Cosmological Constant Problem: Deflation During Inflation

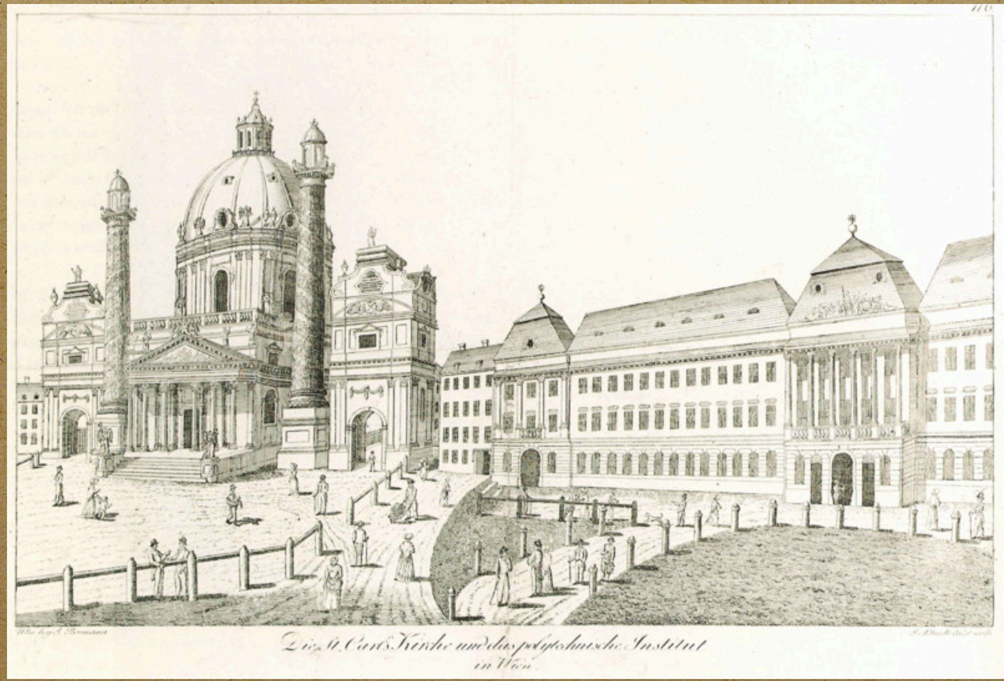
B. Koch

with

F. Canales, C. Laporte, & A. Rincon,

based on:

ArXiv:1812.10526 .



TU Wien

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acknowledge:

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Collaboration

- Angel Rincon, Cristobal Laporte, & Felipe Canales

Content

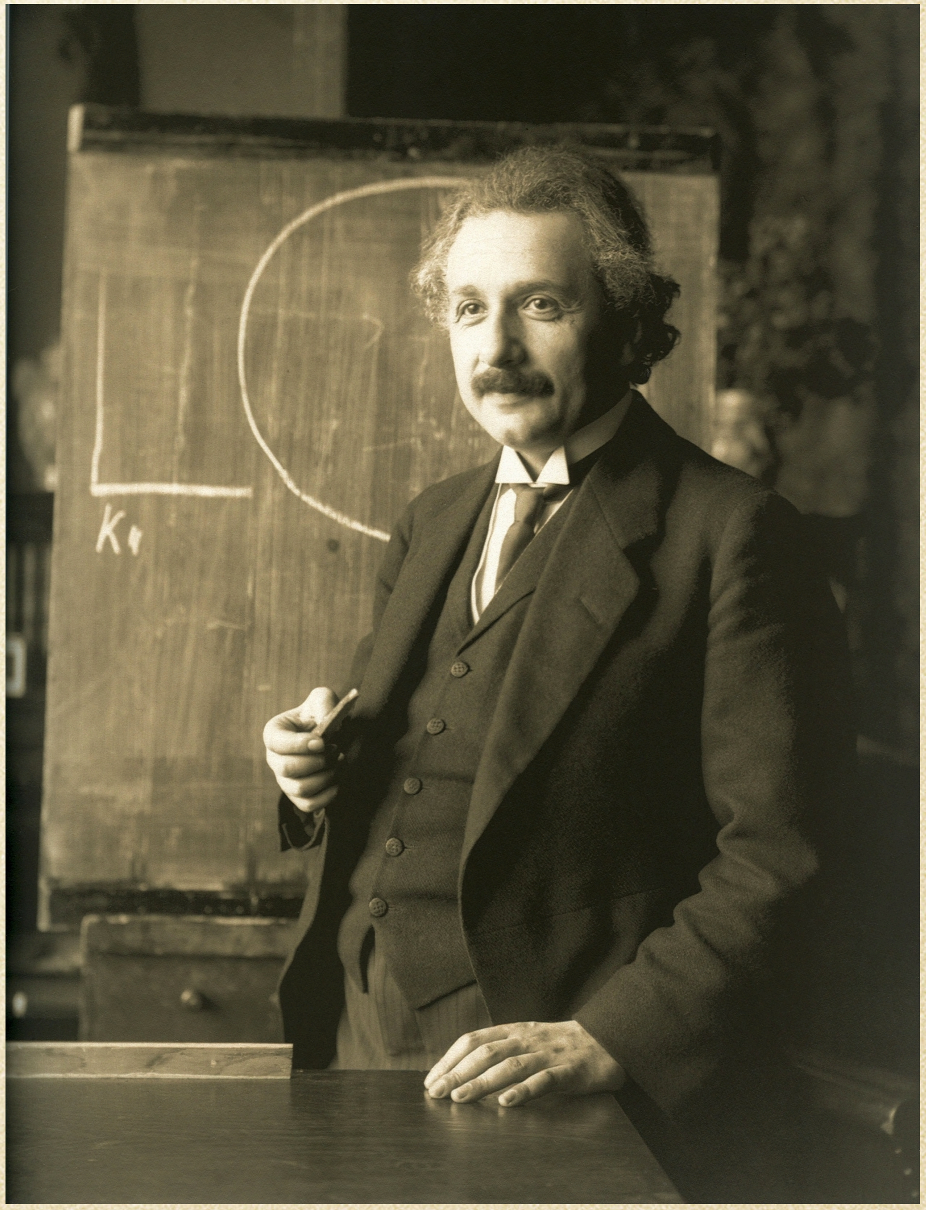
- Cosmological constant problem, status
- Conceptual problem, in evolving Universe
- Scale dependent framework & evolving Universe
- Possible solution: Deflation during inflation
- Link to Asymptotic Safety
- Conclusion

The CCP 1.0

The CCP 1.0

Albert Einstein

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$



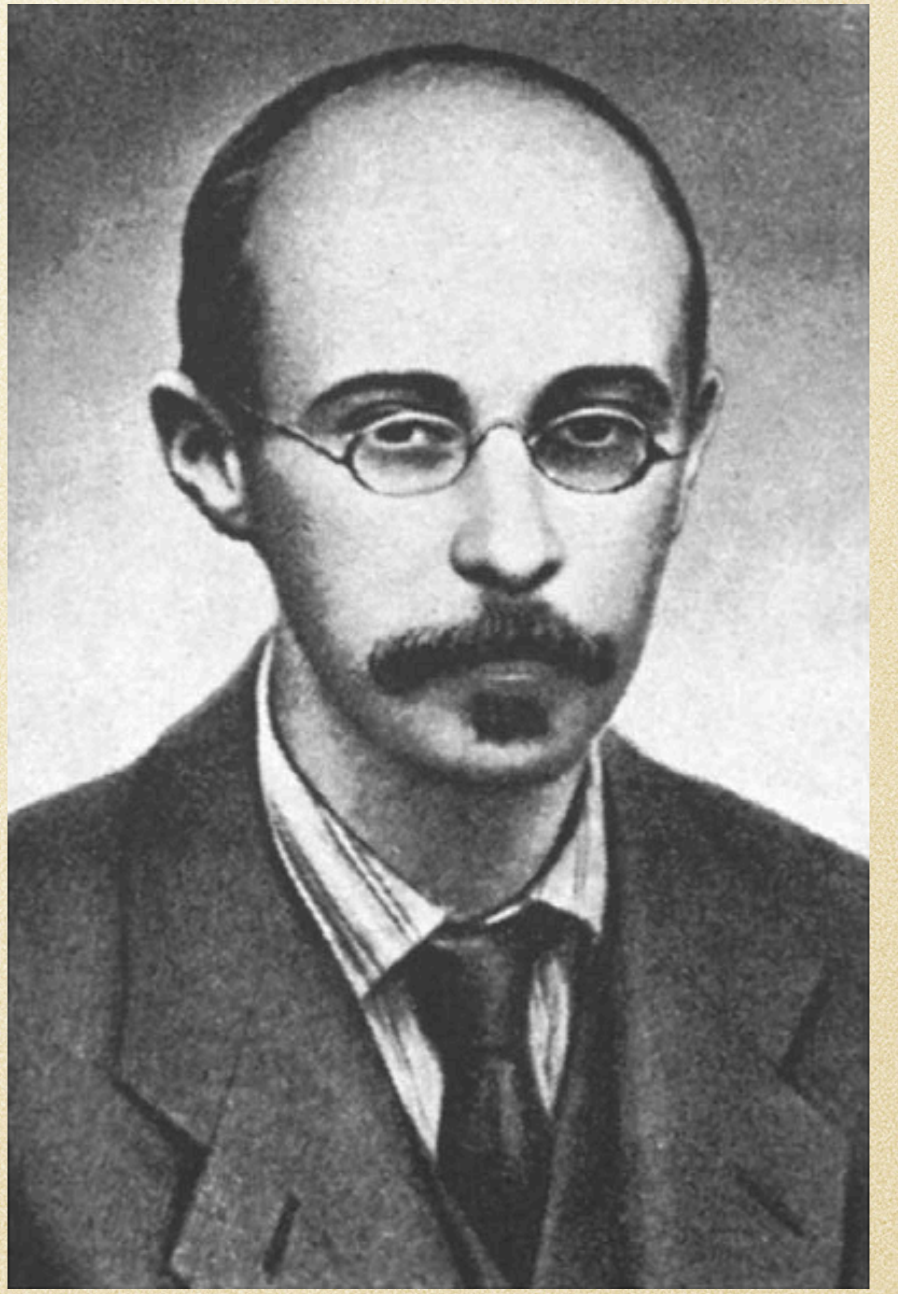
The CCP 1.0

Alexander Friedmann

$$ds^2 = a(t)ds_3^2 - dt^2$$

$$\frac{\dot{a}^2 + k}{a^3} = \frac{1}{3}8\pi G\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$



