Selfconsistent Backgrounds of Effective Action

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Cosmo Pizza
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Outline

I
Introduction

$$\Gamma_k(\phi) \tag{1}$$

In “Pleasantville”

Generating functional

$$Z[J] = \int D\varphi \exp\left(-i \int dx (L(\varphi) - \varphi J)\right) \quad (2)$$

Connected diagrams

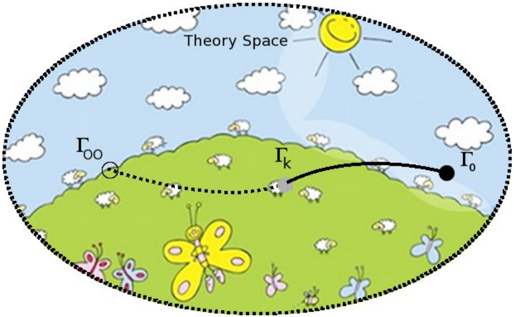
$$W[J] = \ln Z[J] \quad (3)$$

Effective action (1PI)

$$\Gamma[\phi] = sup_J\left(\int J\phi - W[J]\right) \quad (4)$$

What happens if one integrates out only certain Momentum shell ?

$$k \to k + \delta k?$$
$$\Rightarrow \text{RG-Flow}$$



$$\partial_k \Gamma_k = \ldots \qquad (5)$$

Solve:
Running couplings λ_k and effective action

$$\Gamma_k = \Gamma_k(\lambda_k, \phi) \qquad (6)$$

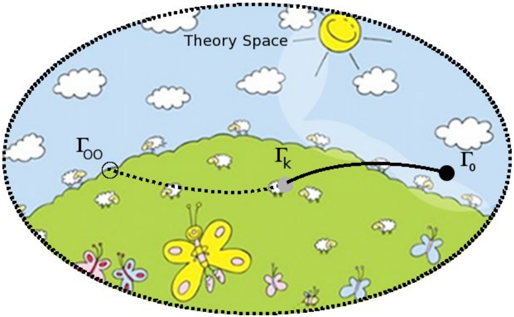
I Introduction Γ_k

Wilsons flow dangers

gauge redundance

$$\phi \rightarrow e^{i\alpha}\phi$$

Fadeev Popov

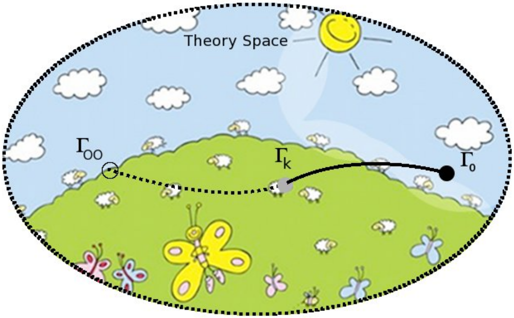


I Introduction Γ_k

Wilsons flow dangers

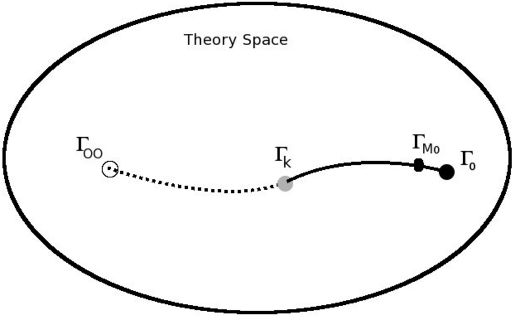
Infrared divergencies

$$k \to 0,\ \Gamma_0 = ?$$



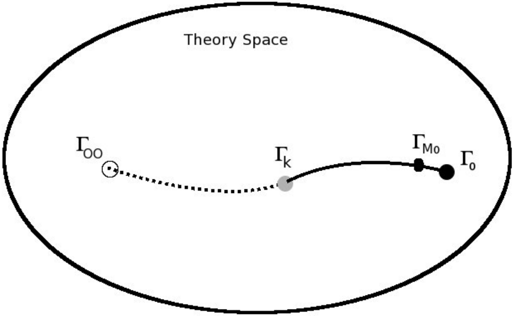
Infrared divergencies

$k \to 0$, $\Gamma_0 =?$



Ultra violet divergencies

$$k \to \infty, \Gamma_\infty = ?$$



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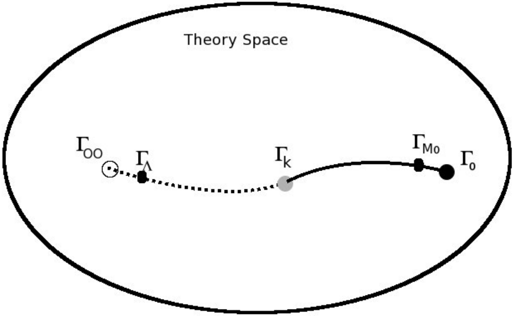


