Black holes in Quantum Einstein Gravity

Benjamin Koch

in colaboraion with

Alfio Bonanno, Carlos Contreras, Alessia Platania, Ignacio Reyes, Paola Rioseco, and Frank Saueressig

bkoch@fis.puc.cl

[a] PUC, Chile

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Outline

- Motivation
- Introduction Asymptotic Safety
- Black holes in Asymptotic Safety
- Cosmic Censorship in Asymptotic Safety
- Conclusion

Motivation

General Relativity

Motivation

$$\hbar$$

Quantization

General Relativity

Motivation

$\hbar$

Quantization

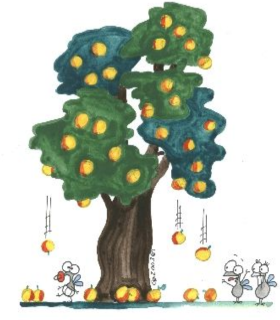
∞

Not renormalizable

General Relativity

Ways out?

$$\hbar \qquad \infty$$



Quantization | Not renormalizable | General Relativity

Ways out?

Motivation

$$\hbar$$

Quantization

String theory

Change degrees of freedom of GR

Motivation

$$\hbar$$

Quantization

*

Triangulation, Spin foams, Horava-Lifshitz

Change degrees of freedom of GR

Something else?

Asymptotic Safety Program

Motivation

$\hbar$

Quantization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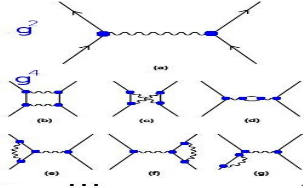
Asymptotic Safety Program

Asymptotic Safety

Asymptotic Safety Program (Weinberg):

$\hbar$



∞

Quantization

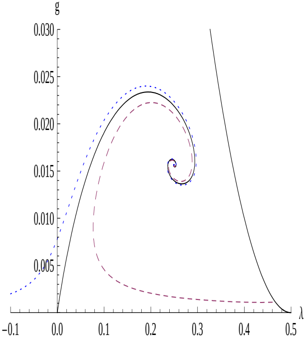
Perturbatively not renormalizable

General Relativity

Asymptotic Safety

Asymptotic Safety Program [∗]:

$\hbar$



Quantization

Functional expansion
⇒ renormalizable

General Relativity

[∗] S. Weinberg, "General Relativity" Cambridge University Press

Asymptotic Safety

Asymptotic Safety Program:

Maybe expansion wrong!
$\rightarrow$ needs the whole functional $\Gamma[\psi]$?
(possible if there are UV-fixed points)

Wetterichs realization [**]

$$\partial_t \Gamma[\psi] = \frac{1}{2} Tr\left[\partial_t R_k ((\Gamma^{(2)}[\psi] + R_k)^{-1})\right] \qquad (1)$$

Flow equation where ψ are fields, $\Gamma^{(2)} = \delta^2\Gamma/\delta\psi^2)$, $t = \ln(k)$, and R_k cut-off function.

$\Rightarrow$ running couplings

[**] M. Reuter, C. Wetterich, Nucl.Phys. B417, 181 (1994)

Asymptotic Safety

Running gravitational couplings [∗]

$$\beta_\lambda = \partial_t \lambda_k = \frac{P_1}{P_2 + 4(d + 2g_k)} \tag{2}$$

$$\beta_g = \partial_t g_k = \frac{2 g_k P_2}{P_2 + 4(4 + 2g_k)}$$

with the dimensionless couplings defined as

$$g_k = k^2 G_k \quad , \quad \lambda_k = \frac{\Lambda_k}{k^2} \tag{3}$$

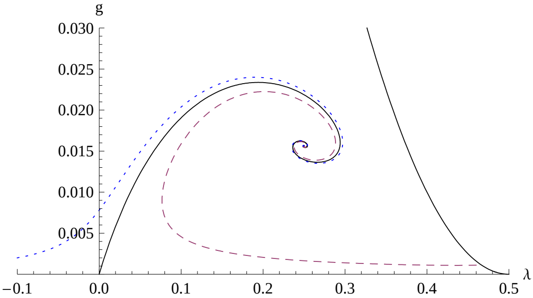
G_0: Newtons constant, Λ_0: Cosmological constant