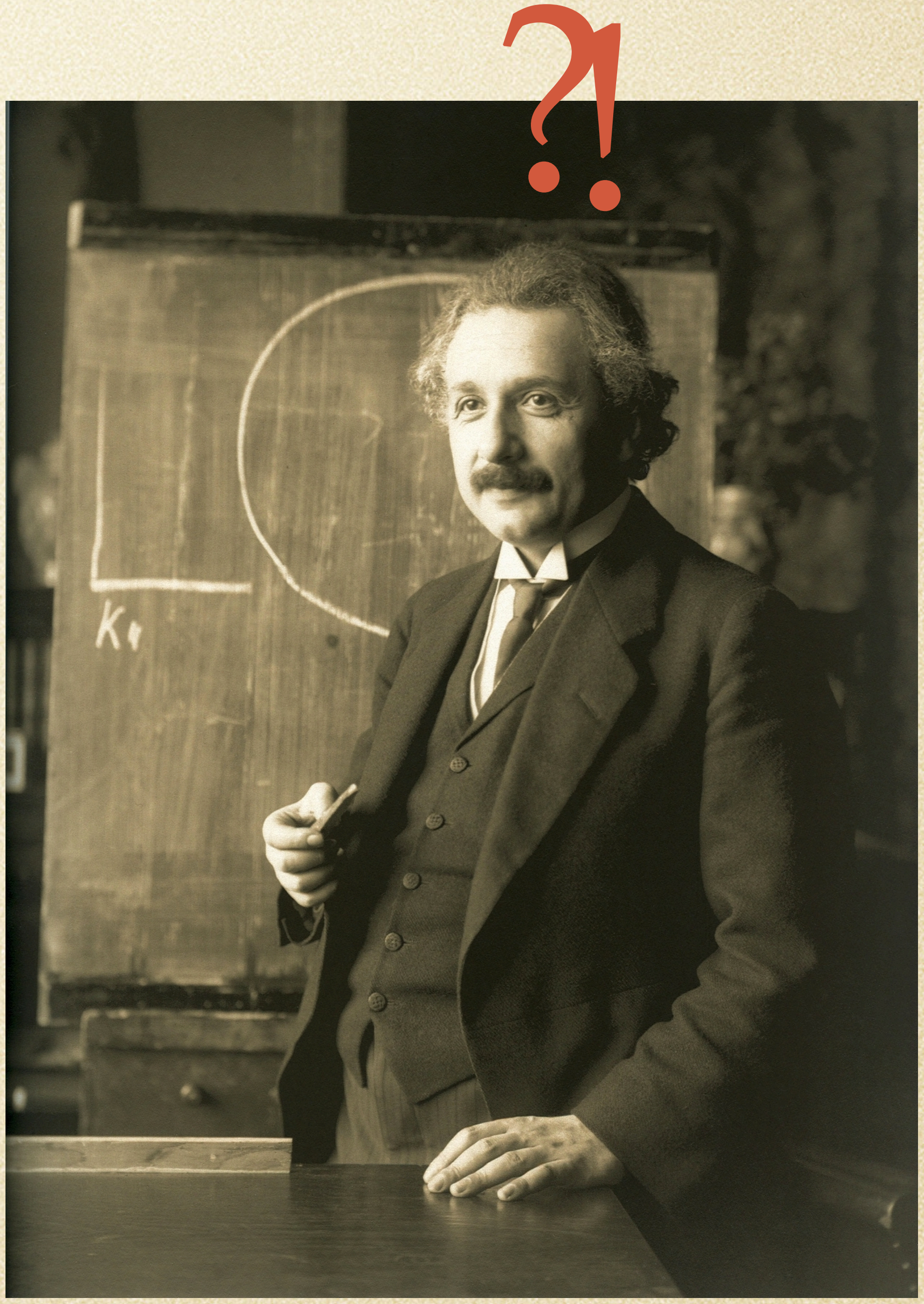
The CCP 1.0

Albert Einstein

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \neq 0$$

$$\Rightarrow \dot{a} \neq 0$$ not static



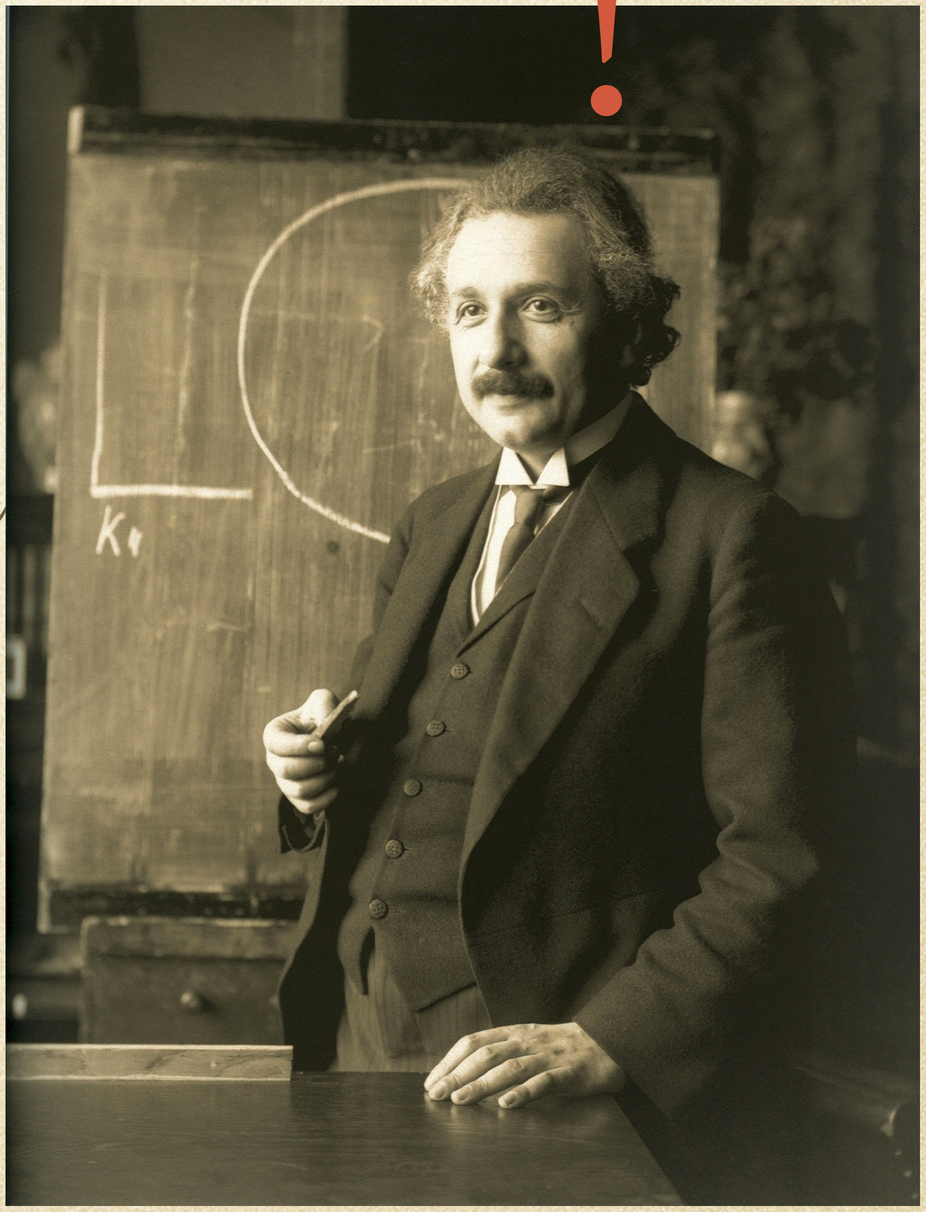
The CCP 1.0

Albert Einstein

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \equiv 0$$

static possible $\dot{a} \equiv 0$



The CCP 2.0

The CCP 2.0

Edwin Hubble, Georges Lemaítre measurement:

ref [1]

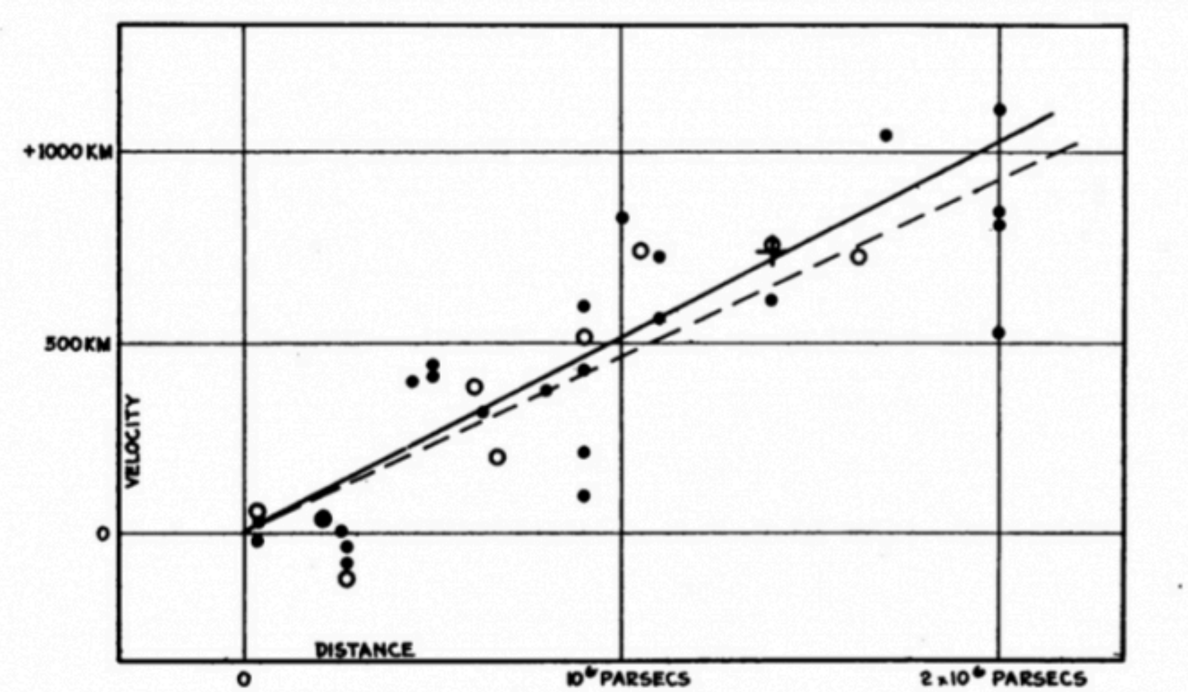


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

1927,1929

The CCP 2.0

Edwin Hubble

measurement:

$$\dot{a} > 0$$ not static

later:

$$\dot{a} = 67.66 \pm 0.42 \frac{km/s}{Mpc}$$

(Planck collaboration 2018)

The CCP 2.0

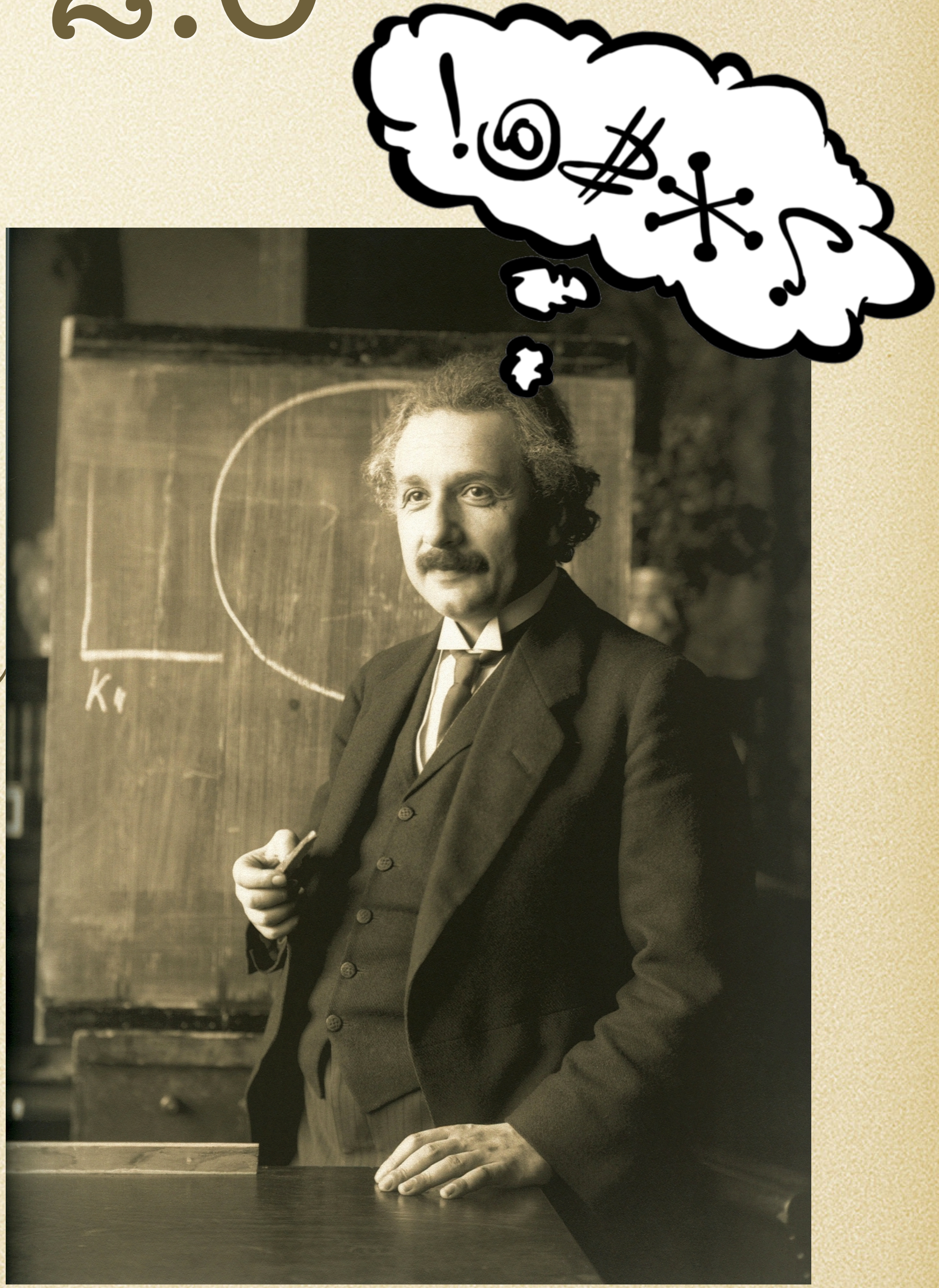
Albert Einstein

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \equiv 0$$

not static $\quad \dot{a} \neq 0$

$\ddot{a} < 0$



"biggest blunder"