I Introduction Γ_k

Wilsons flow dangers

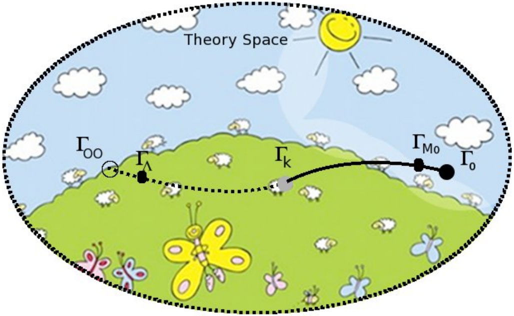
Renormalization:

Live with the problems

Absorb Infinities $\sim \Lambda$ in definition of couplings λ at scale M_0

$$\lambda_{i,0} = \lambda_i(M_0) \tag{7}$$

AFTER renormalization can
pretend to be back in
"Pleasantville"



Coupling flow $\lambda_i(k)$ and effective action Γ_k

At the end effective quantum action:

$$\Gamma_k(\phi_i(x), \partial\phi_i(x), \lambda_n(k)) = \int d^4x \sqrt{-g}\mathcal{L}_k(\phi_i(x), \partial\phi_i(x), \lambda_n(k)) \qquad (8)$$

Quantum background?

$$\frac{\delta\Gamma_k}{\delta\phi_i} = 0 \quad . \qquad (9)$$

“gap equation” solutions?

II
Improving solutions

Improving solutions II

Improving solutions II

$\frac{\delta\Gamma_k}{\delta\phi_i} = 0$ is too hard
so solve first classical eom

$$\frac{\delta S}{\delta\varphi_i} = 0 \quad . \tag{10}$$

and take Γ_k as correction of those solutions

Uehling example: QED textbook

Classical solution

$$V_C = \frac{e^2}{4\pi r} \tag{11}$$

but since the coupling is running:

$$e^2(k) = e^2(k_0)(1 - e_0^2/(6\pi^2)\ln(k/k_0))^{-1} \tag{12}$$

Scale setting:

$$k(r) = \xi/r \quad \text{with condition} \quad k_0 = 1/r_0 \tag{13}$$

Results in Uehling potential

$$V_U(r) = \frac{-e_0^2}{4\pi r}\left(1 - e_0^2/(6\pi^2)\ln(r_0/r) + \mathcal{O}(e_0^4)\right) \tag{14}$$

Improving solutions II

More general consideration

Improving solutions more general:

Classical eom and solution

$$\mathcal{D}(\alpha_0 \phi_i^0) + \sum \lambda_i^{0,abcd}(\phi_a^0 \phi_b^0 \phi_c^0 \phi_d^0) = 0 \quad , \tag{15}$$

Replace solution by scale dependent couplings

$$(\alpha^0, \lambda^0) \rightarrow (\alpha^k, \lambda^k) \quad . \tag{16}$$

Expand around this classical solutions

$$\begin{aligned} \phi_i &= \phi_i^0(r, \alpha_0, \lambda_0, A) + \frac{d}{dk}\phi_i(r, \alpha_k, \lambda_k, A)|_{k=0} \cdot k + \mathcal{O}(k^2) \quad , \\ \alpha_k &= \alpha_0 + \frac{d}{dk}\alpha_k|_{k=0} \cdot k + \mathcal{O}(k^2) \quad , \\ \lambda_k &= \lambda_0 + \frac{d}{dk}\lambda_k|_{k=0} \cdot k + \mathcal{O}(k^2) \quad . \end{aligned} \tag{17}$$

Improving solutions II

More general consideration

Improving solutions more general:

Scale setting

$$k \rightarrow k(r) \quad . \tag{18}$$

(usually just $k \sim 1/r$)

