Theory and Phenomenology at and beyond the Terascale

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Santiago, September 2011

Outline

- The "Daedalus" problem of a unified theory
- Approach: Supersymmetry
- Approach: Large extra dimensions
- Approach: Exact renormalization
- Summary

The Daedalus Problem

The Daedalus Problem

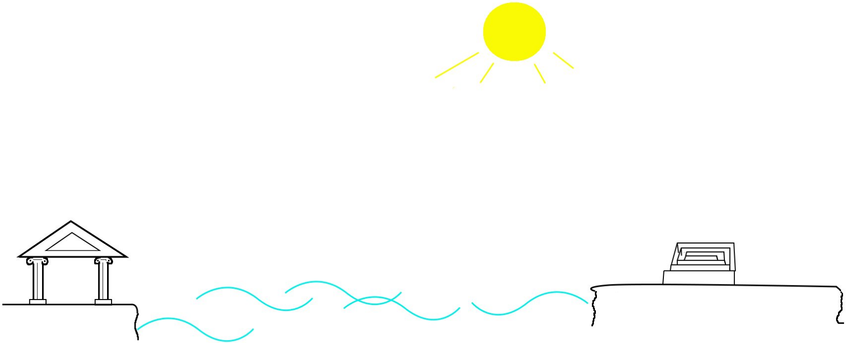
Greek Mythology

The Daedalus Problem

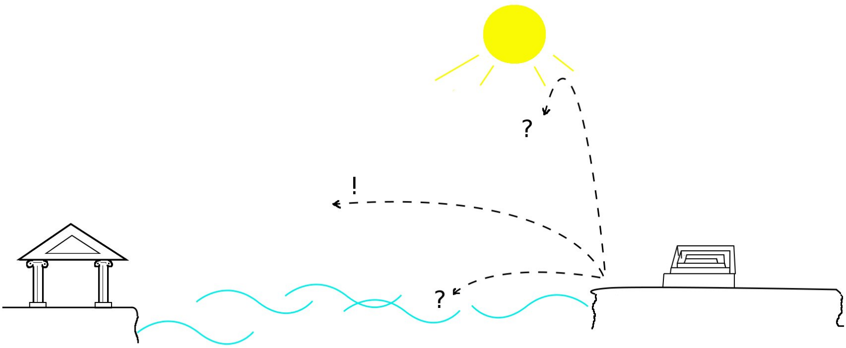
Greek Mythology

The Daedalus Problem

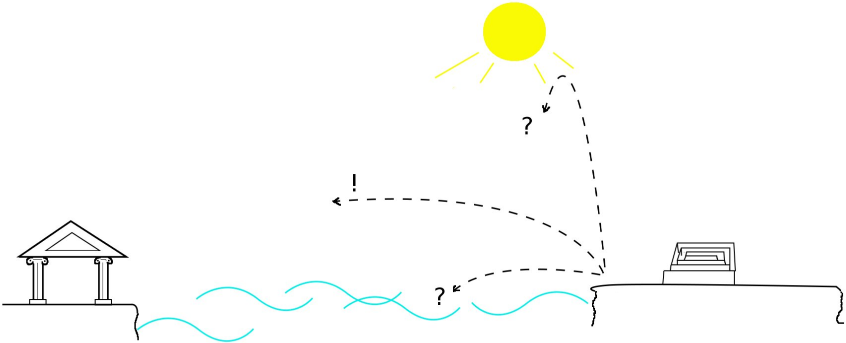
Greek Mythology



The Daedalus Problem
Greek Mythology



Where is the physics?



Where is the physics?

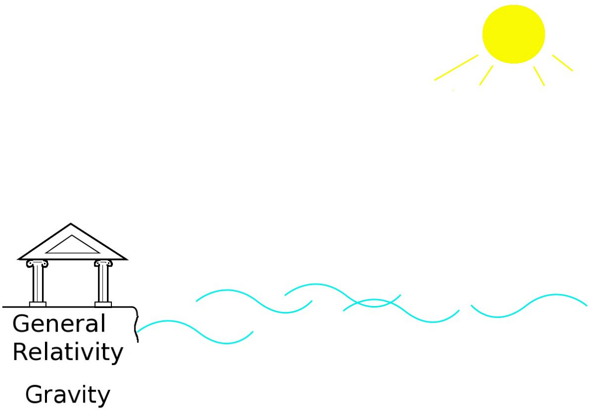
The Daedalus Problem

Physics Analogy

Mechanics

The Daedalus Problem

Physics Analogy



The Daedalus Problem

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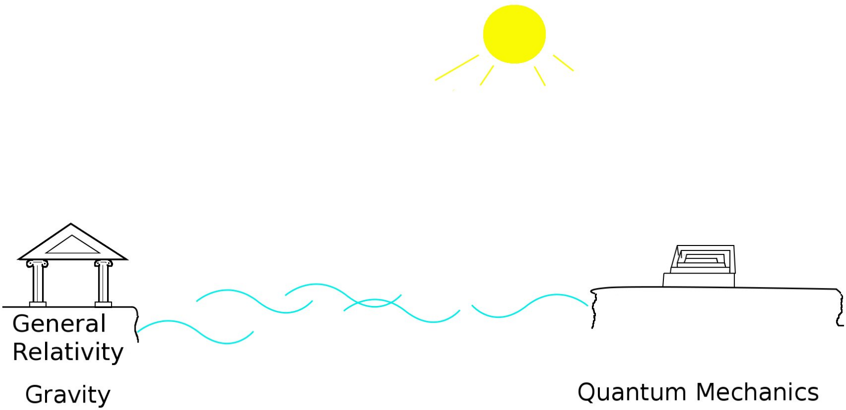

The Daedalus Problem

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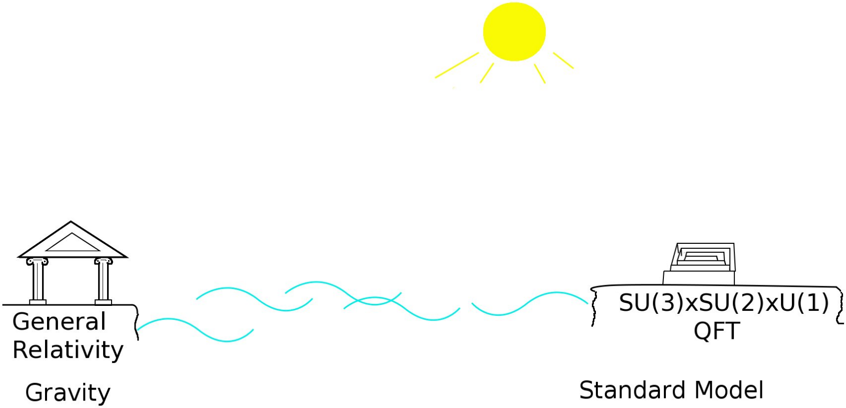

The Daedalus Problem

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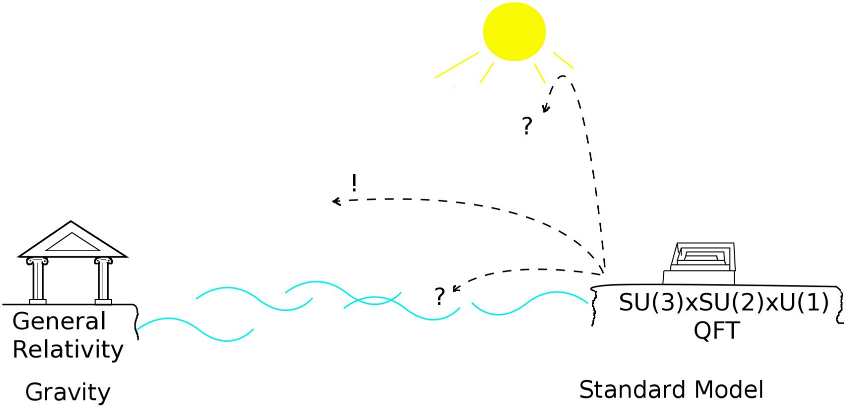

