

Dark Energy

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- 1) **Friedman Robertson Walker metric**
 - a) Flat spatial models
 - b) Kinematics of the FRW metric
- 2) **Standard Cosmology**
 - a) Friedman equations
 - b) Solutions of the Friedman equations
- 3) **Dark energy**
 - a) Supernova 1a
 - b) Other evidence: CMB, BAO



Literature:

- Kolb and Turner, *The Early Universe*, Addison Wesley (1990)
- Scientific Background on the Nobel Prize in Physics 2011, Nobelprize.org
- Frieman, *Lectures on Dark Energy and Cosmic Acceleration*, arXiv:0904.1832



FRW Metric

Flat spatial models



FRW Metric

1 a) Flat spatial models

Distribution of matter is homogeneous and distribution of radiation is isotropic \Rightarrow metric

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (1)$$



FRW Metric

1 a) Flat spatial models

Calculate for the metric (1)

$$\Gamma_{jk}^i = \frac{1}{2} h^{il} (h_{lk,k} + h_{lk,j} - h_{jk,l}), \quad \Gamma_{ij}^0 = \frac{\dot{R}}{R} h_{ij}, \quad \Gamma_{0j}^i = \frac{\dot{R}}{R} \delta_j^i \quad (2)$$

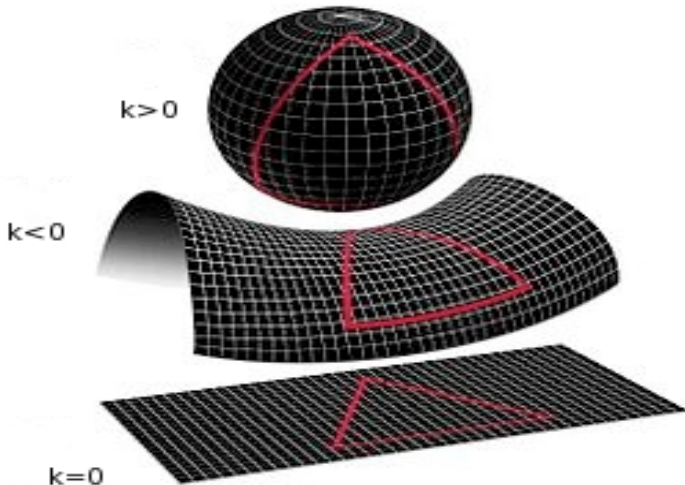
$$R_{00} = -3 \frac{\ddot{R}}{R}, \quad R_{ij} = - \left(\frac{\ddot{R}}{R} + 2 \frac{\dot{R}^2}{R^2} + 2 \frac{k}{R^2} \right) h_{ij}, \quad R = -6 \left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} \right) \quad (3)$$



FRW Metric

1 a) Flat spatial models

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$



FRW Metric

1 b) Particle kinematics

Causal contact (horizon)

Comoving observer at time t receives light from previous emission moment $t = 0$.

⇒ boundary of visible universe

$$d_H(t) = R(t) \int_0^t \frac{dt'}{R(t')} \quad (4)$$



FRW Metric

1 b) Particle kinematics

Red shift

Three momentum $|\vec{u}|$ gets **red** shifted

$$|\vec{u}| \sim \frac{1}{R(t)} \quad (5)$$

Definition of redshift z in Astronomy

$$1 + z = \frac{\lambda_0}{\lambda_1} = \frac{R(t_0)}{R(t_1)}$$



Standard Cosmology

2 a) Friedman equation

Equations of motion

Einstein - Hilbert action gives

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} + \Lambda g_{\mu\nu} \quad (7)$$