[∗] M. Reuter, F. Saueressig, Phys. Rev. D65, 065016 (2002)

D. F. Litim, Phys. Rev. Lett. 92, 201301 (2004) ...

Asymptotic Safety

FRGE solutions:

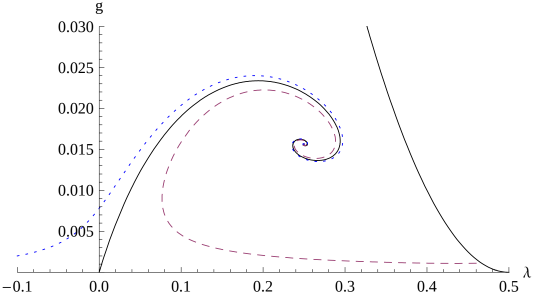


+ UV fixed points λ^* and g^* (Weinbergs hypothesis)
- ∞ couplings, need proof: either irrelevant or finite number UV fixed
- What does this mean for physical systems?

BLACK HOLES

Asymptotic Safety

FRGE solutions:



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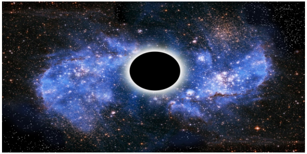
BLACK HOLES

Black Holes

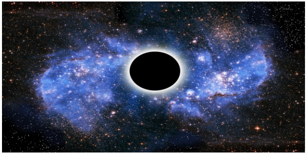
Black Holes



Black Holes

Black Holes

Singularitiy



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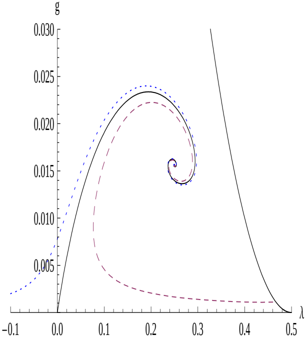
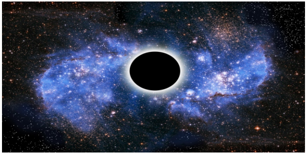


Asymptotic Safety?

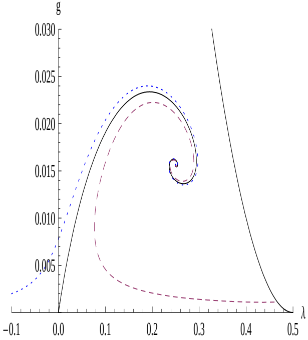


Black Holes

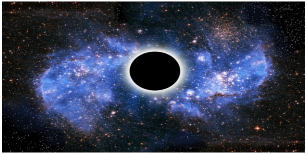
Black holes in Asymptotic Safety

Black Holes

Singularitiy



Asymptotic Safety?



Black Holes

Black holes in Asymptotic Safety

Black Holes

Black holes in Asymptotic Safety:
Two approaches borrowed from QFT

- Improving solutions (Uehling potential textbook QED)
- Improving action and eom (gap equations in QFT)

Improving solutions:
Classical eom's

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi G_k T_{\mu\nu} \tag{4}$$

Classical solution for $ds^2 = f(r)dt^2 + f^{-1}dr^2 + d\Omega$ (with $\Lambda_k \approx 0$)

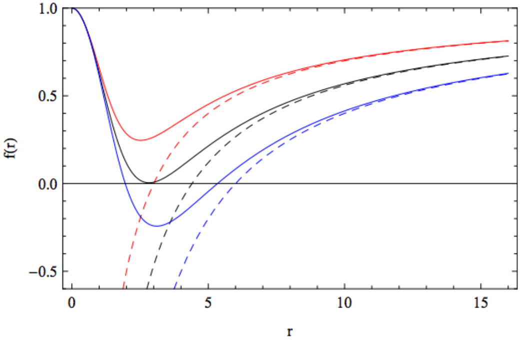
$$f(r) = 1 - \frac{2G_k M}{r} \tag{5}$$

Quantum improvement G_k with $k \neq cte$.