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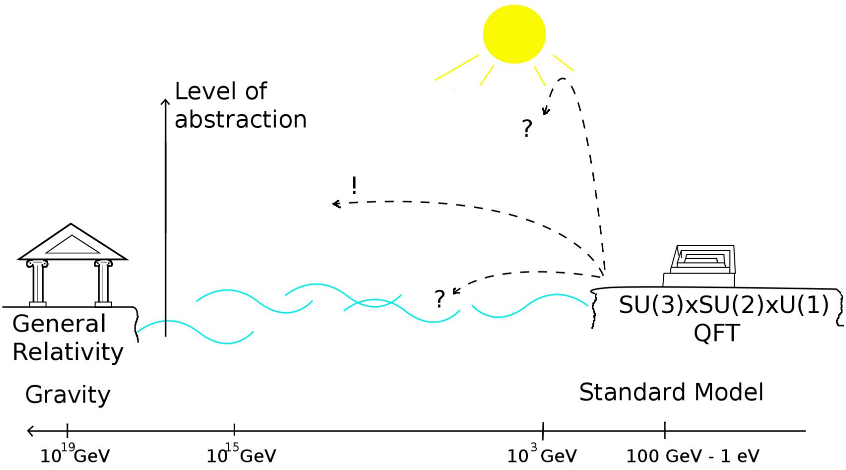

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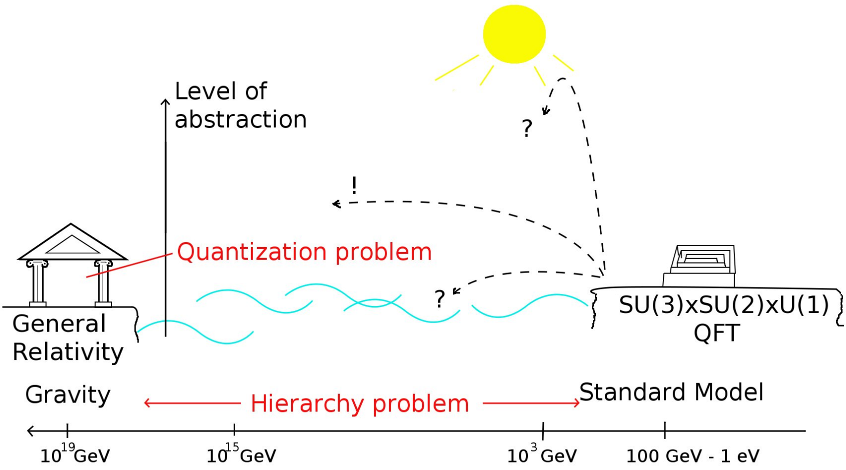

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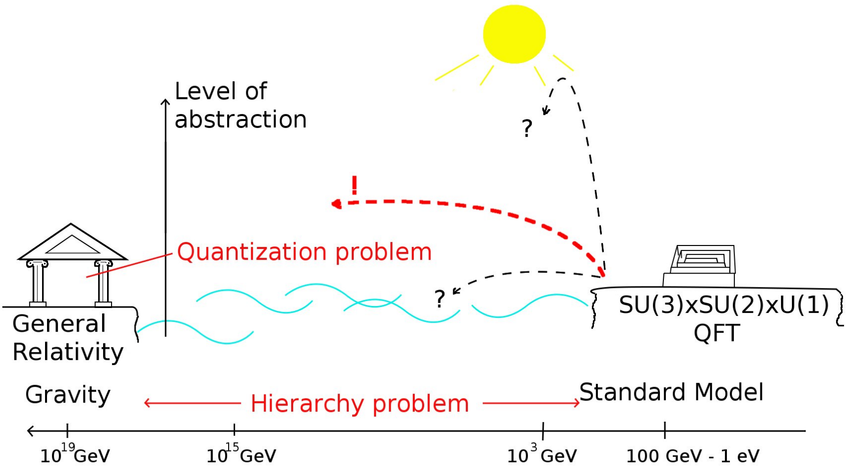
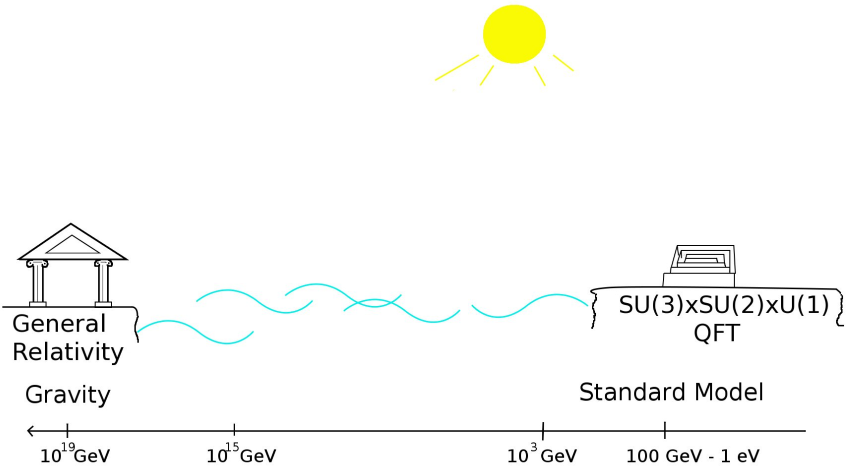


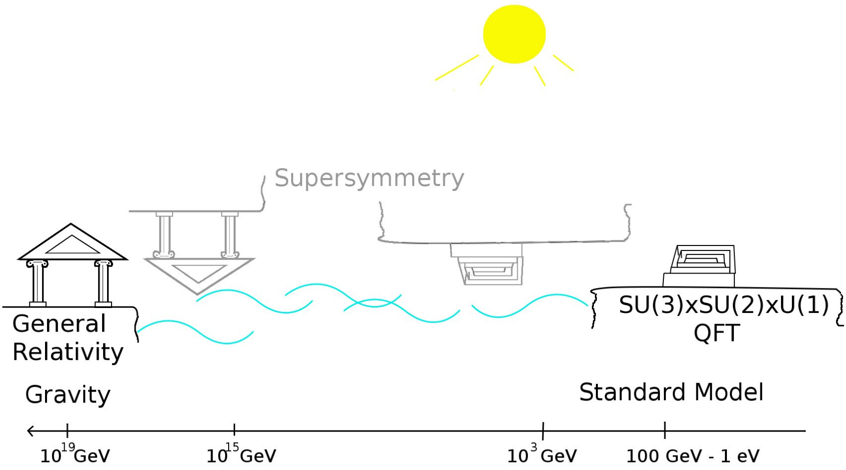
Points to remember

- Hierarchy problem
- Quantization problem
- Stay close to experiment

Approach: Supersymmetry



General Relativity
SU(3)xSU(2)xU(1) QFT
Gravity
Standard Model
10^19 GeV
10^15 GeV
10^3 GeV
100 GeV - 1 eV