Matter stress energy tensor conserved

$$T^{\mu\nu}{}_{;\nu} = 0 \quad (8)$$

assume ideal fluid

$$T^{\mu}_{\nu} = \text{diag}(\rho, -\vec{p})$$



Standard Cosmology

2 a) Friedman equation

Equations of motion

Friedman equations

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad (10)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -8\pi Gp + \Lambda \quad (11)$$

equation of state

$$p = -\frac{d(\rho R^3)}{d(R^3)}$$



Standard Cosmology

2 b) Solutions of the Friedman equations

Friedman equations

Define critical density ρ_c and dimensionless density Ω_i

$$\rho_c = \frac{3H^2}{8\pi G} \quad , \text{ and } \quad \Omega_i = \frac{\rho_i}{\rho_c} \quad (13)$$

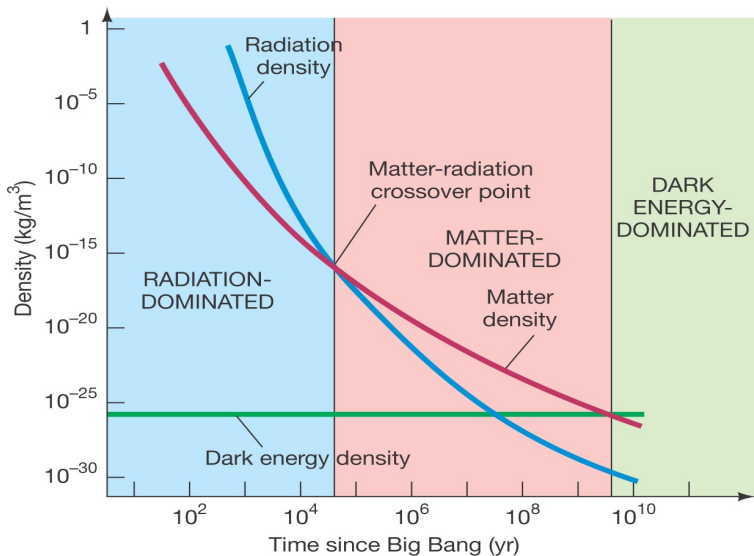
The universe is closed for $\Omega > 1$, flat for $\Omega = 1$, and open for $\Omega < 1$.
Also find

$$\rho = \begin{cases} \sim R^{-3} & \text{for matter} \\ \sim R^{-4} & \text{for radiation} \\ \sim \text{cte.} & \text{for } \Lambda \end{cases}$$



Standard Cosmology

2 b) Solutions of the Friedman equations



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Dark energy

3 a) First evidence

Hubble expansion measured for close Galaxies

$$H_0 = 72 \pm 8 \frac{\text{km}}{\text{s} \cdot \text{Mpc}} \quad (15)$$

Need standard source “candle” in order to translate Luminosity into distance.

$$d_L = \sqrt{\frac{L}{4\pi I}} \quad (16)$$

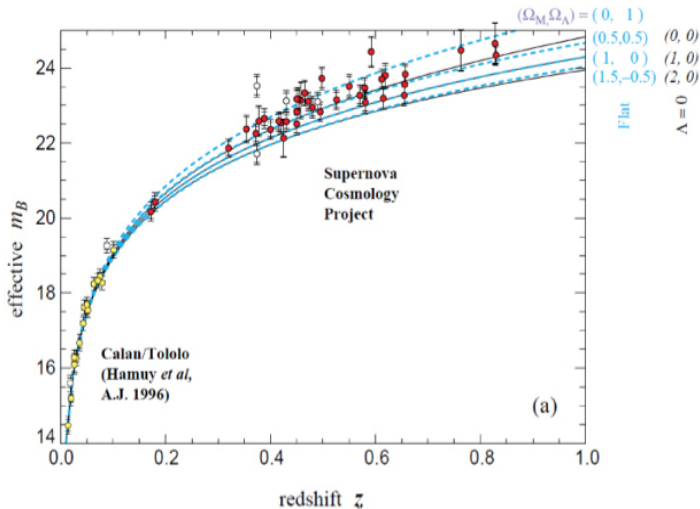
With the model for the universe

$$d_L(z, H_0, \Omega_m, \Omega_\Lambda) = \frac{1+z}{H_0} \int_0^z dz \frac{1}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}$$