$$k = k(r) = \frac{\xi}{d(r)} \tag{6}$$

where $d(r)$ physical cut-off like proper distance $*$
[$*$] A. Bonanno, M. Reuter, Phys. Rev. D62, 043008 (2000)

Improving solutions:[*]

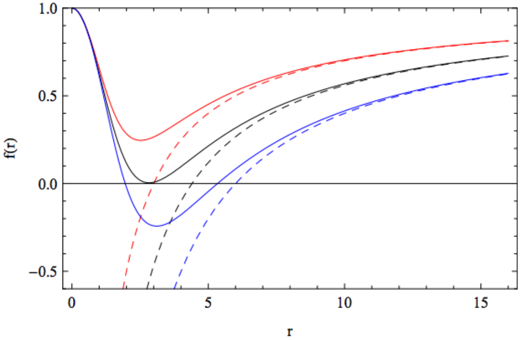


- No Singularity
- Stable remnant
- Similar for different scale setting,
  extra dimensions, charge, or angular momentum but

[*] B.K., F. Saueressig, Int.J.Mod.Phys. A29 (2014) no.8, 1430011

Black Holes

Improving solutions:$_*$



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Improving solutions:$_*$

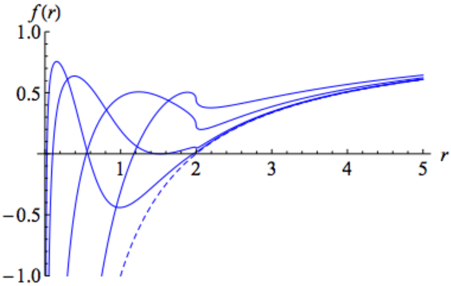
but if one considers

$$\Lambda_k|_{UV} = lim_{k\to\infty} k^2 \lambda^* \tag{7}$$

$\Rightarrow$ the neglected term $\sim \Lambda_k$ in lapse function

$$f(r) = 1 - \frac{2G_k M}{r} + r^2 \Lambda_k \tag{8}$$

can become divergent for $r \to 0_*$



[∗] B.K., F. Saueressig, Class.Quant.Grav. 31 (2014) 015006

Gap equations:[*]

Effective Einstein-Hilbert action

$$\Gamma_k[g_{\mu\nu}] = \int_M d^4x\sqrt{-g}\left(\frac{R-2\Lambda_k}{16\pi G_k}\right) \quad , \tag{9}$$

eom $\delta g_{\mu\nu}$:

$$G_{\mu\nu} = -g_{\mu\nu}\Lambda_k - \Delta t_{\mu\nu} + 8\pi G_k T_{\mu\nu} \quad , \tag{10}$$

with

$$\Delta t_{\mu\nu} = G_k\left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)\frac{1}{G_k} \quad . \tag{11}$$

scale setting δk:[*]

$$\left[R\partial_k\left(\frac{1}{G_k}\right) - 2\partial_k\left(\frac{\Lambda_k}{G_k}\right)\right] = 0 \tag{12}$$

[*] B.K., P. Rioseco, C. Contreras Phys.Rev. D91 (2015) no.2, 025009

Black Holes

Gap equations:
Complicated equations $\Rightarrow$ no analytic BH solution
Trick: Impose Null Energy Condtion

$$\nabla_\mu \Delta t^{\mu\nu} = 0 \tag{13}$$

Trick implies Schwarzschild ansatz $g_{00} = 1/g_{11} = f(r)$
$\Rightarrow$ generalized de Sitter solution, also Reissner Nordstrom, and BTZ:[*]

$$G(r) = \frac{G_0}{\epsilon r + 1} \tag{14}$$

$$f(r) = 1 + 3G_0 M_0 \epsilon - \frac{2G_0 M_0}{r} - (1 + 6\epsilon G_0 M_0)\epsilon r - \frac{\Lambda_0 r^2}{3} + r^2 \epsilon^2 (6\epsilon G_0 M_0 + 1) \ln\left(\frac{c_4(\epsilon r + 1)}{r}\right) \tag{15}$$