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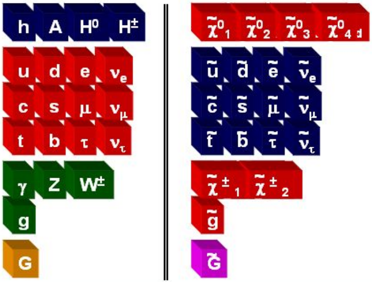
Possible symmetry between bosons and fermions

- Superpartners for known particles
- Improves renormalization behavior (alleviates quantization problem)
- Unifies SM couplings at 10^{15}GeV (alleviates hierarchy problem)
- Provides good dark matter candidates (experiment)
- Many predictions at TeV scale (experiment)



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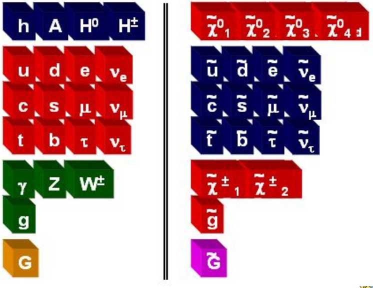

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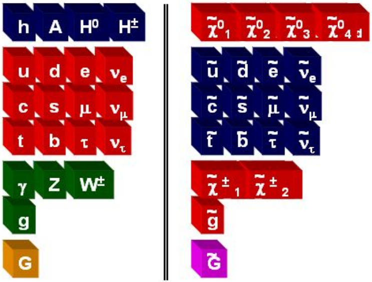

Supersymmetry

Susy General

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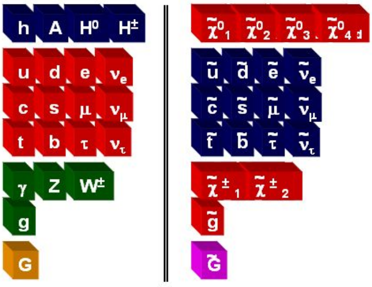

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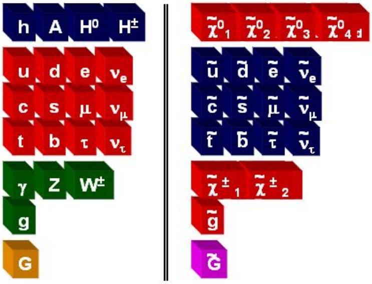
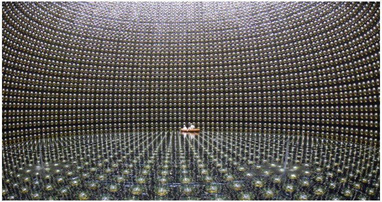


Neutrino oscillation experiments like Super Kamiokande [S1, S2]

Neutrino masses Δm_i
Mixing angles θ_i



[S1] M. A. Diaz, F. Garay, B. Koch, Phys.Rev.D80, 113005 (2009)

[S2] M. A. Diaz, B. Panes, B. Koch, Phys.Rev.D79, 113009 (2009)

Sattelite Fermi-LAT that measures cosmic rays [S3]

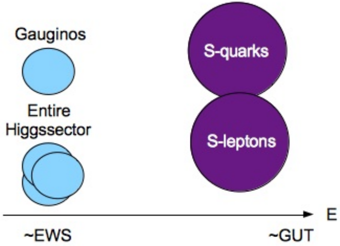
Cosmic γ-ray flux dJ/dE

[S3] M. A. Diaz, S. G. Saenz, B. Koch, Accepted for publication in PRD, (2011)

Supersymmetry: Our Contribution

Partial Split Supersymmetry

We used the model Partial Split Supersymmetry [∗, ∗∗]



[∗] M. A. Diaz, P. Perez, C. Mora, Phys. Rev. D **79**, 013005 (2009)

[∗∗] R. Sundrum, JHEP **1101**, 062 (2011)

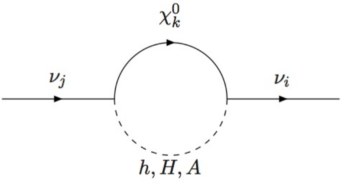
- S-quarks and S-leptons heavy
- Abandon Higgs naturalness
- Keep unification
- Solve proton decay
- Solve FCNC and CP violation

Possible violation of R parity

$$\mathcal{L}_{PSS}^{RpV} = -i\epsilon_i \widetilde{H}_u^T \sigma_2 L_i \; - \; \frac{i}{\sqrt{2}} b_i H_u^T \sigma_2 (\tilde{g}_d \sigma \widetilde{W} - \tilde{g}'_d \widetilde{B}) L_i \; + \; h.c., \quad (1)$$

Mixing of neutralinos induces neutrino mass matrix

At tree level not sufficient but at one loop level:

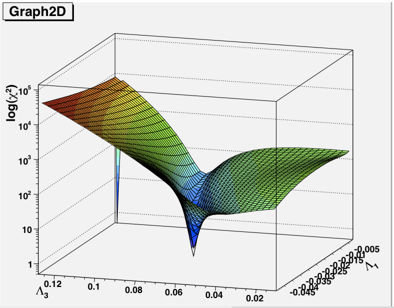


Neutrino mass matrix, where $\Lambda_i = \mu b_i v_u + \epsilon_i v_d$

$$M_{PSS}^{ij} = A\Lambda^i\Lambda^j + B(\epsilon^i\Lambda^j + \epsilon^j\Lambda^i) + C\epsilon^i\epsilon^j \quad (2)$$

Fits ν-masses and ν-angles:

[S2]

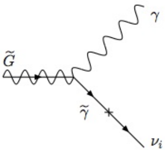


Supersymmetry: Our Contribution

Dark Matter Partial Split Susy → Gravitino

Two body decay:

(Dominant for $m_{3/2}$ small)

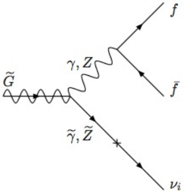


$$\Gamma(\tilde{G} \to \gamma\nu) = \frac{m_{3/2}^3}{32\pi M_P^2} |U_{\tilde{\gamma}\nu}|^2 \qquad (3)$$

with

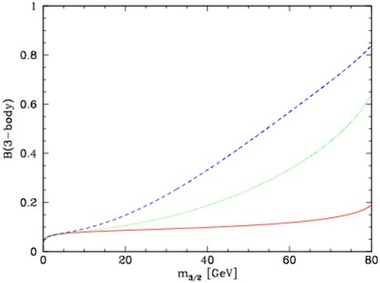
$$U_{\tilde{\gamma}\nu_i} \simeq \frac{\mu}{2(\det M_{\chi^0})} \left(\tilde{g}_d M_1 s_W - \tilde{g}'_d M_2 c_W\right) \Lambda_i$$

Three body decay:



$+ \ldots$

Branching ratio



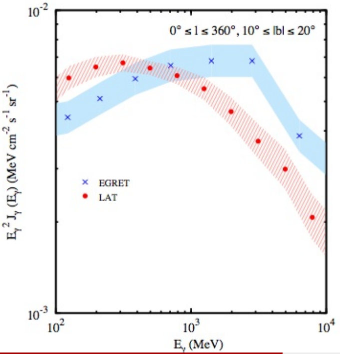
[S3]

Two body decay should induce photon flux

Flux from dark matter halo dominant: (where d_γ constant)

$$E^2 \frac{dJ_{halo}}{dE} = d_\gamma \Gamma(\widetilde{G} \to \gamma\nu) \frac{m_{3/2}}{2} \delta\left(E - \frac{m_{3/2}}{2}\right) \tag{4}$$