Can we do better?
Does it solve improved eom $\frac{\delta \Gamma_k}{\delta \phi_i} = 0$?

$$\mathcal{D}(\alpha \phi_i) + \sum \lambda_i^{abcd}(\phi_a \phi_b \phi_c \phi_d) = 0? \quad , \tag{19}$$

$\rightarrow$ insert expansion

Improving solutions II

More general consideration

Improving solutions more general:

insert expansion

$$
\begin{aligned}
0 \;\; = \;\; & \mathcal{D}(\alpha_0 \phi_i^0) + \sum \lambda_i^{0,abcd}(\phi_a^0 \phi_b^0 \phi_c^0 \phi_d^0) \\
& + \mathcal{D}\left[\left(k \frac{d}{dk} \right) (\alpha_k \phi_i^k)|_{k=0} \right] + \left(k \frac{d}{dk} \right) \sum \lambda_i^{k,abcd}(\phi_a^k \phi_b^k \phi_c^k \phi_d^k)|_{k=0} + \mathcal{O}(
\end{aligned}
$$

Solved by $\phi_i(r,k) = \phi^0(r) \cdot f(k)$ (like QM) if right scale setting:

$$
[\mathcal{D}, k(r)] = 0 \quad . \tag{21}
$$

"Improved scale setting"

Improving solutions II

More general consideration

"Improved scale setting" – example

Spherical symmetry and Laplace:

$$\left[\frac{1}{r^2}\partial_r(r^2\partial_r),\ k(r)\right] = 0 \quad . \tag{22}$$

It is solved by

$$k(r) = \frac{\xi}{r} \quad , \tag{23}$$

Same as dimensional analysis

Completely consistent?

Improving solutions II

More general consideration

"Improved scale setting" - consistent?

classically conserved stress energy tensor

$$\nabla^\mu T^0_{\mu\nu} = 0 \quad , \tag{24}$$

Q-expanded

$$T_{\mu\nu} = T^0_{\mu\nu} + \left(\frac{d}{dk} T^k_{\mu\nu}|_{k=0} \right) \cdot k + \mathcal{O}(k^2) \quad . \tag{25}$$

gives

$$\nabla^\mu T_{\mu\nu} = \nabla^\mu \left(\left(\frac{d}{dk} T^k_{\mu\nu}|_{k=0} \right) \cdot k \right) \equiv 0 \quad . \tag{26}$$

Typically not solved by $[\mathcal{D}, k(r)] = 0$

Improving solutions II

More general consideration

"Improved scale setting" - consistent?

(26) typically not solved by $[\mathcal{D}, k(r)] = 0$
more like $[\nabla_\mu, k(r)] = 0 \rightarrow$ condition too strong

Scale setting only partially consistent– problems with $T^{\mu\nu}$

III
Self-consistent scale setting

Self-consistent scale setting III

Proposal

Effective action and running couplings

$$\Gamma_k(\phi_i(x), \partial\phi_i(x), \lambda_n(k)) = \int d^4x \sqrt{-g}\mathcal{L}_k(\phi_i(x), \partial\phi_i(x), \lambda_n(k)) \quad , \qquad (27)$$

"Gap equations" for quantum background

$$\frac{\delta\Gamma_k}{\delta\phi_i} = 0 \quad . \qquad (28)$$

Scale setting for this background?

Self-consistent scale setting III

Proposal

Scale setting for this background $k = k(r)$?

Idea: Minimal k sensitivity
(just like Callan-Symanzik equations $\frac{d}{dk}\langle T\phi_1(x_1)\phi_2(x_2)\ldots\rangle_k\Big|_{k=k_{opt}} \equiv 0$)

Realization: Promote k to field in Γ_k

$$\Gamma_k(\phi_i(x), \partial\phi_i(x), \lambda_n(k)) \rightarrow \Gamma(\phi_i(x), \partial\phi_i(x), k(x), \lambda_n(k)) \quad . \tag{29}$$

Self-consistent scale setting III

Proposal

“Scale field” $k(x)$

coupled equations of motion

$$\frac{\delta\Gamma}{\delta\phi_i} = 0 \quad , \quad \frac{d}{dk}\mathcal{L}(\phi_i(x), \partial\phi_i(x), k(x), \lambda_n(k))\Big|_{k=k_{opt}} = 0 \quad . \qquad (30)$$

The simultaneous solution of (30) assures optimal scale setting

Self-consistent scale setting III

Proposal

First questions on proposal

- Consistent set of equations?
- Solves $\nabla_\mu T^{\mu\nu} \leftrightarrow k(r)$ problem?
- Examples?

