_{\mu\nu} - \Delta t_{\mu\nu}$

Couplings are connected in RG flow



Couplings are connected in RG flow

We are only interested in SD small modifications of the couplings in the IR:

Expand: $G(k) = G_0(1 + g(k)) = G_0 \left(1 + C_1 G_0 k^2\right) + \mathcal{O}(k^4)$ $\Lambda(k) = \Lambda_0(1 + \lambda(k)) = \Lambda_0 \left(1 + C_3 G_0 k^2\right) + \mathcal{O}(k^4)$

Theorist:Phenomenologist:Experimentalist:predictuse to predictmeasure

Couplings are connected in RG flow

We are only interested in SD small modifications of the couplings in the IR: Expand: $G(k) = G_0(1 + g(k)) = G_0 (1 + C_1 G_0 k^2) + \mathcal{O}(k^4)$ $\Lambda(k) = \Lambda_0(1 + \lambda(k)) = \Lambda_0 (1 + C_3 G_0 k^2) + \mathcal{O}(k^4)$

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In density $\rho_{\Lambda}(k) = \frac{c^4 \Lambda(k)}{8\pi G(k)} = \rho_0 + k^2 c^4 \frac{(C_1 - C_3)}{8\pi} \Lambda_0 + \mathcal{O}(k^4)$



Derivatives of Modified Einstein equations $G_{\mu\nu} = 8\pi G(k)T_{\mu\nu} - \Lambda(k)g_{\mu\nu} - \Delta t_{\mu\nu}$

Does Casimir/SD affect Newton?

Trace trick

$$R^{\mu}_{\nu} = 8\pi \frac{G(k)}{c^4} \left(T^{\mu}_{\nu} - \frac{1}{2} g^{\mu}_{\nu} T \right) + \Lambda(k) g^{\mu}_{\nu} + G(k) \left(\frac{1}{2} g^{\mu}_{\nu} \nabla^2 + \nabla^{\mu} \nabla_{\nu} \right) \frac{1}{G(k)}$$

 $G(k(\overrightarrow{x}))$

With non-relativistic matter

$$\left(T^{\mu}_{\nu} - \frac{1}{2}g^{\mu}_{\nu}T\right) = \frac{\rho_{M}}{2}\operatorname{diag}(-1, 1, 1, 1)$$

• Modified Einstein equations $G_{\mu\nu} = 8\pi G(k)T_{\mu\nu} - \Lambda(k)g_{\mu\nu} - \Delta t_{\mu\nu}$ Derivatives of $G\left(k(\vec{x})\right)$

Weak field and weak SD expansion...

$$\begin{split} ds^2 &= -\left(1 + 2\epsilon_{\Phi}\Phi(r,\theta,\phi)\right)c^2dt^2 + \left(1 - 2\epsilon_{\Phi}\Psi(r,\theta,\phi)\right)dr^2 + \left(1 + 2\epsilon_{\Phi}\Xi(r,\theta,\phi)\right)r^2d\Omega^2 + \mathcal{O}(\epsilon_{\Phi}^2)\\ G(k) &= \epsilon_{\Phi}\left(G_0 + \epsilon_G\Delta G(k) + \mathcal{O}(\epsilon_{\Phi}^2)\right) \end{split}$$

 $\Lambda(k) \to \epsilon_{\Phi} \Lambda(k)$

$R_0^0 \text{ component}$ $\overrightarrow{\nabla}^2 \Phi(r,\theta,\phi) = \frac{4\pi}{c^4} G_0 \rho_M(r,\theta,\phi) + \frac{\epsilon_G}{\epsilon_\Phi} \frac{\overrightarrow{\nabla}^2 \Delta G(k)}{2G_0} - \Lambda(k) + \mathcal{O}(\epsilon_\Phi,\epsilon_G)$

Solution...