Compare to observed photon flux:



⇒ Constraint on gravitino lifetime

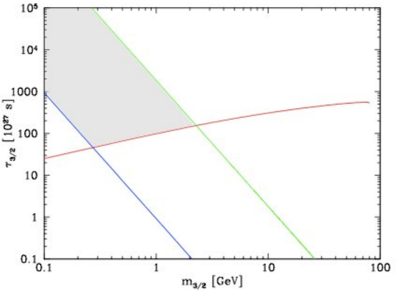
$$\left(\frac{\tau_{3/2}}{10^{27}\ \mathrm{s}}\right) > B\frac{0{,}851}{p}\left(\frac{m_{3/2}}{1\ \mathrm{GeV}}\right)^{0{,}41}$$

B: two body branching ratio
p: detector efficiency at $E = p_{3/2}$

← FermiLAT, PRL 103,251101(2009)

Demand:

- Reproduce all neutrino masses and mixings (blue-green)
- Dark matter $m_{3/2}$ that agrees with γ flux (red-∞)



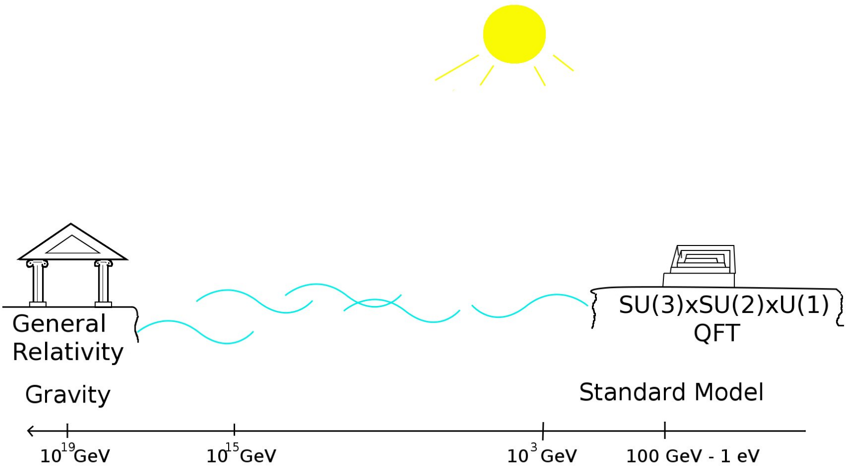
(a) *Allowed region for* $M_1 = 100$ *GeV.*

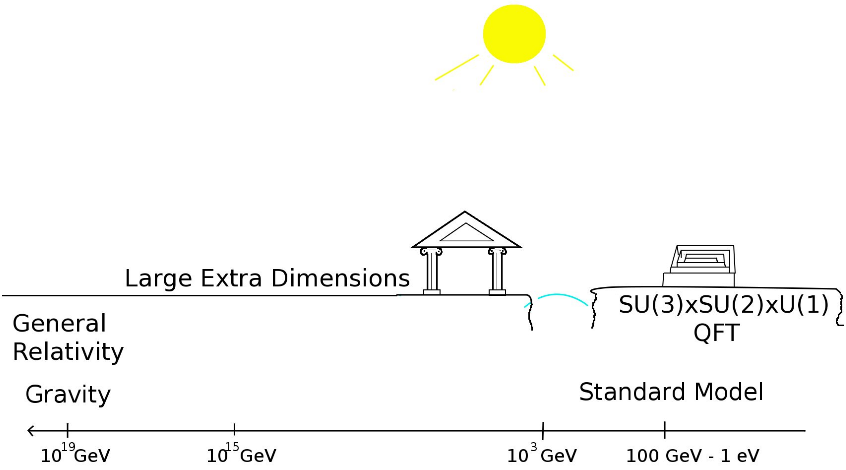
Maximal value for $m_{3/2}$ (Low) [s3]

Found surprising and testable prediction

Approach:
Large Extra Dimensions

Large Extra Dimensions
LXD General





Large Extra Dimensions
SU(3)xSU(2)xU(1)
QFT
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Idea:
Gravity looks weaker than it is. Hidden dimensions d cause this effect

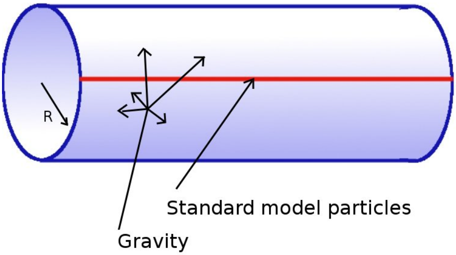
$$G_N = \frac{1}{M_{Pl}^2} \quad (5)$$

True gravity scale M_f in $d+4$ dimensions [*]

$$M_{Pl}^2 = M_f^{2+d} R^d \quad (6)$$

Allowed by experiment:

$$\begin{aligned} M_f &\geq 1{,}5\, TeV \quad (7) \\ R &\leq 1 \mu m \\ d &\geq 2 \end{aligned}$$



Also less simplistic models [**]

[*] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys.Lett. B436, 257-263 (1998)

[**] L. Randall, R. Sundrum, Phys.Rev.Lett. 83 3370-3373, 4690-4693 (1999)

If really

$$M_f \approx TeV \approx M_Z \approx 0{,}1\, TeV \tag{8}$$

- Explains hierarchy
- Does not solve quantization
- A lot of observables at $\sim$TeV

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LXD: Our Contribution

Link to Experiment 1

Spectrum of cosmic rays (like Auger observatory)

Predict deviation from expected high energy spectrum [X1]

[X1] B. Koch, H. Drescher, M. Bleicher, Astropart.Phys. 25, 291-297 (2006)

LXD: Our Contribution

Gravitational Radiation from Cosmic Rays

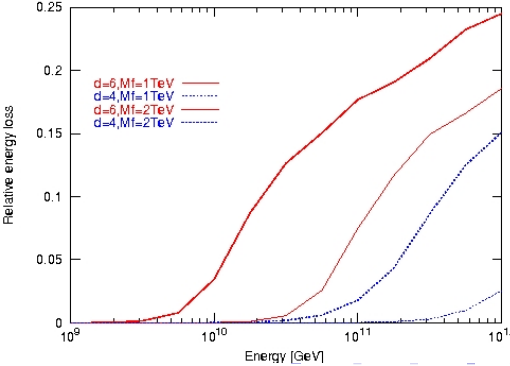
If $M_f \sim TeV$ elastic scattering of cosmic rays $\Rightarrow$ gravitational radiation

$$\frac{dE}{dk_d d\vec{k}} = \frac{t}{2^{d-1}\pi^{d/2}\Gamma(d/2)M_f^{d+2}} \frac{k_d^{d-2}\vec{k}^2(2k_d^2 + 3\vec{k}^2)}{(k_d^2 + \vec{k}^2)^2} \ . \tag{9}$$

Allows to calculate average relative energy loss

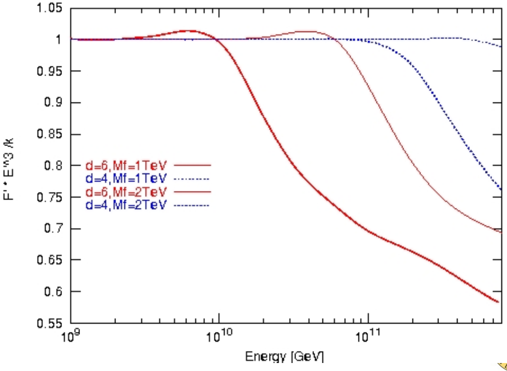
Using Glauber hadron profile

$$\frac{dE}{dx}(s,d) = \frac{\int_0^{\sqrt{s}/2} dt \, \frac{d\sigma^0_{\text{hA}}}{dt} E(t,s,d)}{\lambda \int_0^{\sqrt{s}/2} dt \, \frac{d\sigma^0_{\text{hA}}}{dt}} \tag{10}$$