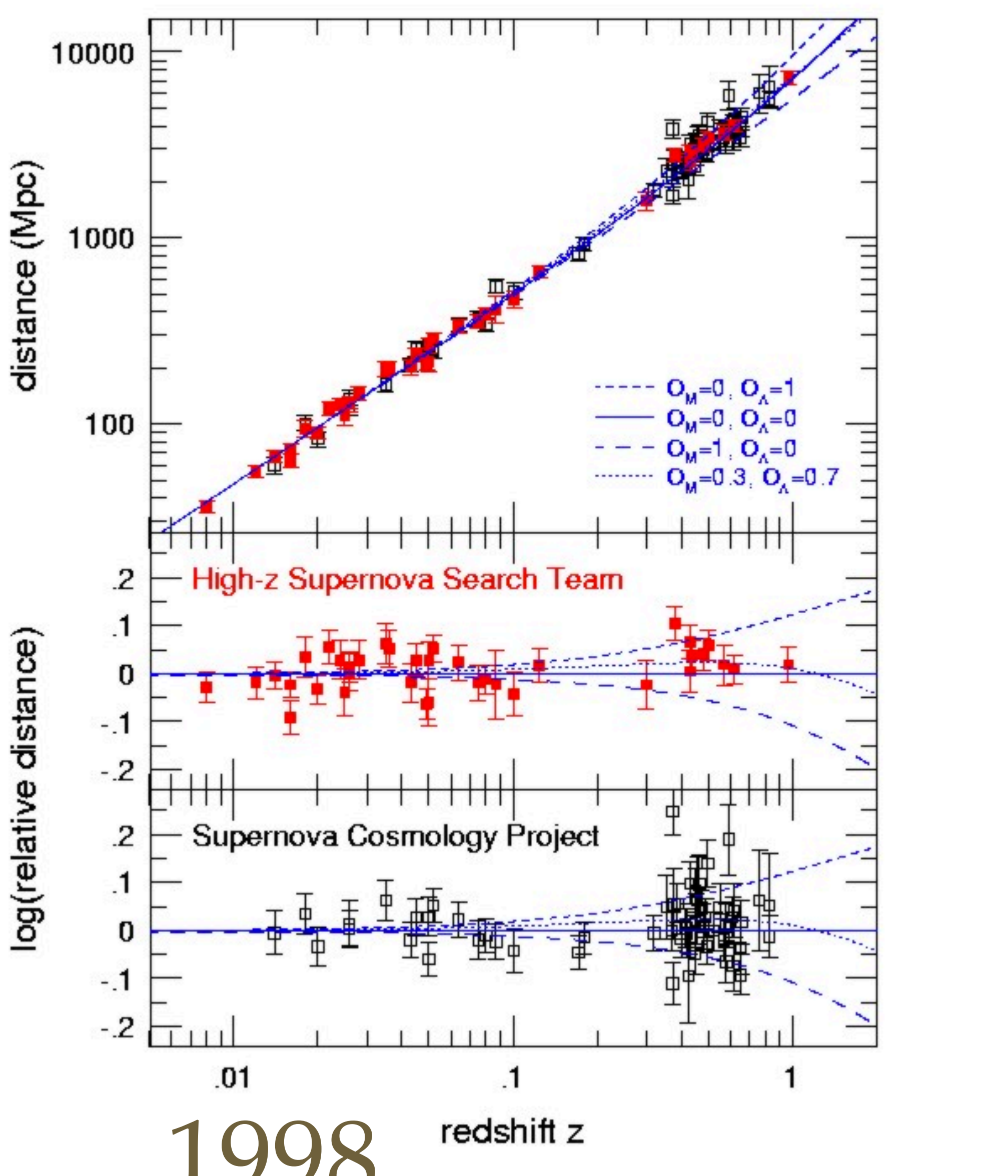
The CCP 3.0

The CCP 3.0

S. Perlmutter, A. Riess, B. Schmidt

mea

ref [2]



1998

$\dot{a} \neq 0$

$\dot{a}$ 0

$\ddot{a} > 0$!

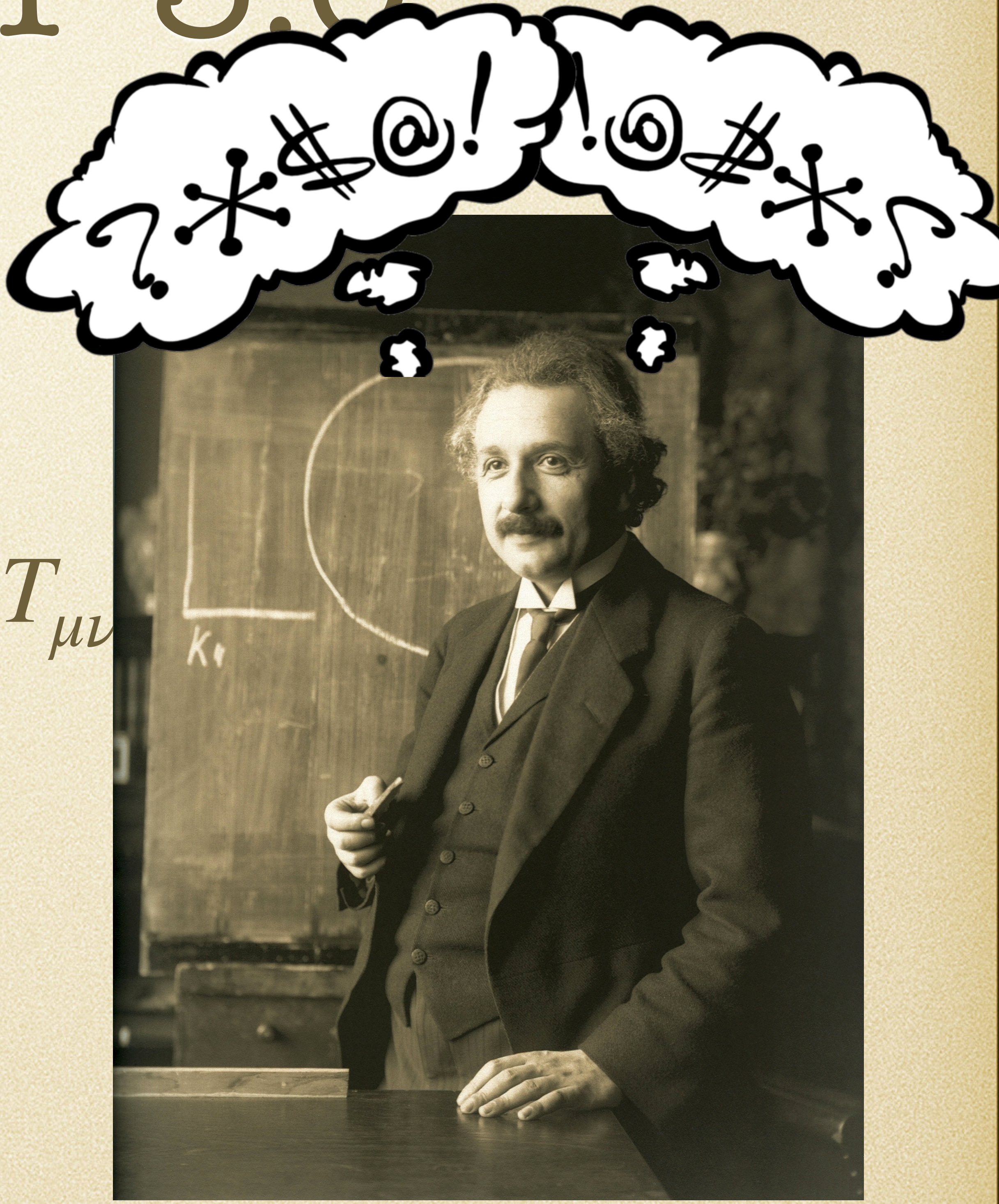
The CCP 3.0

Albert Einstein

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

$$\Lambda > 0 \Rightarrow \ddot{a} > 0$$



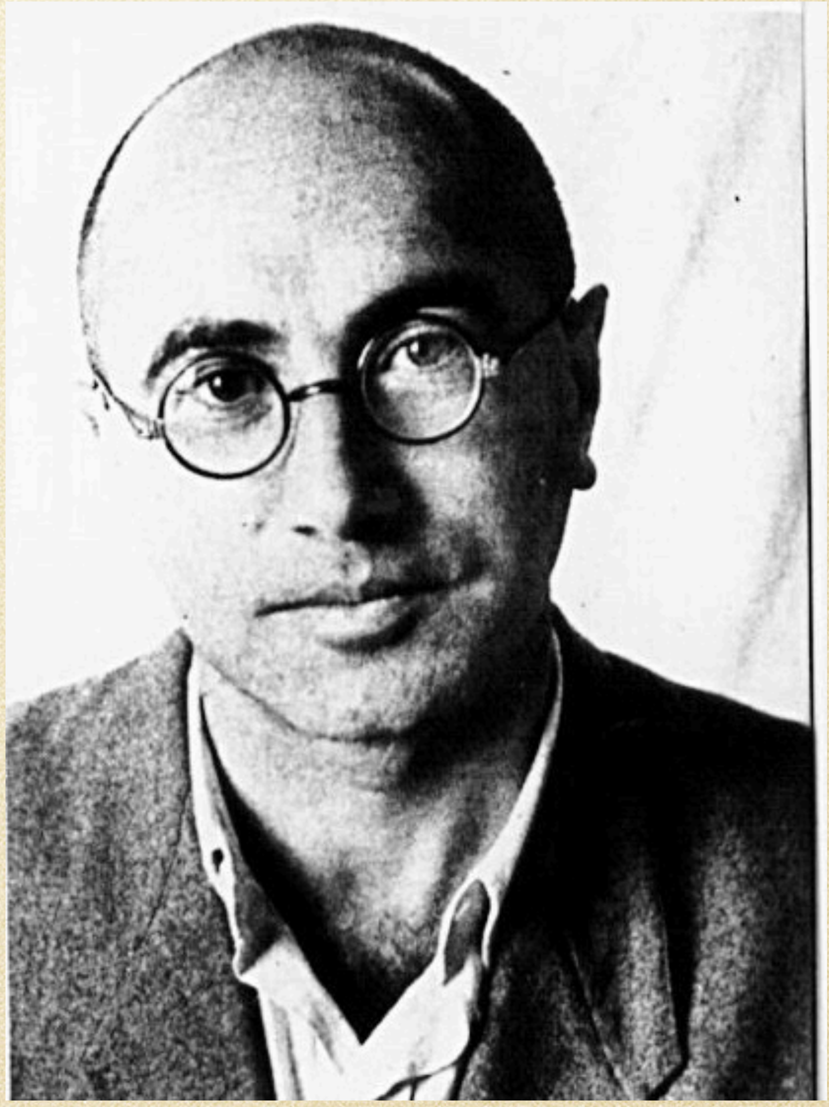
"???"