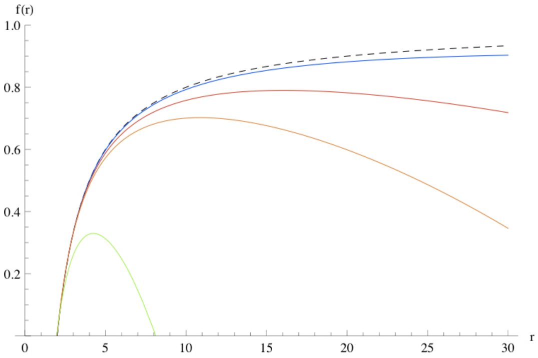
$$\ldots$$

Constants of integration: G_0, M_0, Λ_0, ϵ, c_4

[*] B.K., P. Rioseco, Class.Quant.Grav. 33 (2016) 035002,

B.K. I. Reyes, A. Rincon, Class.Quant.Grav. 33 (2016) no.22, 225010.

Gap equations:



- Has singularity ...

Fair to say:

Question of singularity is still open!

What is the problem with such singularities?

Black Holes

Fair to say:

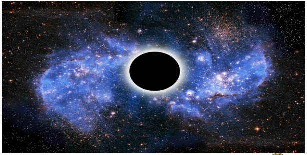
Question of singularity is still open!

What is the problem with such singularities?

Black Hole Formation

Black Hole Formation

Remember classical BH

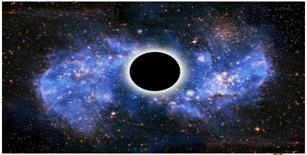


Black Holes

Black Hole Formation

Remember classical BH

Singularitiy



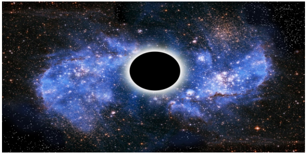
Black Holes

Black Hole Formation

Remember classical BH

Singularities

Censorship hypothesis



Black Holes

dressed singularity might not be the problem

$\Rightarrow$

study naked singularities (e.g. BH formation)

Black Hole Formation

Remember classical BH

Singularities

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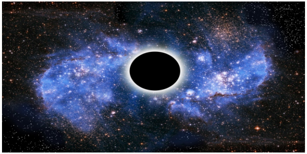

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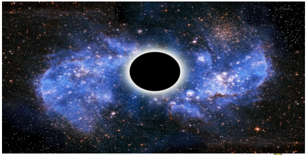
Black Hole Formation

Classical Kuroda-Papapetrou model

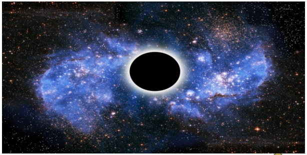


Black Hole formation

Black Hole Formation

Classical Kuroda-Papapetrou model

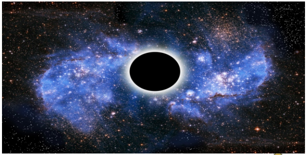
Singularity



Black Hole formation

Singularity

Censorship hypothesis comes “late”

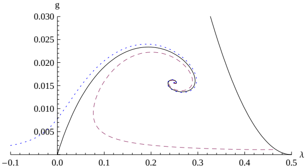


Black Hole formation

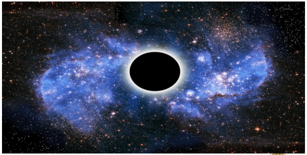
Black Hole Formation

Classical Kuroda-Papapetrou model

Singularity



Censorship “late”, AS can help?