Standard Cosmology

3 a) First evidence



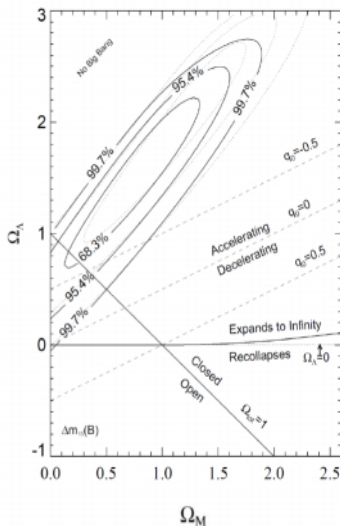
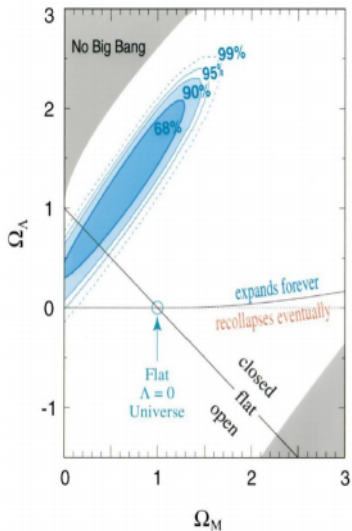
SN 1a is very good standard candle

Vary parameters Ω_Λ and Ω_m



Standard Cosmology

3 a) First evidence



Standard Cosmology

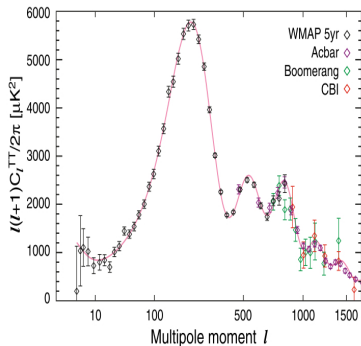
3 a) Further evidence

The Cosmic Microwave Background

Waves in early Baryon- γ plasma

$$d_0 \approx c_s t_S \quad (18)$$

evolve until today with $H(t)$

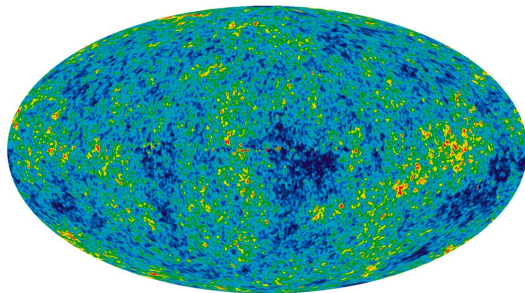


Standard Cosmology

3 a) Further evidence

The Baryon Acoustic Oscillations

Same waves in early Baryon plasma give density fluctuations $\Delta\rho$ which are seed for Galaxy formation.



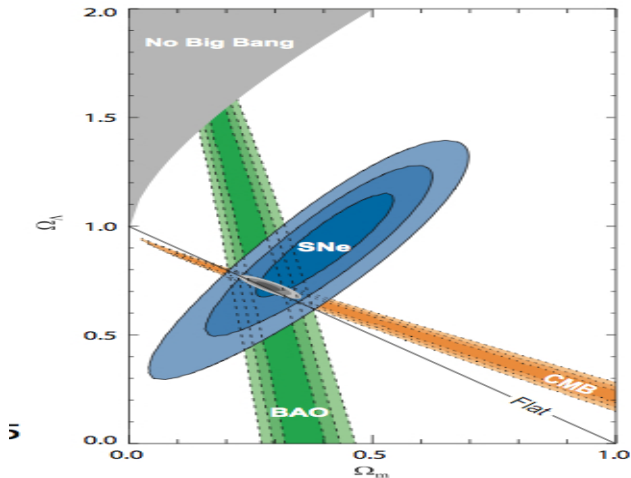
Fluctuations in CMB



Standard Cosmology

3 a) Further evidence

Putting the evidence together



Thank You!