The CCP 3.0

Yakov Zeldovich

Quantum fluctuations predict value of Λ

1967



The CCP 3.0

Steven Weinberg

Quantum fluctuations predict value of Λ

Problem since 1998 ref [3]

The CCP 3.0

Quantum fluctuations
predict value of Λ

$$\left(\frac{\Lambda}{G}\right)_{eff} \sim M^4(1 + \ldots)$$

ref [3]

effective quantum value

different methods

cut-off mass scale of QFT

use: $G \sim \dfrac{1}{M_P^2}$

The CCP 3.0

Quantum fluctuations
predict value of Λ

$$(\Lambda)_{eff} \sim \frac{M^4}{M_P^2}(1 + \ldots)$$

effective
quantum value

different
methods

cut-off
mass scale of QFT

Planck mass

The CCP 3.0

Quantum fluctuations predict value of Λ

Highest physical mass scale

$$(\Lambda)_{eff} \sim \frac{M^4}{M_P^2}(1 + \ldots)$$

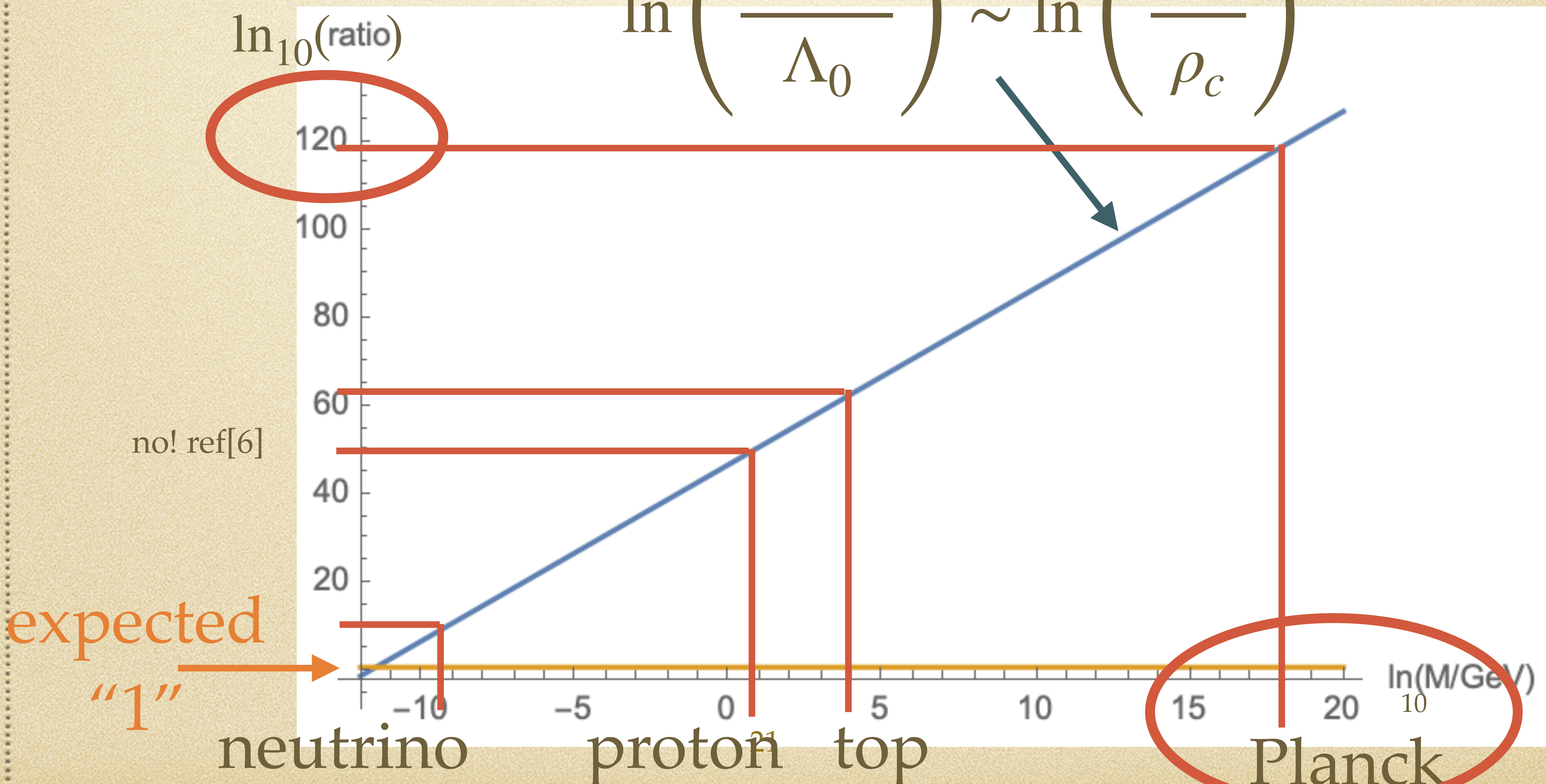
Observed value

$$\Lambda_o = \frac{\rho_c}{M_P^2} \approx \frac{10^{-47} GeV^4}{M_P^2}$$

observed critical energy density

The CCP 3.0

Problem as a ratio: $\ln\left(\frac{(\Lambda)_{eff}}{\Lambda_0}\right) \sim \ln\left(\frac{M^4}{\rho_c}\right)$



The CCP 3.0

Problem as a ratio:

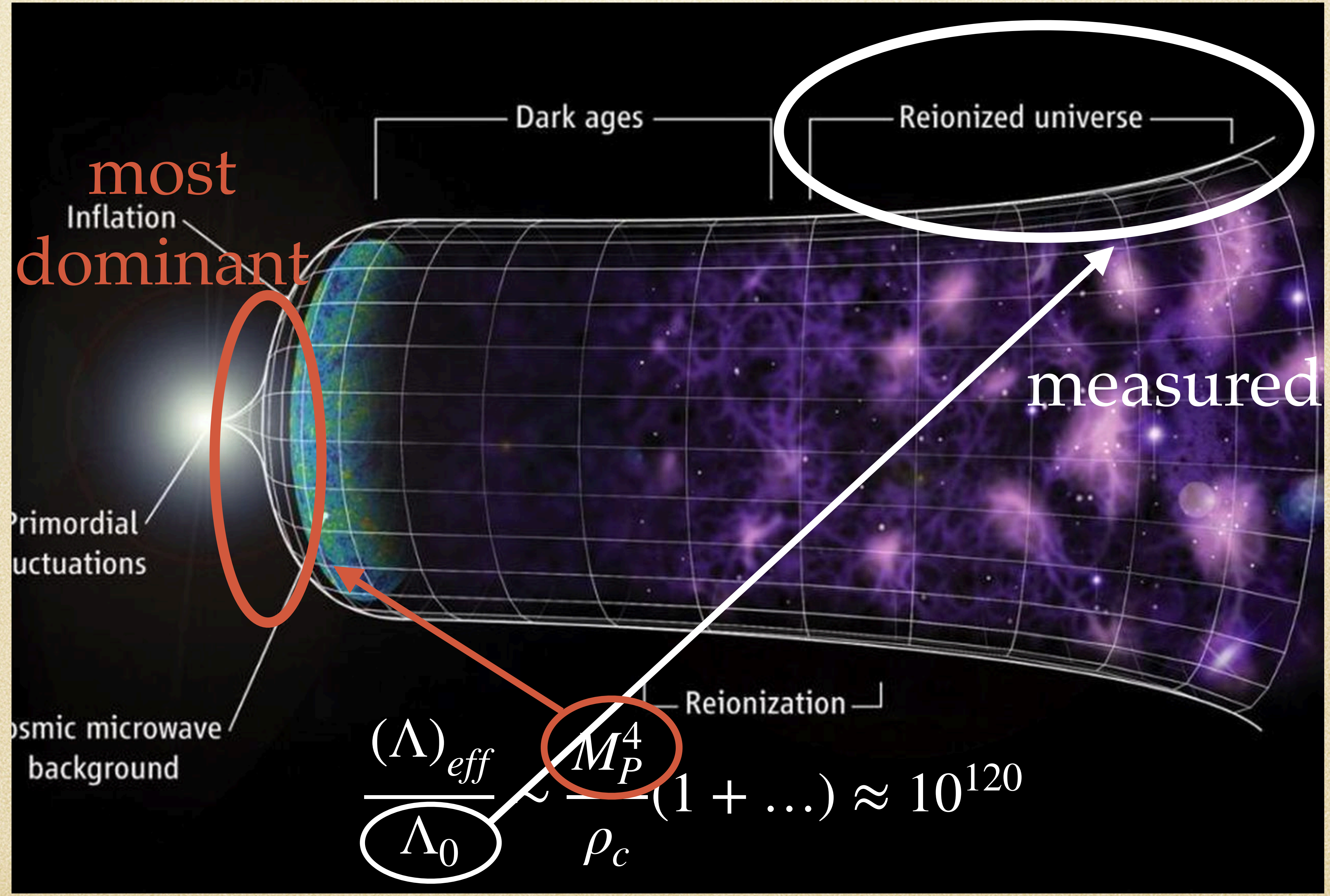
$$\frac{(\Lambda)_{eff}}{\Lambda_0} \sim \frac{1}{G_N \cdot \Lambda_0} \sim \frac{M_P^4}{\rho_c} \approx 10^{120}$$

we try to address
this problem

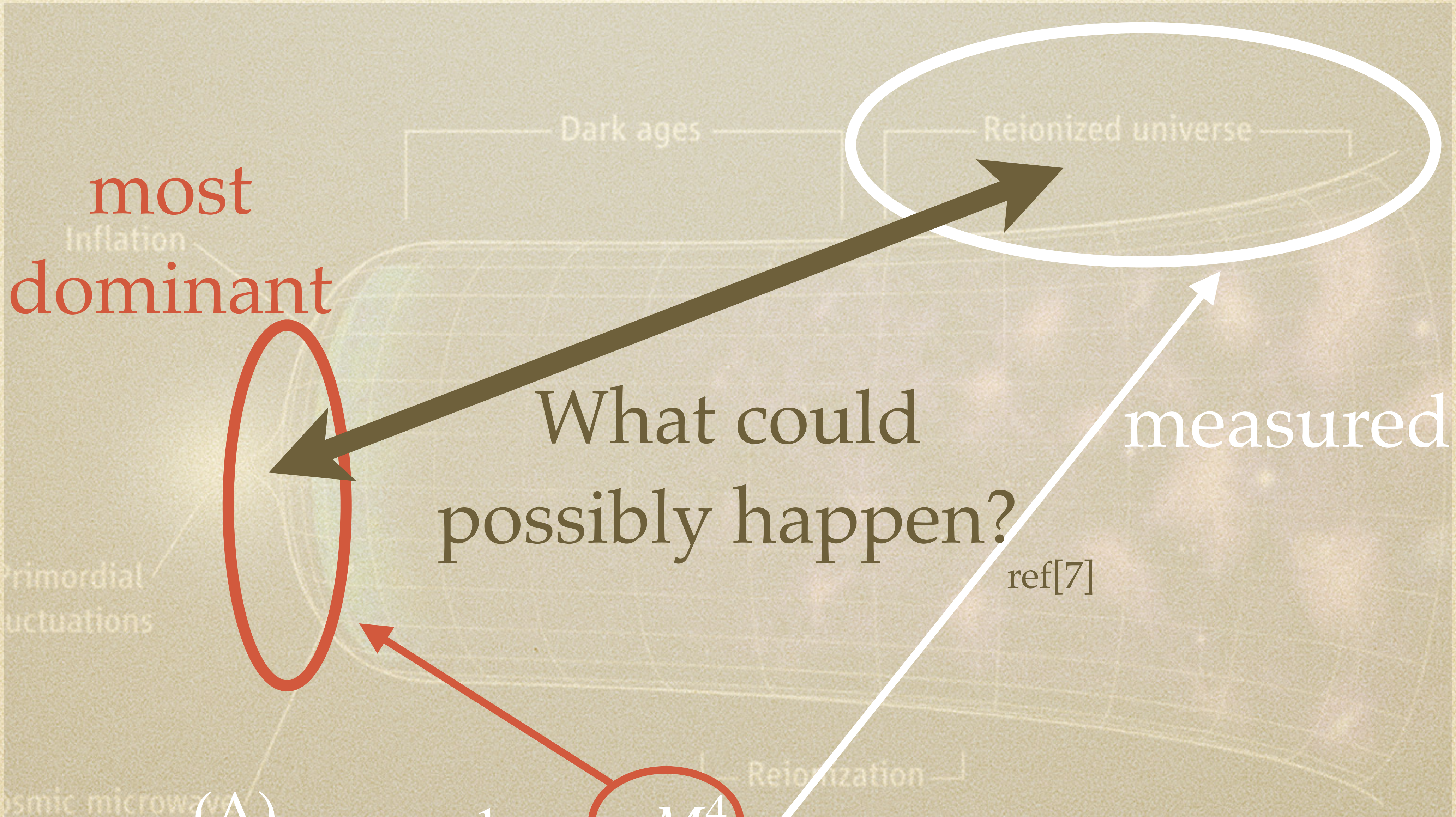
assuming there are quantum fluctuations
of gravity associated to the Planck scale

Evolving Universe Issue

Evolving Universe Issue



Evolving Universe Issue



$$\frac{(\Lambda)_{eff}}{\Lambda_0} \sim \frac{1}{G_N\Lambda_0} \sim \frac{M_P^4}{\rho_c}(1+\ldots) \approx 10^{120}$$

Scale Dependent Framework

Gravity as classical theory

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{G_N} - 2\frac{\Lambda_0}{G_N} \right)$$

Gravity as effective QFT

$$\Gamma_k = \int d^4x \sqrt{-g} \left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k} \right) + \dots$$

Scale Dependent Framework

Gravity as effective QFT

$$\Gamma_k = \int d^4x \sqrt{-g} \left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k} \right) + \ldots$$

Non renormalizable?