Energy loss → missed in spectrum Monte Carlo

LXD's can provoke strong miss interpretation of actual cosmic ray flux. [x1]



LXD: Our Contribution

Link to Experiment 2

Large Hadron Collider (LHC) [X2, X3]

Cross sections $\frac{d\sigma}{dE d\Omega}$

event rates N_i [R2, R3]

[X2] T. Humanic, B. Koch, H. Stoecker, Int.J.Mod.Phys. E16, 841-852, (2007)

[X3] B. Koch, M. Bleicher, S. Hossenfelder, JHEP 0510, 053 (2005)

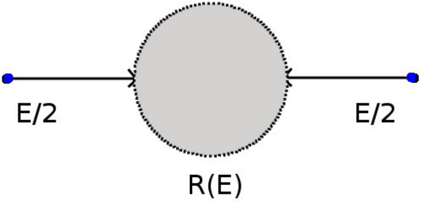
LXD: Our Contribution

Mini Black Holes

The energy of every collision defines event horizon (R_H black hole)

Usually very small
But LXD:

$$R_H^{d+1} = \frac{16\pi}{(d+2)A_{d+2}} \frac{M}{M_f^{d+2}} . \quad (11)$$



Can be large!
Integrate cross section
$\sigma(E) \approx R_H^2 \theta(E - M_f) \Rightarrow$
Possibly many black holes produced at TeV energy

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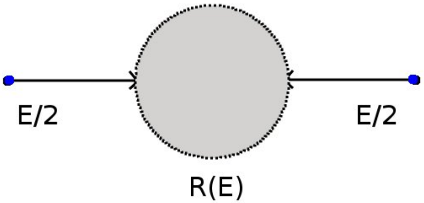

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Integrate cross section
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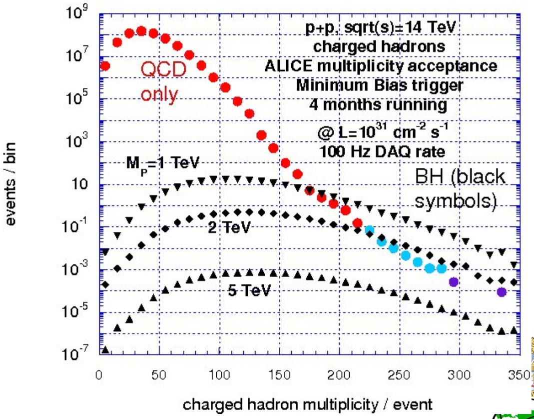
Possibly many black holes produced at TeV energy

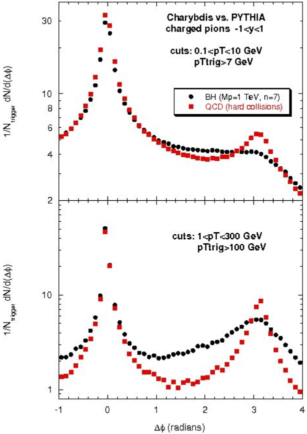
Analyzed observable: Multiplicity

Black holes radiate with low temperature
⇒ Higher multiplicities
in Monte Carlo simulation



Analyzed observable: Asymmetry

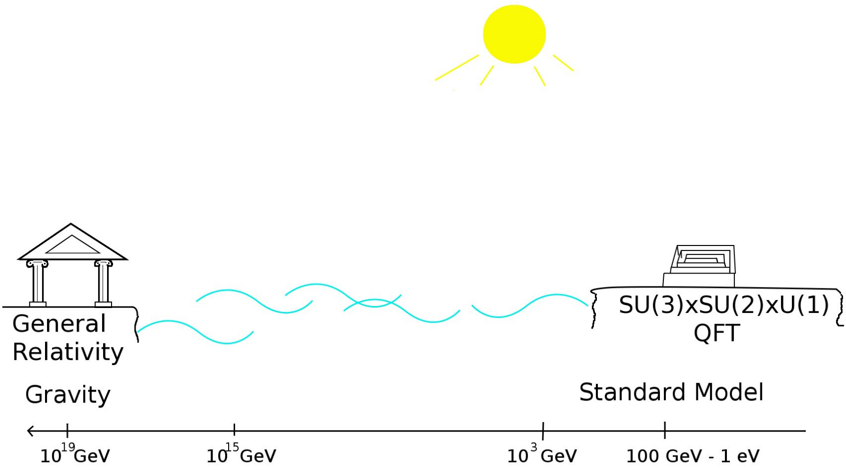
Black holes replace back to back jets $\Rightarrow$ crate angular asymmetry



Approach:
Exact Renormalization Group

Exact Renormalization Group

ERGE General



Exact Renormalization Group

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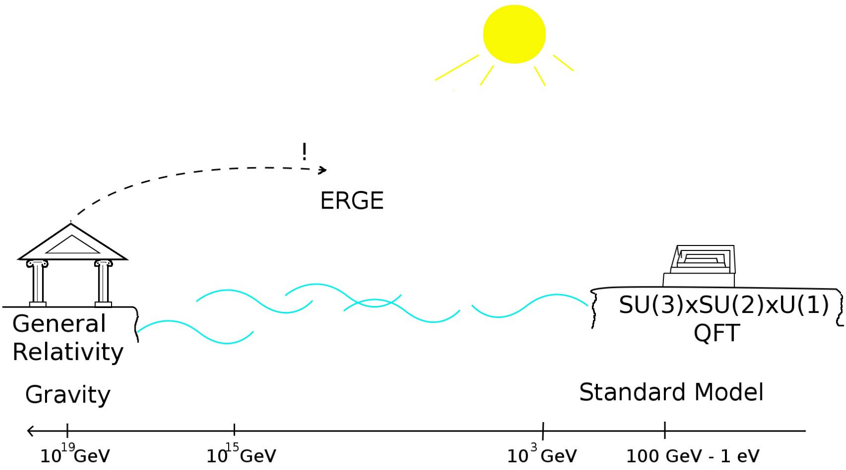

What exactly is the quantization problem?

"Gravity is not renormalizable"

What is renormalizable?

"Well ..."

Exact Renormalization Group

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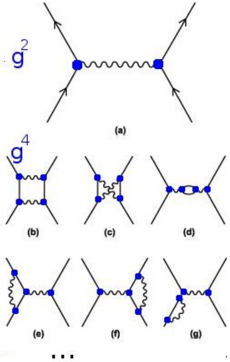
What is renormalizable?

Feynman method:
Power expansion in coupling g

$$\text{Result} = c_1 \cdot g^2 + c_2 \cdot g^4 \cdot \infty + \dots \qquad (12)$$

Problem ∞ canceled by N adjustments
(N=small for any order g^m)

$$\text{Result}' = c_1 \cdot g^2 + c_2' \cdot g^4 + \dots \qquad (13)$$



Gravity: $N_G \to \infty$ for $g \to \infty$

What is renormalizable?