IV
Examples

Examples IV

ϕ^n

Effective action

$$\Gamma_k = \int d^4x\sqrt{-g}(\alpha(k)(\partial\phi)^2 - \lambda_n(k)\phi^{2n})/2 \tag{31}$$

Black board:

eom $\delta\phi$,

eom k

conservation $\nabla_\mu T^{\mu\nu}$

Examples IV
ϕ^n loops

ϕ^n scale setting
at : 1,2,3,4, 5 loop α_k, λ_k, and m_k ...

Examples IV

Gravity and gauge

Effective action

$$\Gamma_k[g_{\mu\nu}, A_\alpha] \;=\; \int_M d^4x\sqrt{-g}\left(\frac{R-2\Lambda_k}{16\pi G_k} - \frac{1}{4e_k^2}F_{\mu\nu}F^{\mu\nu}\right) \quad , \qquad (32)$$

Einstein-Hilbert-Maxwell

Equations of motion

eom $\delta g_{\mu\nu}$:

$$G_{\mu\nu} = -g_{\mu\nu}\Lambda_k - \Delta t_{\mu\nu} + 8\pi G_k T_{\mu\nu} \quad , \tag{33}$$

with

$$\Delta t_{\mu\nu} = G_k \left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu \right) \frac{1}{G_k} \quad . \tag{34}$$

and

$$T_{\mu\nu} = F_\nu{}^\alpha F_{\mu\alpha} - \frac{1}{4} g_{\mu\nu} F_{\mu\nu} F^{\mu\nu} \quad . \tag{35}$$

eom δA_μ:

$$D_\mu \left(\frac{1}{e_k^2} F^{\mu\nu} \right) = 0 \quad , \tag{36}$$

Conservation and symmetry

Symmetry diff:

$$\nabla^\mu G_{\mu\nu} = 0 \tag{37}$$

Symmetry $U(1)$:

$$\nabla_{[\mu} F_{\alpha\beta]} = 0 \quad . \tag{38}$$

Consistency:

Using everybody ... one can actually show that

Scale setting eom δk:

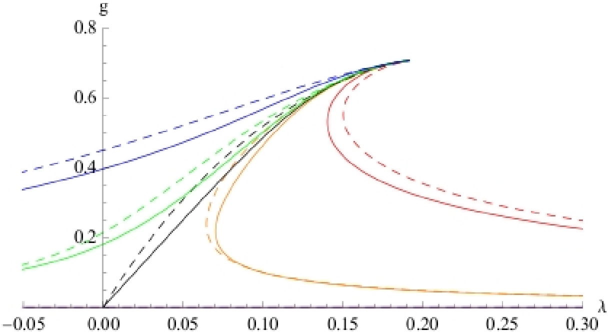
$$\left[R\nabla_\mu\left(\frac{1}{G_k}\right)-2\nabla_\mu\left(\frac{\Lambda(k)}{G_k}\right)-\nabla_\mu\left(\frac{4\pi}{e_k^2}\right)F_{\alpha\beta}F^{\alpha\beta}\right]\cdot(\partial^\mu k)=0 \quad . \tag{39}$$

is actually self-consistent consequence of eoms and conservation laws

Examples IV

Selfconsistent EH-Black hole solution

Found self-consistent Einstein Hilbert black hole solution $\Lambda(r), v.s. G(r)$ with scale setting $k(r)$



compared to RG flow $\Lambda(k), v.s. G(k)$ in "Functional Renormalization Group" (no scale setting no black hole)

Examples IV

Selfconsistent RN-Black hole solution

Found self-consistent Reissner Nordström black hole solution $e(r), v.s. G(r)$ with scale setting $k(r)$

no comparison to “Functional Renormalization Group” yet ...

V
Summary and Outlook

Summary

- Selfconsistent quantum backgrounds $\delta\Gamma_k/\delta\phi_i = 0$
- Improving solutions: Derivative scale setting $[\mathcal{D}, k] = 0$
- Self-consistent scale setting $\delta\Gamma_k/\delta\phi_i = 0$ and $\delta\Gamma_k/\delta k = 0$

Summary

Thank you