Yes, but …

Could still be predictive QFT
(Asymptotic Safety)

Asymptotic Safety in a nutshell

$$\Gamma_k = \int d^4x \sqrt{-g} \left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k} \right) + \ldots$$

- Idea: works if non trivial UV-fixed points for finite number of couplings (S.W)

Asymptotic Safety in a nutshell

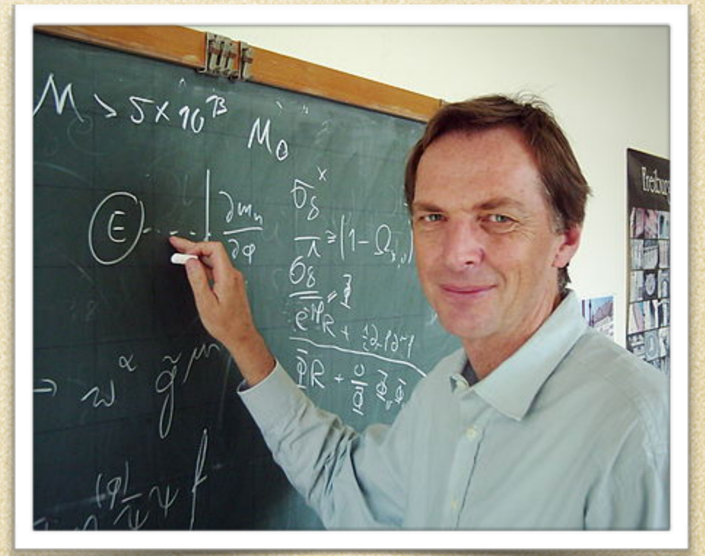
$$\Gamma_k = \int d^4x \sqrt{-g} \left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k} \right) + \ldots$$

- Tool: Functional renormalization group equation

$$\partial_k \Gamma_k = \frac{1}{2} Tr \left(\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right)$$

two point function

regulator



C. Wetterich

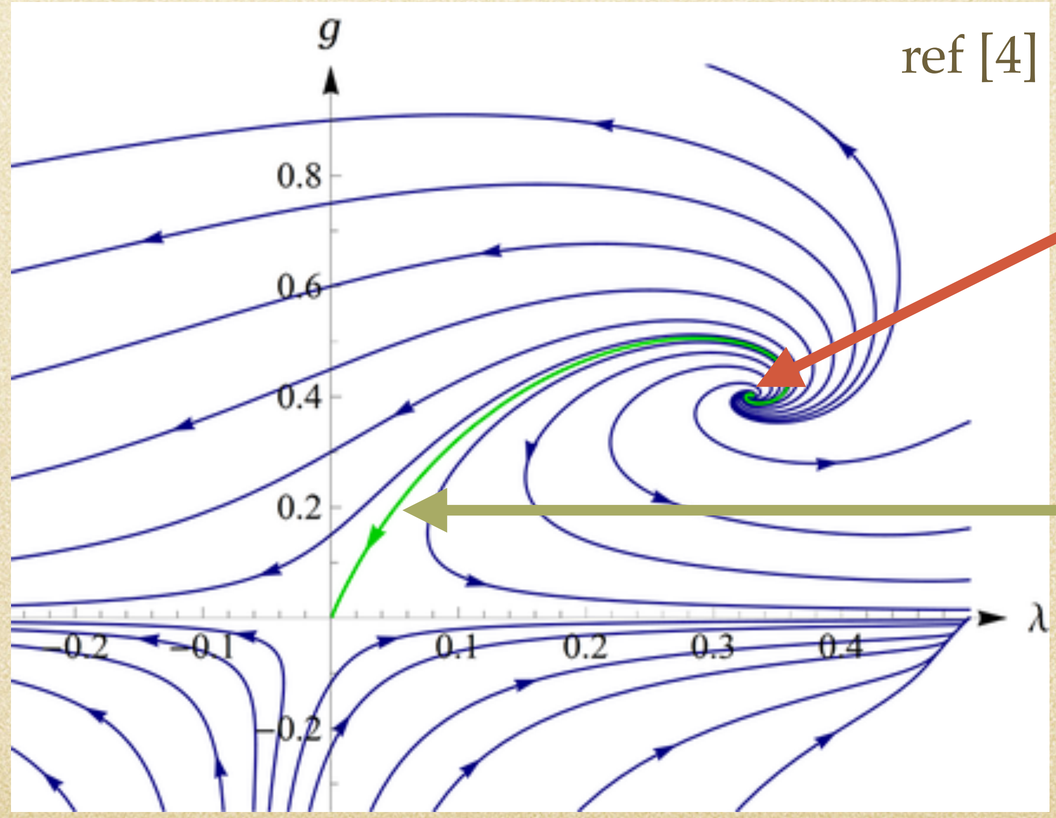
Asymptotic Safety in a nutshell

$$\Gamma_{k} = \int d^4x \sqrt{-g} \left(\frac{R}{G_{k}} - 2\frac{\Lambda_{k}}{G_{k}} \right) + \ldots$$

- Results: Plenty of evidence supporting idea

$$G_k = \frac{g_k}{k^2}$$

$$\Lambda_k = \lambda_k k^2$$



UV FP

separatrix

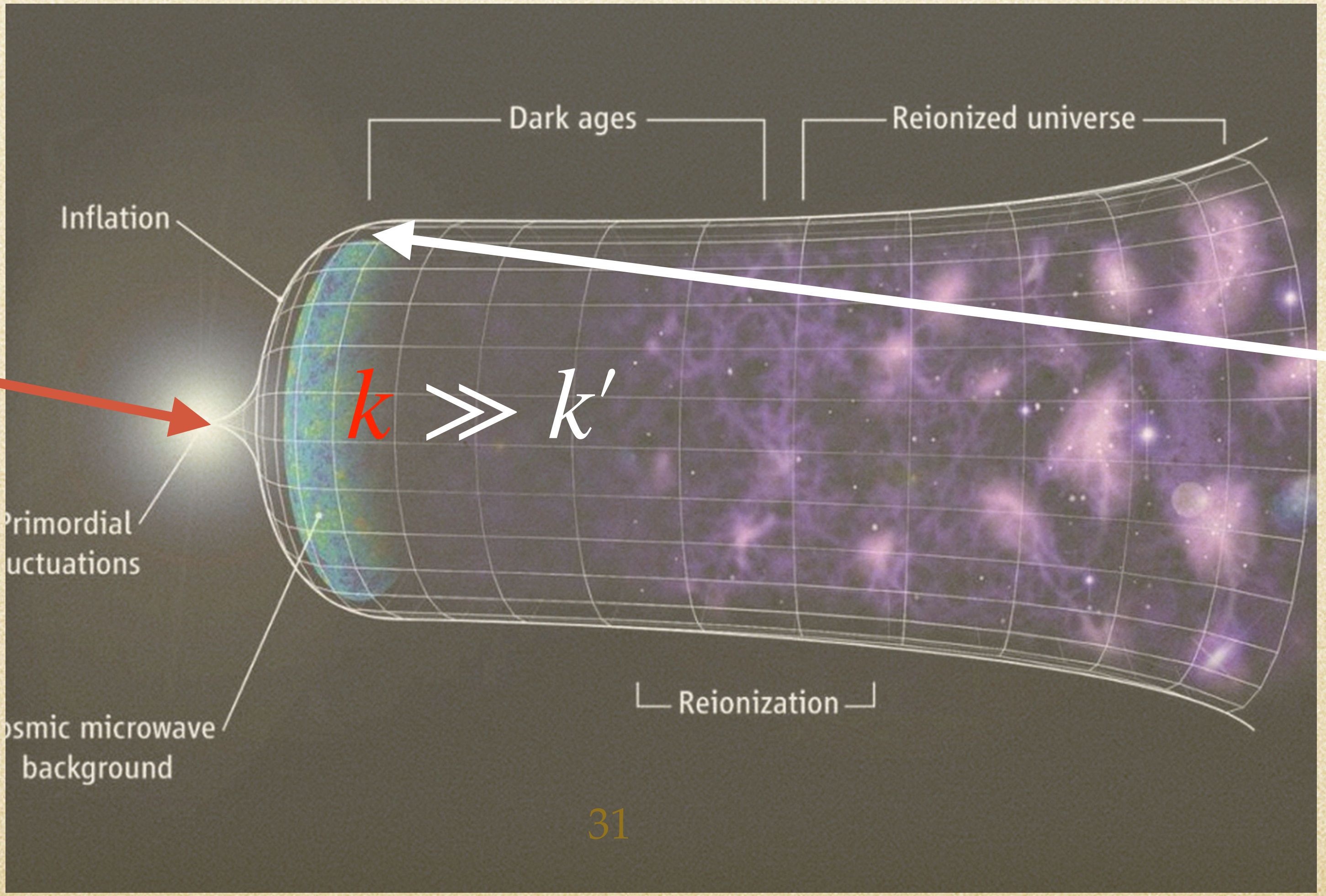
Scale Dependent Framework

OK, assume $$\Gamma_k = \int d^4x \sqrt{-g} \left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k} \right)$$

Implications for CCP 3.0?

Maybe:

$$\frac{1}{G_k \cdot \Lambda_k} \approx 1$$



$k \gg k'$

$$\frac{1}{G_{k'} \cdot \Lambda_{k'}} \neq 1$$

lets see:

Deflation During Inflation

Deflation During Inflation

$$\Gamma_k = \int d^4x\sqrt{-g}\left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k}\right)$$

Need to solve gap equations

$$G_{\mu\nu} = -\Lambda_k g_{\mu\nu} - \Delta t_{\mu\nu}$$

not constant!

with

$$\Delta t_{\mu\nu} = G_k\left(g_{\mu\nu}\nabla^\alpha\nabla_\alpha - \nabla_\mu\nabla_\nu\right)\frac{1}{G_k}$$

Deflation During Inflation

Assume homogenous background

$$ds^2 = -dt^2 + a(t)\left(\frac{1}{1-\kappa r^2}dr^2 + r^2 d\Omega_2^2\right)$$

spatial curvature

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda_k}{3} = \frac{1}{3}\rho_{SD}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda_k = -p_{SD}$$

scale dependence

Deflation During Inflation

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda_k}{3} = \frac{1}{3}\rho_{SD}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda_k = -\,p_{SD}$$

scale dependence

Since k=k(t) $\Rightarrow \quad G_k = G(t) \quad \& \quad \Lambda_k = \Lambda(t)$

$$\frac{1}{3}\rho_{SD} = \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right) \quad \& \quad -p_{SD} = -2\left(\frac{\dot{G}}{G}\right)^2 + \frac{\ddot{G}}{G} + 2\left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

Deflation During Inflation

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda(t)}{3} = \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda(t) = -2\left(\frac{\dot{G}}{G}\right)^2 + \frac{\ddot{G}}{G} + 2\left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

Problem: 2 equations

3 unknown functions: a(t), G(t), $\Lambda(t)$

Solution: Impose energy condition!

Deflation During Inflation

Null Energy Condition (NEC):

$$\Delta t_{\mu\nu}\ell^{\mu}\ell^{\nu} = 0$$

where

$$\frac{d\ell^{\mu}}{dt} + \Gamma^{\mu}_{\alpha\beta}\ell^{\alpha}\ell^{\beta} = 0 \;\longrightarrow\; \ell^{\mu} = c_0\frac{1}{a}\left(1, \frac{1}{\sqrt{1-\kappa r^2}}\frac{1}{a}, 0, 0\right)$$

thus

$$-2\left(\frac{\dot{G}}{G}\right)^2 + \left(\frac{\ddot{G}}{G}\right) - \left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right) = 0$$

Deflation During Inflation

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda(t)}{3} = \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right) \qquad \text{2 gap}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda(t) = -2\left(\frac{\dot{G}}{G}\right)^2 + \frac{\ddot{G}}{G} + 2\left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

$$+\left(\frac{\ddot{G}}{G}\right) - \left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right) = 2\left(\frac{\dot{G}}{G}\right)^2 \qquad \text{1 NEC}$$

3 unknowns, 3 equations $\Rightarrow$ solve!