Black Hole formation

Classical Vaidya metric

$$ds^2 = -f(r,v)\cdot dv^2 + 2dvdr + r^2 d\Omega^2 \tag{16}$$

with advanced ingoing null coordinate v.
Null geodesics:

$$\frac{dr}{dv} = \frac{1}{2}\left(1 - \frac{2G_0 m(v)}{r}\right) . \tag{17}$$

$$f(r,v) = 1 - \frac{2G_0 m(v)}{r} \tag{18}$$

Mass inflow modeled by:

$$m(v) = \begin{cases} 0 & v < 0 \\ \lambda v & 0 \leq v < \bar{v} \\ \bar{m} & v \geq \bar{v} \end{cases} . \tag{19}$$

Horizons:

- High mass inflow

$$\lambda > \lambda_c = \frac{1}{16 G_0} \tag{20}$$

$\Rightarrow$ Singularity at $r = 0$ always covered by an horizon

- Low mass inflow

$$\lambda < \lambda_c = \frac{1}{16 G_0} \tag{21}$$

$\Rightarrow$ Singularity at $r = 0$ can be naked

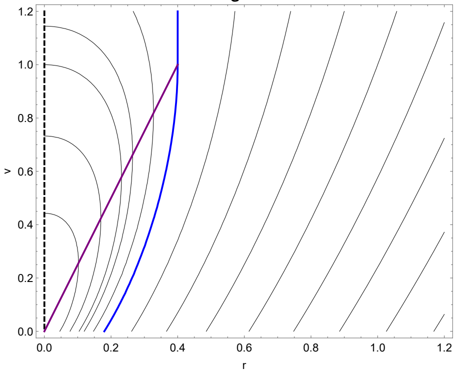
Can be seen in phase diagram:

Black Hole Formation

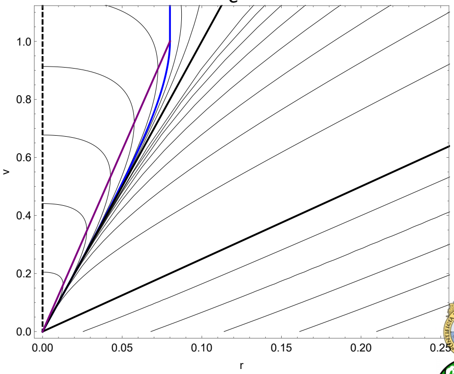
Classical Kuroda-Papapetrou model

Phase diagram:

$\lambda > \lambda_c$



$\lambda < \lambda_c$



Singularity covered

Singularity naked

blue apparent horizon, purple event horizon

Improved Vaidya metric

$$ds^2 = -f_k(r,v) \cdot dv^2 + 2dvdr + r^2 d\Omega^2 \qquad (22)$$

with

$$f_k(r,v) = 1 - \frac{2G_k m(v)}{r} \qquad (23)$$

Identify IR cut-off scale with scale imposed by infalling radiation

$$k \sim T \sim \rho^{1/4} \qquad (24)$$

ξ : proportionality constant, ρ given from classical field equations ($G_{v,v}$)

$$\frac{\dot{m}(v)}{4\pi r^2} = \rho(v,r). \qquad (25)$$

Thus,

$$f_k(r,v) = 1 - \frac{2\lambda G_0 v}{r + \alpha\sqrt{\lambda}} \quad , \qquad \text{with} \quad \alpha = \frac{\xi^2 G_0}{\sqrt{4\pi g_*}}, \qquad (26)$$

Note:[∗]

- Improved lapse function $f_k(r,v)$ is well defined in the limit $r \to 0$

$$lim_{r\to 0}\, f_k(r,v) = 1 - \frac{\sqrt{16\pi\lambda}}{\omega\,\xi^2}\, v \tag{27}$$

- However singular curvatures in $r \to 0$ e.g.
$R = -\frac{G_0\sqrt{\lambda}v}{\alpha r^2} + O(1/r^2)$, $\quad K = \frac{16 G_0\sqrt{\lambda}v}{\alpha^2 r^4} + O(1/r^3)$.
- One might invent cut-off identification without singularity, but don't want to do reverse engineering
- Like in all improving solutions schemes (27) does not solve eoms

[∗] B. Bonanno, B.K., A. Platania, arXiv:1610.05299.