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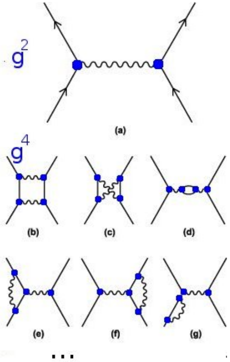

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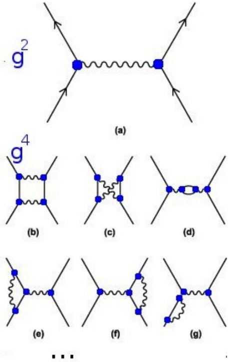

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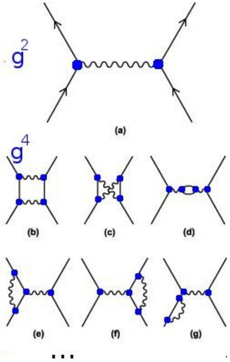
(N=small for any order g^m)

$$\text{Result}' = c_1 \cdot g^2 + c_2' \cdot g^4 + \dots \qquad (13)$$



Gravity: $N_G \to \infty$ for $g \to \infty$

Weinbergs Idea [*]

Maybe expansion wrong!
→ needs the whole functional $\Gamma[\psi]$?
(possible if there are UV-fixed points)

Wetterichs realization [**]

$$\partial_t \Gamma[\psi] = \frac{1}{2} Tr \left[\partial_t R_k ((\Gamma^{(2)}[\psi] + R_k)^{-1}) \right] \tag{14}$$

Flow equation where ψ are fields, $\Gamma^{(2)} = \delta^2\Gamma/\delta\psi^2$), $t = \ln(k)$, and R_k cut-off function.

⇒ running couplings

[*] S. Weinberg, "General Relativity" Cambridge University Press
[**] M. Reuter, C. Wetterich, Nucl.Phys. B417, 181 (1994)

Running gravitational couplings [*]

$$\begin{aligned} \beta_\lambda &= \partial_t \lambda_k = \frac{P_1}{P_2 + 4(d + 2g_k)} \\ \beta_g &= \partial_t g_k = \frac{2g_k P_2}{P_2 + 4(4 + 2g_k)} \end{aligned} \tag{15}$$

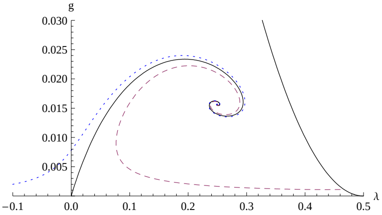
with the dimensionless couplings defined as

$$g_k = k^2 G_k \quad , \quad \lambda_k = \frac{\Lambda_k}{k^2} \tag{16}$$

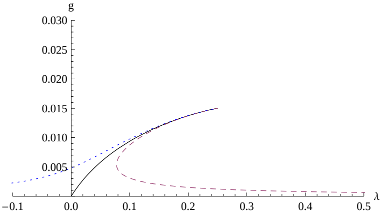
G_0: Newtons constant, Λ_0: Cosmological constant

[*] D. F. Litim, Phys. Rev. Lett. 92, 201301 (2004)

ERGE solutions:



Numerical solution of (15), [R1]



Analytical approximation of (15) using $g, \lambda \ll 1$, [R1]

We use analytical approximation

$$\lambda(g) = \frac{g^* \lambda^*}{g} \left((5+e)\left[1 - g/g^*\right]^{3/2} - 5 + 3g/(2g^*)(5 - g/g^*) \right)$$

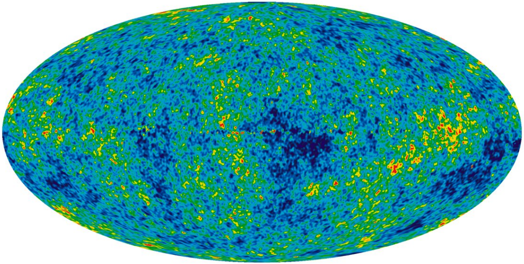
$$g(k) = \frac{k^2}{1 + k^2/g^*} \quad ,$$

With the UV fixed points λ^* and g^*

ERGE: Our Contribution

Link to Experiment 1

WMAP-satellite measured microwave temperature of the sky.



Variations of only $\frac{1}{100{,}000}$,
even for causally disconnected regions (horizon problem)

Explanation:

- Usually one invents new field “inflaton”
- We used ERGE [*R1*]

[*R1*] B. Koch, I. Ramirez, Class.Quant.Grav. 28, 055008 (2011)

ERGE: Our Contribution

Early Universe

Homogeneous background

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad . \tag{17}$$

Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{a_0^4\rho_r}{a^4} + \frac{a_0^3\rho_m}{a^3}\right) + \frac{\Lambda}{3} \tag{18}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left(\frac{a_0^4\rho_r}{a^4} + \frac{a_0^3\rho_m}{2a^3}\right) + \frac{\Lambda}{3} \quad . \tag{19}$$

- Works in late universe
- Fails in early universe (horizon problem)
- Other issues ...

Homogeneous background

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad . \tag{20}$$

Modified Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_k}{3}\left(\frac{a_0^4\rho_r}{a^4} + \frac{a_0^3\rho_m}{a^3}\right) + \frac{\Lambda_k}{3} - \frac{\kappa}{a^2} + \frac{\dot{G}_k \dot{a}}{G_k a} \quad , \tag{21}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G_k}{3}\left(\frac{a_0^4\rho_r}{a^4} + \frac{a_0^3\rho_m}{2a^3}\right) + \frac{\Lambda_k}{3} + \frac{\dot{G}_k \dot{a}}{2 G_k a} + \frac{G_k \ddot{G}_k - 2\dot{G}_k^2}{2 G_k^2} \tag{22}$$

- Works in late universe
- Good in early universe, solves horizon problem
- Shares other problems and open questions

ERGE: Our Contribution

Early Universe

UV solution of modified Friedmann equations

$$a = C \cdot t \qquad (23)$$

Implies that Hubble horizon

$$h_H = \frac{1}{t_f - t_i} \int_{t_f}^{t^i} \frac{c}{\dot{a}} = \frac{c}{C} \quad . \qquad (24)$$

is smaller than causal horizon

$$h_c = \int_{t_i}^{t_f} dt \frac{c}{a(t)} = \frac{c}{C} \left[\ln \left(\frac{t_f}{t_i} \right) \right] \quad . \qquad (25)$$

$h_C > h_H \Rightarrow$ Solves horizon problem

ERGE: Our Contribution

Link to Experiment 2

Large Hadron Collider (LHC)

Cross sections $\frac{d\sigma}{dEd\Omega}$
event rates N_i [R2, R3]

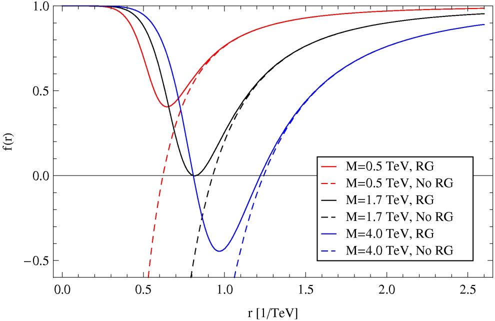
[**R2**] T. Burschil, B. Koch, JETP Lett. 92, 4 (2010)

[**R3**] B. Koch Phys.Lett. B. 663, 334 (2008)

Running fundamental scale M_f [∗]

$$\tilde{M}_f^{d+2}(k) = M_f^{d+2}\left[1+\left(\frac{k}{tM_f}\right)^{d+2}\right] \qquad (26)$$