Deflation During Inflation

Solution:

$$a(t) = a_i e^{\frac{t}{\sqrt{\Lambda_0/3}}}$$ still inflation

$$G(t) = \frac{G_0}{1 + \xi a(t)}$$

$$\Lambda(t) = \Lambda_0 \left[\frac{1 + 2\xi a(t)}{1 + \xi a(t)} \right]$$

3 integration constants: G_0, Λ_0, ξ

controls SD

Deflation During Inflation

Solution:

$$\lim_{\xi\to 0} a(t) = a_i e^{\frac{t}{\sqrt{\Lambda_0/3}}}$$

$$\lim_{\xi\to 0} G(t) = G_0$$

$$\lim_{\xi\to 0} \Lambda(t) = \Lambda_0$$

controls SD

3 integration constants: G_0, Λ_0, ξ

Deflation During Inflation

Solution:

$$a(t) = a_i e^{\frac{t}{\sqrt{\Lambda_0/3}}}$$

$$G(t) = \frac{G_0}{1 + \xi a(t)}$$

$$\Lambda(t) = \Lambda_0 \left[\frac{1 + 2\xi a(t)}{1 + \xi a(t)} \right]$$

3 integration constants: G_0, Λ_0, ξ

What does this mean for the CCP?

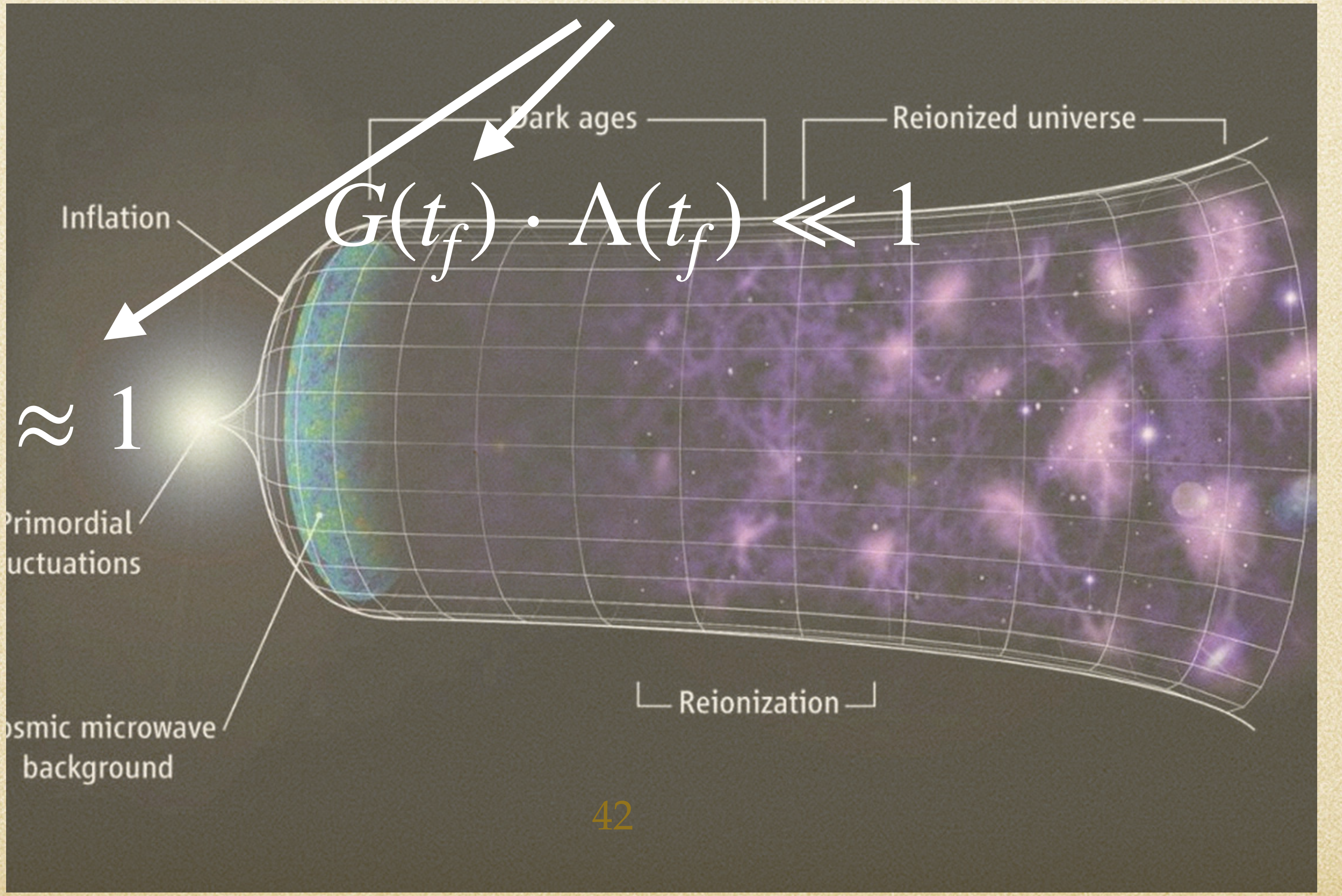
Deflation During Inflation

What does this mean for the CCP?

$$G_k \cdot \Lambda_k = G(t) \cdot \Lambda(t)$$

$$G(t_f) \cdot \Lambda(t_f) \ll 1$$

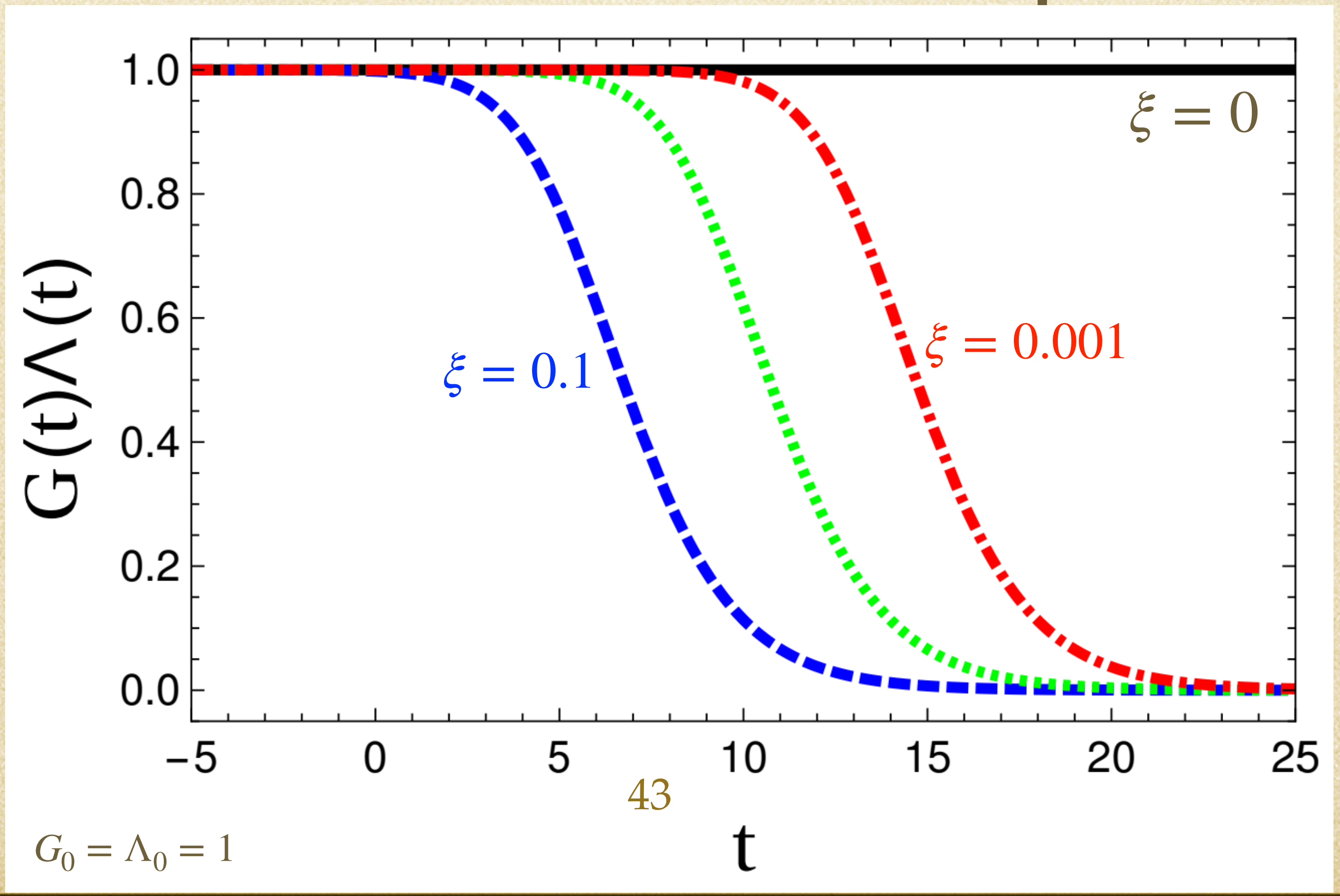
$$G(t_i) \cdot \Lambda(t_i) \approx 1$$



Deflation During Inflation

What does this mean for the CCP?

$$G(t) \cdot \Lambda(t) = \frac{G_0}{1 + \xi a(t)} \cdot \Lambda_0 \left[\frac{1 + 2\xi a(t)}{1 + \xi a(t)} \right]$$



Looks good, conditions?

Deflation During Inflation

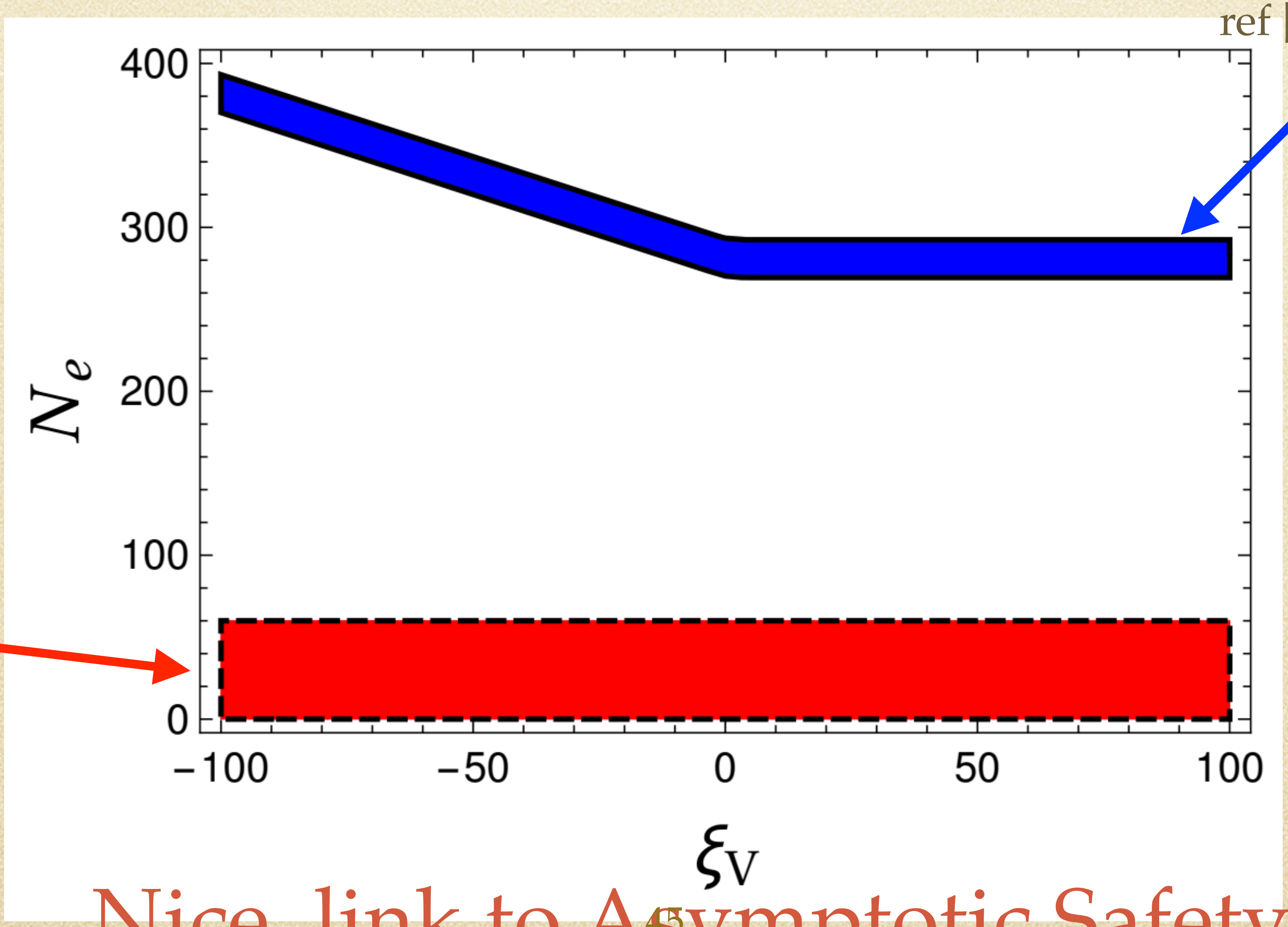
CCP conditions on parameters

- Initial a $a(t_i) = 1$
- Initial CCP $\Lambda(t_i) \cdot G(t_i) = 1$
- Final G $G(t_f) = G_N$
- Final CCP $G(t_f) \cdot \Lambda(t_f) = 10^{-(120\pm 5)}$
- Flatness $N_e \geq 60; \quad t_f - t_i = N_e\sqrt{\Lambda_0/3}$