• Modified Einstein equations $G_{\mu\nu} = 8\pi G(k)T_{\mu\nu} - \Lambda(k)g_{\mu\nu} - \Delta t_{\mu\nu}$

Derivatives of $G(k(\vec{x}))$

 $\frac{\nabla^2 \rho_C(\vec{x})}{4(C - C)}$

Solution

$$\Phi(\overrightarrow{x}) = \frac{G_0}{c^4} \int_{V_1} d^3 x_1 \frac{\widetilde{\rho}_M(\overrightarrow{x}_1)}{|\overrightarrow{x} - \overrightarrow{x}_1|} + \mathcal{O}(\epsilon_{\Phi})$$



For extended objects...

 $\alpha c^2 C_1$

 $G(k(\vec{x}))$

 $\alpha c^2 C_1 \frac{\nabla^2 \rho_C(\vec{x})}{c^4 (C_1 - C_1) \Lambda}$

Derivatives of Modified Einstein equations $G_{\mu\nu} = 8\pi G(k)T_{\mu\nu} - \Lambda(k)g_{\mu\nu} - \Delta t_{\mu\nu}$

with

 $\tilde{\rho}_M = \rho_m +$

For extended objects: Newton

$$\overrightarrow{\mathcal{F}}_{G,12} = -\overrightarrow{\mathcal{F}}_{G,21} = G_0 \int_{V_2} d^3 x_2 \int_{V_1} d^3 x_1 \frac{\widetilde{\rho}_M(\overrightarrow{x}_1) \widetilde{\rho}_M(\overrightarrow{x}_2) (\overrightarrow{x}_2 - \overrightarrow{x}_1)}{|\overrightarrow{x}_2 - \overrightarrow{x}_1|^3}$$

 $^2 V^2 G(k)$

- Modified Einstein equations &
- Couplings are connected in RG flow

⇒ Gravitational attraction between plates changes

$$\alpha c^2 C_1 \frac{\overrightarrow{\nabla}^2 \rho_C(\overrightarrow{x})}{c^4 (C_1 - C_3) \Lambda_6} \longrightarrow \overrightarrow{\mathcal{F}}_{G,12} \neq \overrightarrow{F}_{G,12}$$

Hypothesis can be tested by experiment: Verify, or set bounds on α ...

Gravitational attraction between plates changes

What that means

Change ρ_Q

Hypothesis

 $\alpha c^2 C_1 \frac{\overrightarrow{\nabla}^2 \rho_C(\overrightarrow{x})}{c^4 (C_1 - C_2) \Lambda c} \rightarrow \overrightarrow{\mathcal{F}}_{G,12} \neq \overrightarrow{F}_{G,12}$

ρ_{Λ} not measurable



SD & flow

 $\rho_M \to \tilde{\rho}_M$ measurable

Towards experiment

Cannex planned experiment



Towards experiment

Results (preliminary estimate):

$$1 \ll (8\pi G_0^2) \frac{\int_{a/2}^D dz \,\rho_M(z)}{\int_{a/2}^D dz c^2 \,\overrightarrow{\nabla}^2 G}$$

 $\alpha \frac{C_1}{C_1 - C_3} \ll 10^{-32}$

 $\alpha < small$



heading towards ultra small

²⁸ potentially excludes models

Interpretation

$$\alpha \frac{C_1}{C_1 - C_3} \ll 10^{-32}$$

A. ρ_0 contribution to ρ_Λ strongly suppressed ($\alpha \ll 1$) B. $\Lambda(k)$ has very weak RG coupling to G(k)C. Effective Einstein equations have additional fields, contributions, stuff, leading to cancellations... For each interpretation many possible subcategories, e.g.

- B. 1. Λ is not a coupling but a field 2. *G* is not a coupling but a field

 - 3. RG group is not universal
 - 4. Hierarchy in QG parameters: $C_3 \gg C_1$
 - 5. ...