Improve black hole solution



$$ds^2 = f(r)dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega_{d+2} \qquad (27)$$

[R2]

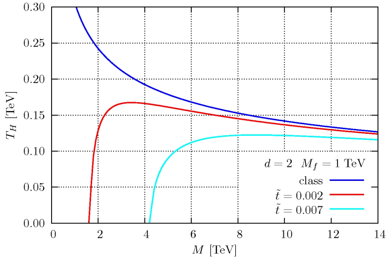
[∗] J. Hewett and T. Rizzo, JHEP 0712, 009 (2007)

ERGE: Our Contribution

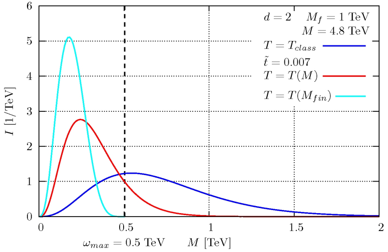
Black Holes in Extra Dimensions

Temperature [R2]

$$T_H = \frac{1}{4\pi}(\partial_r f(r))\Big|_{r=Horizon} \quad (28)$$



Radiation spectrum [R2]



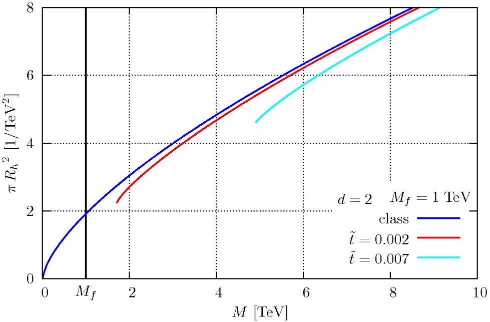
$$I(\omega, T_H) = N\frac{\omega^3}{\exp(\omega/T_H) + s} \quad , \quad (29)$$

$$M_{fin} = \sqrt{M^2 + m_\omega^2 - 2E_\omega M} \quad . \quad (30)$$

$$T_H = T_H(M_{fin}) \quad , \quad (31)$$

Cross section [R2, R3]

$$\tilde{\sigma}(\sqrt{s}) = \pi \tilde{R}_H^2 \theta(\sqrt{s} - M_c) \quad (32)$$

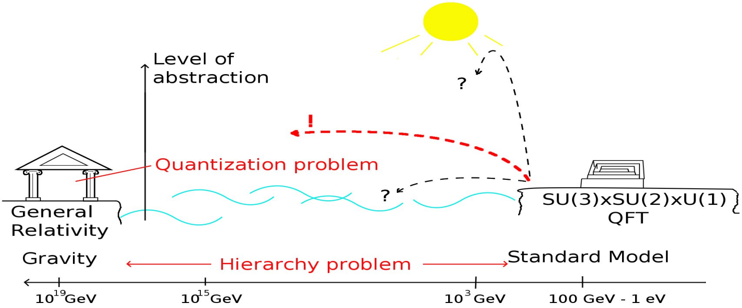


Black hole cross sections for $d = 2$ and $M_f = 1$ TeV, varying $\tilde{t}$

Much less black holes than in the usual estimate

Summary

- **Introduced problems of unification**
- Studied three different approaches
- Compared to various experiments
- Obtained predictions or limits



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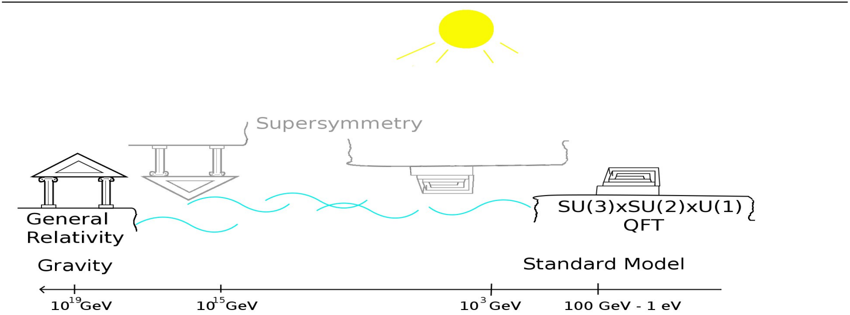

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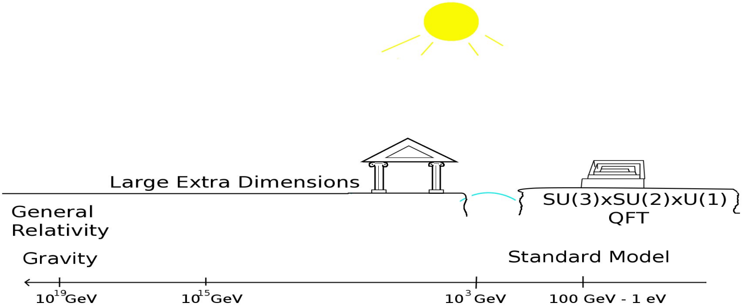

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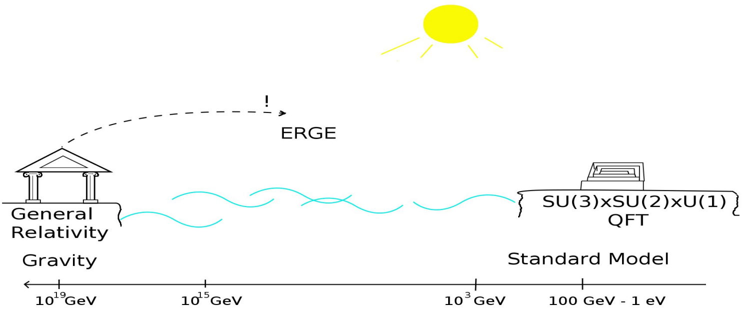

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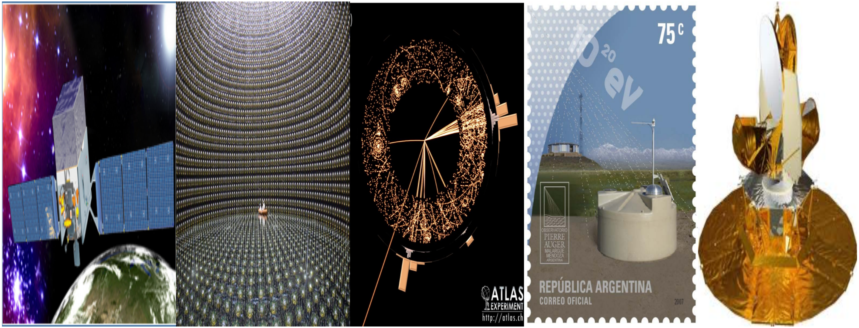

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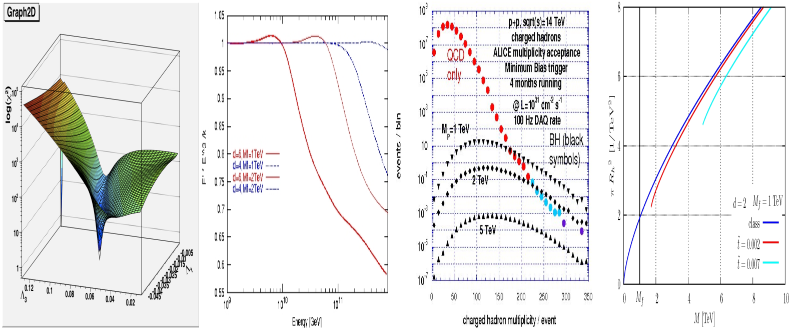


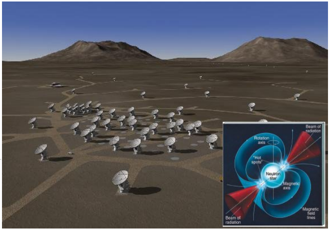
No prediction confirmed?