Deflation During Inflation

CCP conditions on parameters

ref [0]



solves CCP

flatness excluded

$\xi = e^{\xi_V}$

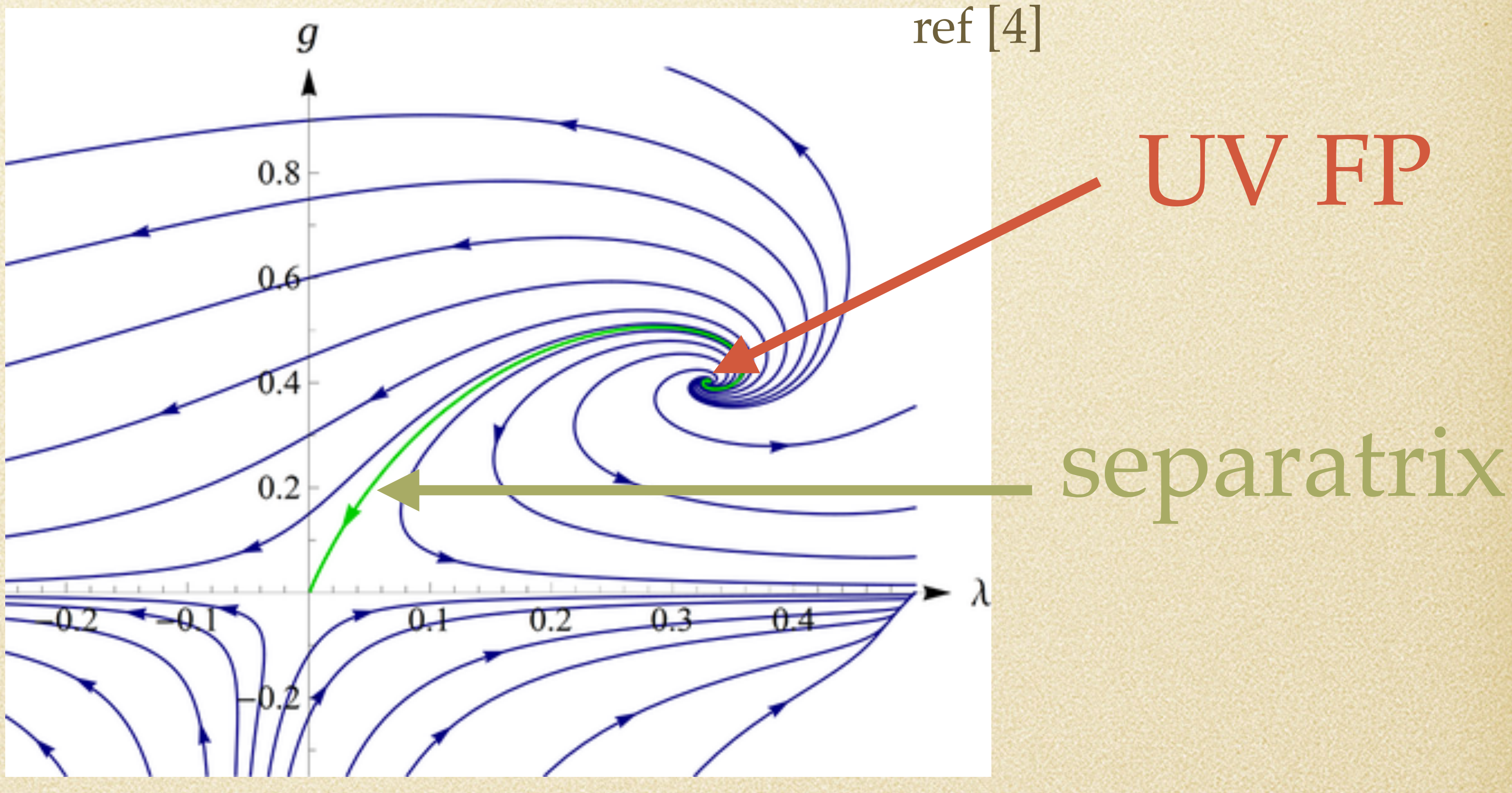
Nice, link to Asymptotic Safety?

Link to AS?

Remember:

$$G_k = \frac{\hat{g}_k}{k^2}$$

$$\Lambda_k = \hat{\lambda}_k k^2$$



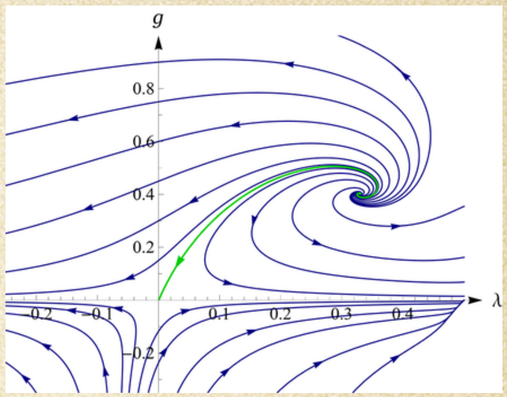
with

ref [5]

$$\hat{g}(\hat{t}) = \frac{g_0 e^{2\hat{t}}}{1 + g_0\left(e^{2\hat{t}} - 1\right)/g^*} \qquad \hat{\lambda}(\hat{t}) = \frac{g^*\lambda_0 + e^{-2\hat{t}}\left(e^{4\hat{t}} - 1\right) g_0 \lambda^*}{1 + g_0\left(e^{2\hat{t}} - 1\right)/g^*}$$

$$\hat{t} = \log(k/k_0)$$

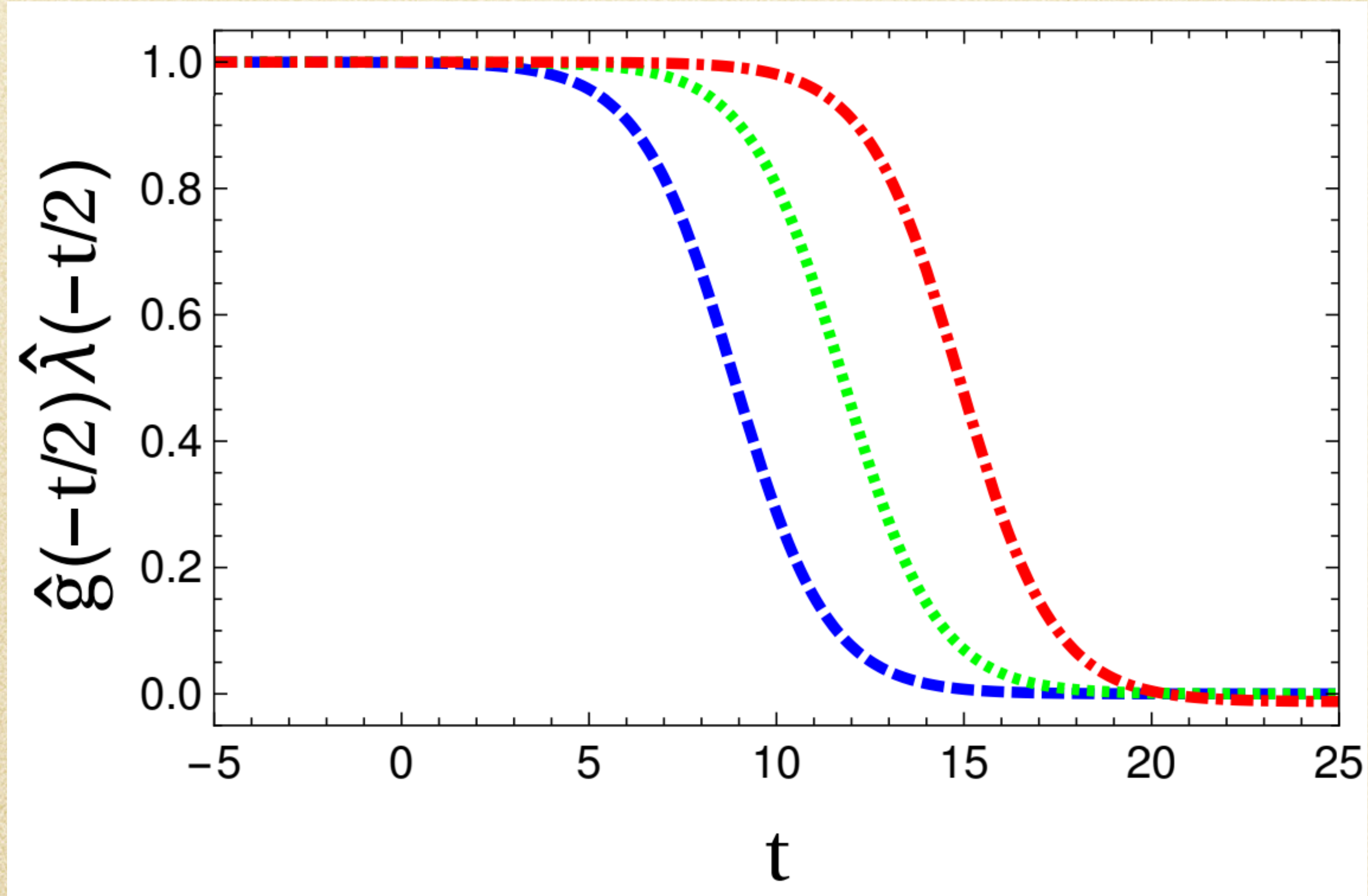
Link to AS?



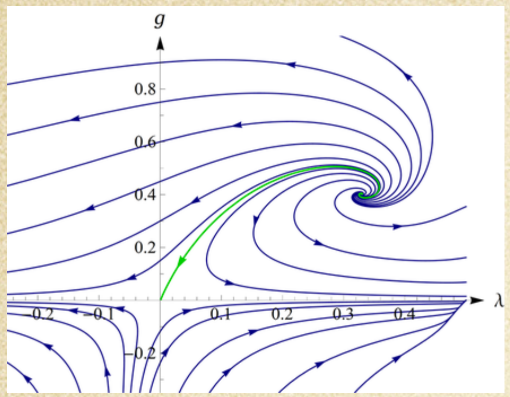
For CCP need:

$$G_k \cdot \Lambda_k = \frac{\hat{g}_k}{k^2} k^2 \hat{\lambda}_k = \hat{g}_k \cdot \hat{\lambda}_k$$

insert & plot



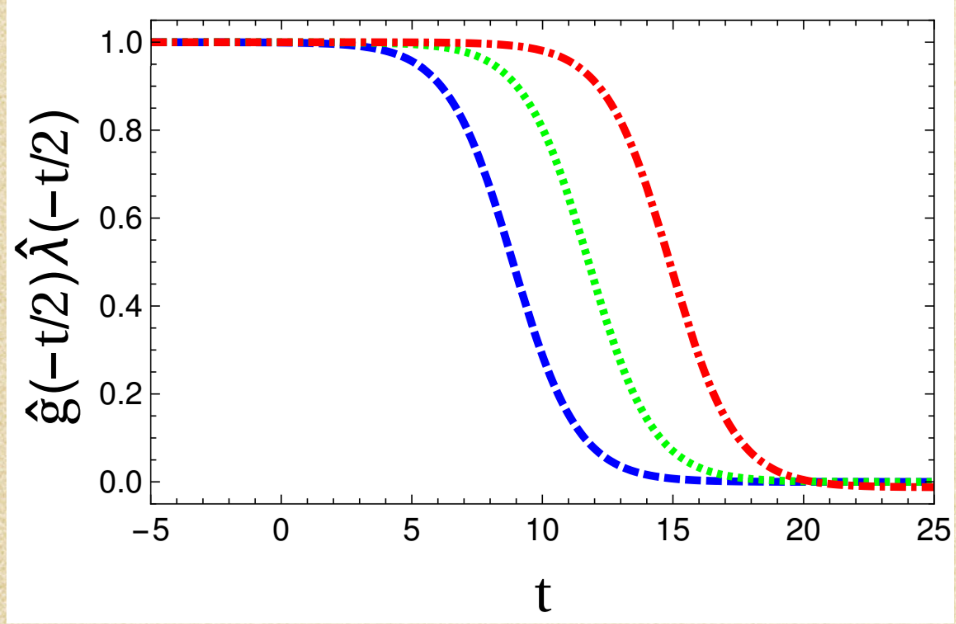
looks familiar?