Black Hole Formation

AS improved Kuroda-Papapetrou model

From (27) apparent horizon shifted by the constant $\alpha\sqrt{\lambda}$

$$r_{AH}(v) = 2\,m(v)\,G_0 - \alpha\sqrt{\lambda} = 2\,m(v)\,G_0 - \frac{G_0\,\xi^2}{g_*}\sqrt{\frac{\lambda}{4\pi}}, \qquad (28)$$

from $r_{AH} \geq 0$ and matching to improved Schwarzschild $\rightarrow$ minimum "time" $\bar{v}$ of irradiation, necessary to actually form a black hole

$$r_S = 2\,\lambda\bar{v}\,G_0 - \alpha\sqrt{\lambda} \geq 0 \qquad \Rightarrow \qquad \bar{v} \geq v_{\min}(\lambda) \equiv \frac{\xi^2}{2\,g_*}\sqrt{\frac{1}{4\pi\lambda}}. \qquad (29)$$

Null geodesics from

$$\dot{r}(v) = \frac{1}{2}\left(1 - \frac{2\,\lambda v\, G_0}{r(v) + \alpha\sqrt{\lambda}}\right), \tag{30}$$

Integrating (e.g. for $\lambda \leq \frac{1}{16\, G_0}$) gives implicit equation

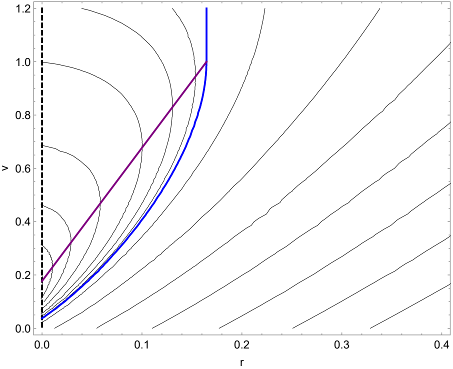
$$\frac{|r(v) + \alpha\sqrt{\lambda} - \mu_- v|^{\mu_-}}{|r(v) + \alpha\sqrt{\lambda} - \mu_+ v|^{\mu_+}} = \tilde{C} \tag{31}$$

with two constant solutions

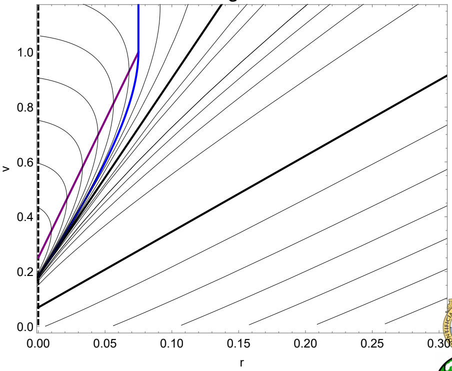
$$r_\pm(v) = -\alpha\sqrt{\lambda} + \mu_\pm\, v, \tag{32}$$

Phase diagram:

$\lambda > \lambda_c$



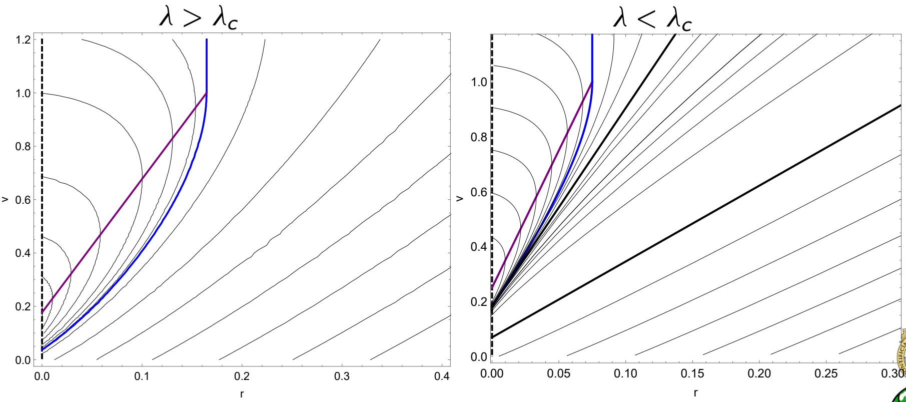
$\lambda < \lambda_c$



blue apparent horizon, purple event horizon

Singularity always naked but how bad is it?