Interpretation

A: ($\alpha \ll 1$) Implications for the CCP

 $\Upsilon_0 \equiv \frac{\rho_{\Lambda_0}}{\rho_{Q,0}(\kappa)} = \frac{\Lambda_0 c^3 \hbar^3}{8\pi G_0 \kappa_0^4} = \begin{cases} 10^{-121} & \text{for } \kappa_0 = c \sqrt{\frac{c\hbar}{G_0}} \\ 10^{-55} & \text{for } \kappa_0 = cm_Z. \end{cases}$ Problem comes from the ambition $\rho_{\Lambda} = \Upsilon(\rho_Q) \cdot \rho_Q,$ Casimir can contribute to both $\rho_O = \rho_{O,0} + \beta \cdot \rho_C$ $\rho_{\Lambda} = \rho_{\Lambda_0} - \alpha \cdot \rho_C$ hypothesis, Should be $\beta = 1$, but who knows ... 30 α

Interpretation

A: $(\alpha \ll 1)$ Implications for the CCP Look at changes of the CCP $\Upsilon'_0 \equiv \frac{d\Upsilon(\rho_Q)}{d\rho_C}\Big|_{\rho_C=0}$

 $\alpha = \Upsilon'_0 + \beta \Upsilon_0 \qquad \Rightarrow$ Measure changes in CCP

Under construction



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Thank You!



Backup stuff

Scale-Dependent & FRG

Scale Dependent Framework

Gravity as effective QFT

$$\Gamma_{k} = \int d^{4}x \sqrt{-g} \left(\frac{R}{G_{k}} - 2\frac{\Lambda_{k}}{G_{k}} \right) + \dots$$

Non renormalizable? Yes, but ... Could still be predictive QFT (Asymptotic Safety)



Asymptotic Safety in a nutshell $\Gamma_{k} = \int d^{4}x \sqrt{-g} \left(\frac{R}{G_{k}} - 2 \frac{\Lambda_{k}}{G_{k}} \right) + \dots$

• Idea: works if non trivial UV-fixed points for finite number of couplings (S.W)



Asymptotic Safety in a nutshell $\Gamma_{k} = \int d^{4}x \sqrt{-g} \left(\frac{R}{G_{k}} - 2 \frac{\Lambda_{k}}{G_{k}} \right) + \dots$

Tool: Functional renormalization group equation

$$\partial_k \Gamma_k = \frac{1}{2} Tr \left(\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right)$$

two point regulator function



C. Wetterich

Asymptotic Safety in a nutshell $\Gamma_{k} = \int d^{4}x \sqrt{-g} \left(\frac{R}{G_{k}} - 2 \frac{\Lambda_{k}}{G_{k}} \right) + \dots$

Results: Plenty of evidence supporting idea



Albert Einstein

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



NP 1921





Albert Einstein

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \neq 0$$
$$\Rightarrow \dot{a} \neq 0 \text{ not static}$$



Albert Einstein

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \equiv 0$$
static possible $\dot{a} \equiv 0$



The CCP 2.0

The CCP 2.0

Edwin Hubble, Georges Lemaítre

measurement:





The CCP 2.0

Edwin Hubble measurement: not static $\dot{a} > 0$ later: *Mpc* (Planck collaboration 2018)

The CCP 2.0, Albert Einstein $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda_{\mu\nu} = 8\pi G T_{\mu\nu}$ K. $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + -\frac{\Lambda}{3}$ a not static $\dot{a} \neq 0$ $\ddot{a} < 0$ "biggest blunder"

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S. Perlmutter, A. Riess, B. Schmidt, & others





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Yakov Zeldovich

Quantum fluctuations predict value of Λ

1967



Steven Weinberg

Quantum fluctuations predict value of Λ ref [3] Problem since 1998







Quantum fluctuations predict value of Λ

Highest physical mass scale

 $(\Lambda)_{eff} \sim \frac{M^4}{M_P^2} (1 + \dots)$

Observed value

 $\Lambda_o = \frac{\rho_c}{M_P^2} \approx \frac{10^{-47} GeV^4}{M_P^2}$

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observed critical energy density



Problem as a ratio:

$$\frac{(\Lambda)_{eff}}{\Lambda_0} \sim \frac{1}{G_N \cdot \Lambda_0} \sim \frac{M_P^4}{\rho_c} \approx 10^{120}$$

we try to address this problem

assuming there are quantum fluctuations of gravity associated to the Planck scale