Yes one!

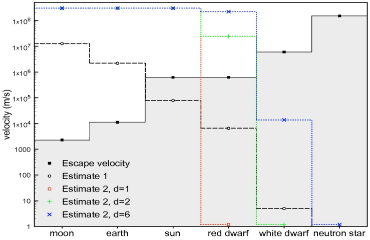
No prediction confirmed?

Yes one!

Using cosmic rays (Auger ...) & neutron stars (ALMA ...)



We found [*A1*]



⇒ Prediction:

Mini BHs are

- Not there or
- Not dangerous

[*A1*] B. Koch, M. Bleicher, H. Stoecker, Phys.Lett. B672, 71-76 (2009)

LHC runs since 2009

We are still here

Prediction confirmed!

LHC runs since 2009

We are still here

Prediction confirmed!

LHC runs since 2009

We are still here

Prediction confirmed!

Thank you!

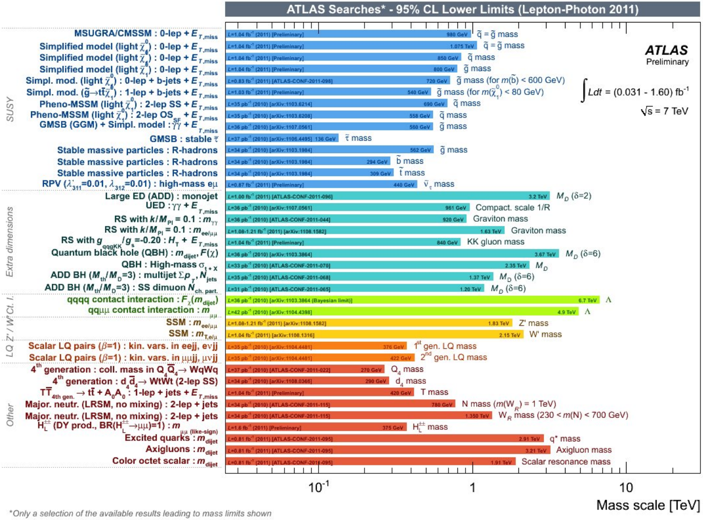
BBC news 27.08.2011*:

- ... LHC results put supersymmetry theory “on the spot”.
- ... simplest version of the theory has in effect bitten the dust.
- ... experts working in the field are “disappointed” by the results – or rather, the lack of them.
- and so on ...

What is behind that?

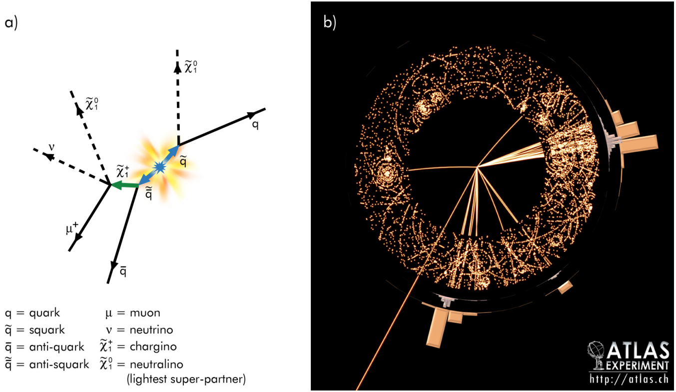
*http://www.bbc.co.uk/news/mobile/science-environment-14680570?SThisFB

Thousands of models ⇒ nature decides ⇒ ideally there is only one!



Large number of observables have been studied

Example:

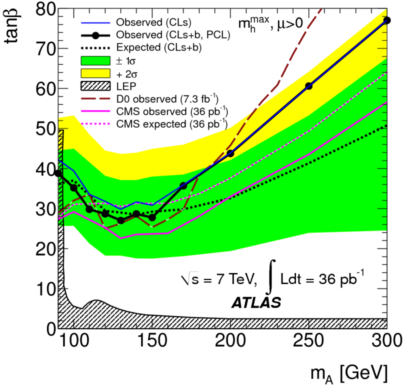


S-quark, anti s-quart production and observable at LHC

Backups: Supersymmetry

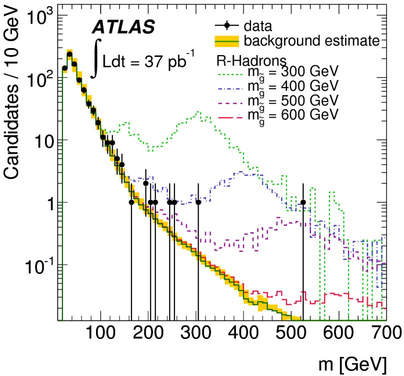
Results

Constraints on parameterspace for the MSSM Higgs sector



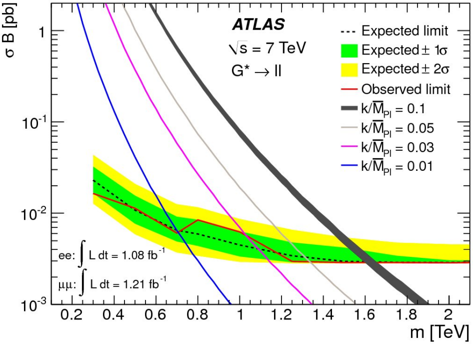
CERN-PH-EP-2011-104

Search for superpartners in the di-lepton channel



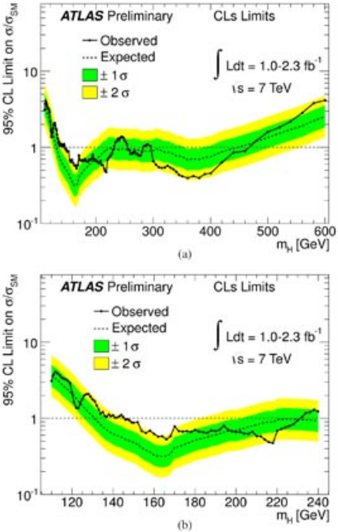
CERN-PH-EP-2011-077

Constraints on Randall Sundrom graviton mass for various values of $k/M_{\bar{P}l}$



CERN-PH-EP-2011-123

Constraints and evidence on SM Higgs



today on www.atlas.ch

Backups: Supersymmetry Our Contribution

Connecting Neutrino Model with Gravitino DM

Link due to neutrino photino mixing:

$$\underbrace{\Gamma(\widetilde{G} \to \gamma \nu)}_{\text{determines } \gamma\text{–flux}} \sim |U_{\widetilde{\gamma}\nu_i}|^2 \simeq \underbrace{\left(\frac{\mu}{2(\det M_{\chi^0})} \left(\tilde{g}_d M_1 s_W - \tilde{g}'_d M_2 c_W \right) \Lambda_i \right)^2}_{\text{parameters of neutrino model}}$$

Numerical parameter scan, values of $U_{\widetilde{\gamma}\nu_i}$

M_1	$\lvert U_{\widetilde{\gamma}\nu}\rvert^2$(min)	$\lvert U_{\widetilde{\gamma}\nu}\rvert^2$(max)
100 GeV	2×10^{-16}	4×10^{-13}
300 GeV	2×10^{-17}	3×10^{-14}
500 GeV	1×10^{-17}	1×10^{-14}