Link to AS?

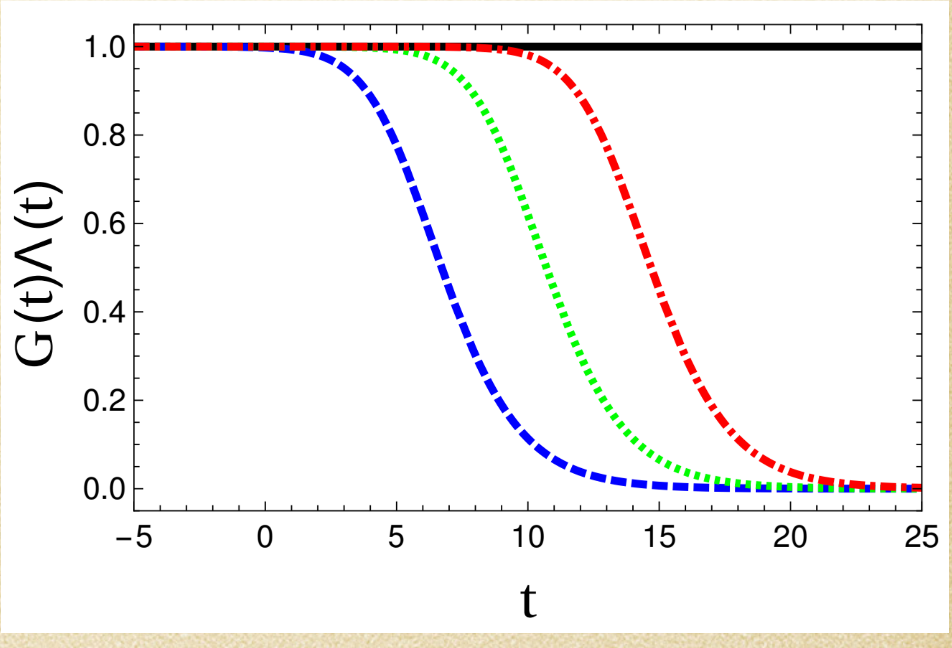
AS renormalization flow

$$G_k \cdot \Lambda_k = \frac{\hat{g}_k}{k^2} k^2 \hat{\lambda}_k = \hat{g}_k \cdot \hat{\lambda}_k$$



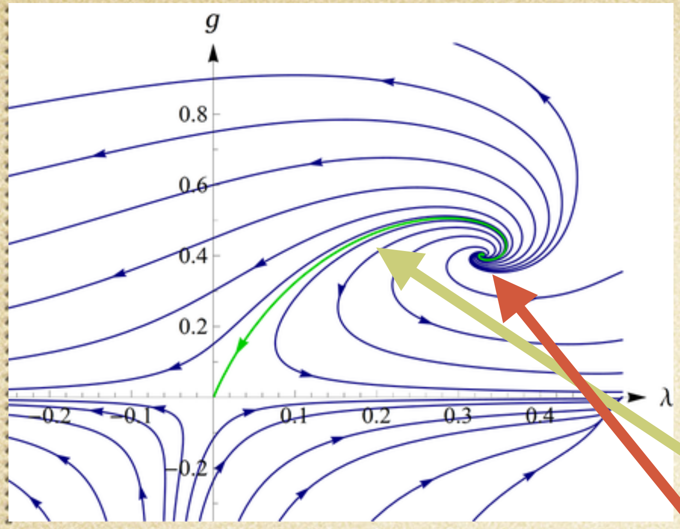
SD & NEC

$$G(t) \cdot \Lambda(t)$$



looks familiar!

why, how?



Link to AS?

Remember:

$$\hat{g}(\hat{t}) \cdot \hat{\lambda}(\hat{t}) = \frac{g_0 e^{2\hat{t}}}{1 + g_0\left(e^{2\hat{t}} - 1\right)/g*} \cdot \frac{g*\lambda_0 + e^{-2\hat{t}}\left(e^{4\hat{t}} - 1\right) g_0\lambda*}{1 + g_0\left(e^{2\hat{t}} - 1\right)/g*}$$

Approximate to UV FP & separatrix

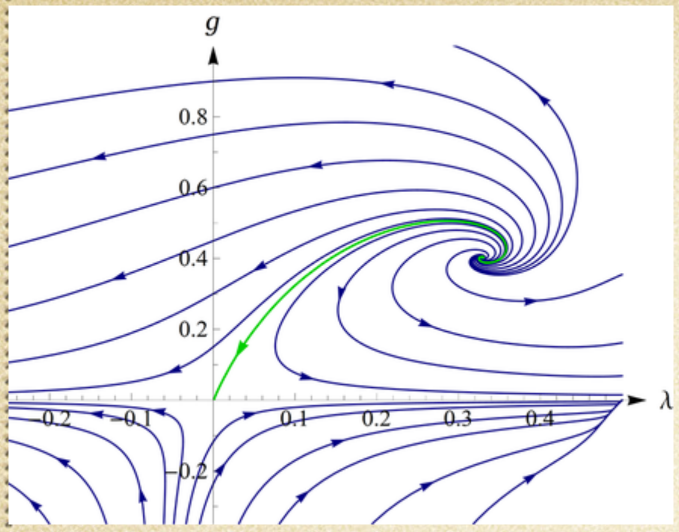
$$\hat{g}(\hat{t})\hat{\lambda}(\hat{t}) = g*\lambda*\left(\frac{g*\lambda_0}{g_0\lambda*} + e^{2\hat{t}}\right)\left(e^{2\hat{t}} + \frac{g*}{g_0}\right)^{-2} \equiv G(t)\cdot\Lambda(t)$$

For

$$g*\lambda* \to G_0\Lambda_0$$

$$g_0 \to G_0/(a_i\xi)$$

$$\hat{t} \to -t/(2\tau)$$



Link to AS?

Comments on matching:

$$\hat{g}(\hat{t})\hat{\lambda}(\hat{t}) \equiv G(t) \cdot \Lambda(t)$$

AS RG

$$g^*\lambda^* \to G_0\Lambda_0$$
$$g_0 \to G_0/(a_i\xi)$$
$$\hat{t} \to -t/(2\tau)$$

SD & NEC

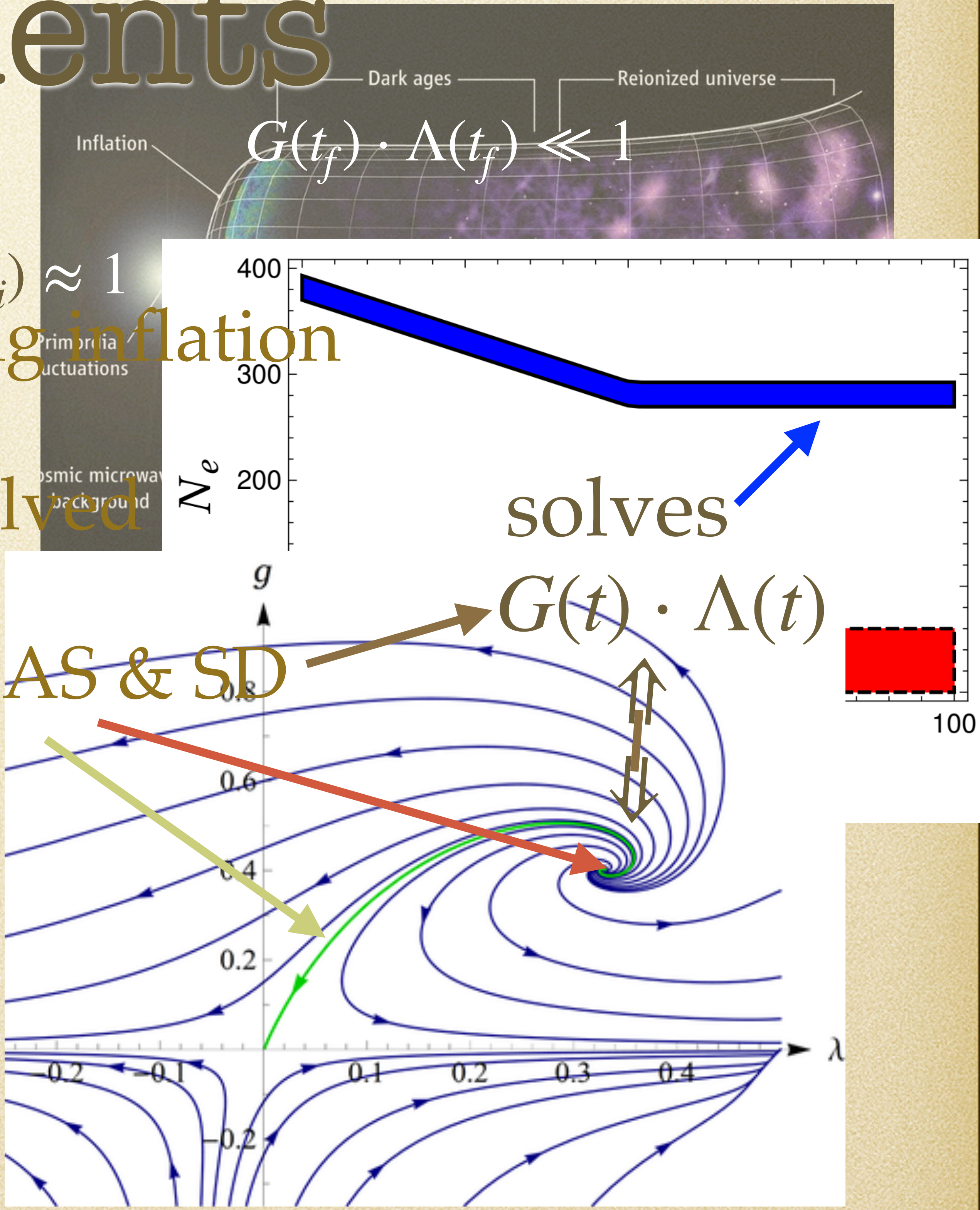
- Non trivial "coincidence"
- Works for many flow truncations
- UV FP @ inflation makes sense
- Separatrix special flow trajectory
- scale setting makes sense $\frac{k}{k_0} = e^{-t/(2\tau)}$

Concluding Comments

$G(t_f) \cdot \Lambda(t_f) \ll 1$

$G(t_i) \cdot \Lambda(t_i) \approx 1$

- CCP 3.0: watch out SD during inflation
- Showed: with SD CCP3 is solved
- Beautiful matching between AS & SD
- Outlook: Post inflation?



solves

$G(t) \cdot \Lambda(t)$

Thank You!

Literature

0) F. Canales, B. Koch, C. Laporte, A. Rincon arXiv:1812.10526
1) E. Hubble, Proceedings of the National Academy of Sciences of USA, Volume 15, Issue 3, pp. 168-173
2) Supernova Search Team (A. G. Riess (UC, Berkeley, Astron. Dept.) et al.). May 1998. 36 pp. Published in Astron.J. 116 (1998) 1009-1038
3) R. J. Adler, B. Casey and O. C. Jacob, Am. J. Phys. 63, 620 (1995);
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