Phase diagram:



blue apparent horizon, purple event horizon

Singularity always naked but how bad is it?

Nature of the singularity (how bad is it?)

Study geodesics as dynamical system[*]

$$\begin{cases} \frac{\mathrm{d}v(t)}{\mathrm{d}t} = N(r,v) \\ \frac{\mathrm{d}r(t)}{\mathrm{d}t} = D(r,v) \end{cases} , \tag{33}$$

where t is a parameter and the functions $N(r,v)$ and $D(r,v)$ are defined as

$$N(r,v) = 2\,r \qquad D(r,v) = r - 2\,M(r,v). \tag{34}$$

Singularities are fixed points (e.g. $r = 0$ and $M(0,v) = 0$)

Expand near the singularity

$$\begin{cases} \frac{\mathrm{d}v(t)}{\mathrm{d}t} = \dot{N}_{\mathrm{FP}}\,(v - v_{\mathrm{FP}}) + N'_{\mathrm{FP}}\,(r - r_{\mathrm{FP}}) \\ \frac{\mathrm{d}r(t)}{\mathrm{d}t} = \dot{D}_{\mathrm{FP}}\,(v - v_{\mathrm{FP}}) + D'_{\mathrm{FP}}\,(r - r_{\mathrm{FP}}) \end{cases} . \tag{35}$$

[*] M. D. Mkenyeleye, R. Goswami, and S. D. Maharaj, Phys. Rev. D 90, 064034 (2014).

Nature of the singularity classified by eigenvalues of the stability matrix J of the system (35)

$$\chi_{\pm} = \frac{1}{2}\left(\mathrm{Tr}J \pm \sqrt{(\mathrm{Tr}J)^2 - 4\,\det J}\right), \tag{36}$$

where

$$\mathrm{Tr}J = \dot{N}_{\mathrm{FP}} + D'_{\mathrm{FP}} = 1 - 2\,M'_{FP} \tag{37}$$
$$\det J = \dot{N}_{\mathrm{FP}} D'_{\mathrm{FP}} - \dot{D}_{\mathrm{FP}} N'_{\mathrm{FP}} = 4\,\dot{M}_{FP}. \tag{38}$$

Strength of the singularity is

$$S = \frac{\dot{M}_{FP} X_{FP}^2}{2} = 0. \tag{39}$$

where $X_{FP} \equiv lim_{(r,v)\to \mathrm{FP}} \frac{v(r)}{r}$.
$\Rightarrow$ singularity is **integrable** “harmless”.
Interesting:

- $S \to 0$ does not depend on cut-off identification as long as

$$lim_{k\to\infty} G_k = 0 \tag{40}$$

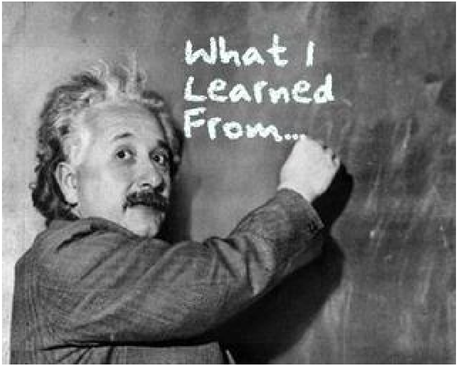
Summary

Summary

- Quantum gravity and Asymptotic Safety
- Black holes in AS: singularity unsure
- Naked singularities e.g. Kuroda-Papapetrou model
- AS improved Kuroda-Papapetrou model

Summary

Take home messages:



- Important test QG candidate with problematic solutions of GR
- In different attempts, the singularity might go away or persist
- Even if naked singularities don't go away in AS, at least they become integrable

Thank you

Thank you !