Black holes and running couplings

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collaboration with

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Outline

- Exact Renormalization Group for gravity (ERG)
- The ERG flow and "improved" black holes
- Going beyond "improved"
- Flow induced by new solution
- Status ... conclusion

Exact Renormalization Group: Motivation

ERG General

The quantization problem of gravity

What is the quantization problem?

"Gravity is not renormalizable"

What is renormalizable?

"Well ... ask Claus Kiefer"

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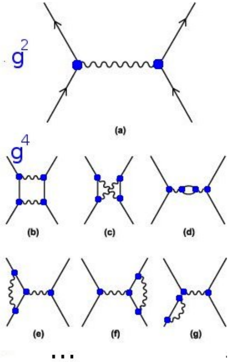
Exact Renormalization Group

ERG General

What is renormalizable?

Feynman method:
Power expansion in coupling g



$$\text{Result} = c_1 \cdot g^2 + c_2 \cdot g^4 \cdot \infty + \dots \qquad (1)$$

Problem ∞ canceled by N adjustments
(N=small for any order g^m)

$$\text{Result}' = c_1 \cdot g^2 + c_2' \cdot g^4 + \dots \qquad (2)$$

Gravity: $N_G \to \infty$ for $g \to \infty$

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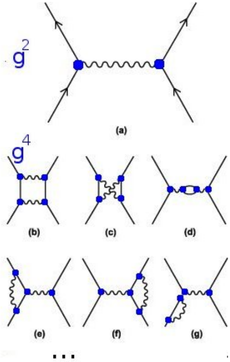

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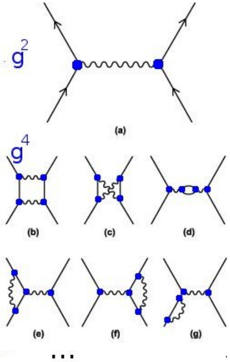

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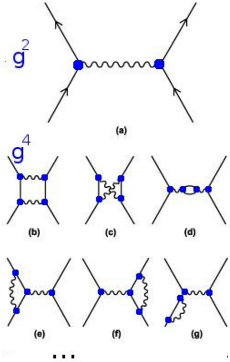


Gravity: $N_G \to \infty$ for $g \to \infty$

Weinbergs Idea [∗]

Maybe expansion wrong!
→ needs the whole functional $\Gamma[\psi]$?
Important: Existence of non-trivial UV-fixed points
(Issues → Ilya Shapiro)

Wetterichs realization [∗∗]

$$\partial_t \Gamma[\psi] = \frac{1}{2} Tr \left[\partial_t R_k \cdot (\Gamma^{(2)}[\psi] + R_k)^{-1} \right] \tag{3}$$

Flow equation where ψ are fields, $\Gamma^{(2)} = \delta^2\Gamma/\delta\psi^2$), $t = \ln(k)$, and R_k cut-off function.

⇒ running couplings

[∗] S. Weinberg, "General Relativity" Cambridge University Press

[∗∗] M. Reuter, C. Wetterich, Nucl.Phys. B417, 181 (1994)

Exact Renormalization Group

ERG for Gravity

Define dimensionless couplings

$$g_k = k^2 G_k \qquad \lambda_k = \frac{\Lambda_k}{k^2} \tag{4}$$

G_0: Newtons constant, Λ_0: Cosmological constant

With Wetterichs equation one can get running gravitational couplings [*]

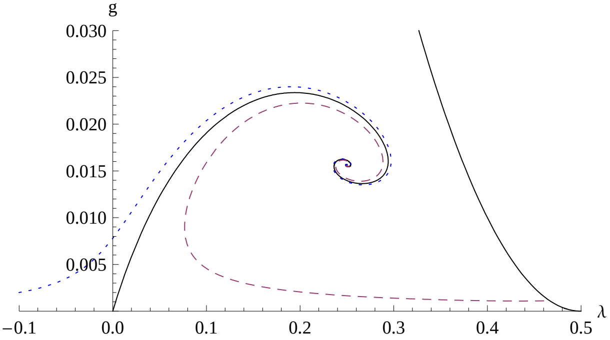
$$\begin{aligned} \beta_\lambda &= \partial_t \lambda_k = \frac{P_1}{P_2 + 4(d + 2g_k)} \\ \beta_g &= \partial_t g_k = \frac{2g_k P_2}{P_2 + 4(4 + 2g_k)} \end{aligned} \tag{5}$$

[*]Reuter ..., but here use: D. F. Litim, Phys. Rev. Lett. 92, 201301 (2004)

Exact Renormalization Group: Flow

ERG flow

Solve numerically:



Numerical solution of (5), [R1]

Would be nice to have analytical expression to work with ...

Exact Renormalization Group: Flow

ERG flow

Expand beta functions for small couplings $g, \lambda \ll 1$:

$$\beta_g = g(k)(2 - 24g(k)) \tag{6}$$

$$\beta_\lambda = 12g(k) - 2\lambda(k) \tag{7}$$

Solve

$$g_{ERG}(k) = \frac{k^2 G_0 g_U^*}{g_U^* + G_0 k^2} \tag{8}$$

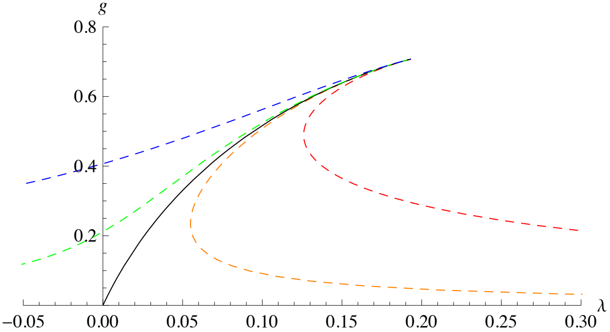
$$\lambda(k)_{ERG} = \lambda_U^* + \frac{1}{k^2}\Lambda_0 - \frac{g_U^* \lambda_U^*}{G_0 k^2} \text{Log}\left[\left(1 + G_0 \frac{k^2}{g_U^*}\right)\right] \tag{9}$$

with fixed points g_U^* and λ_U^* used as free parameters.

Exact Renormalization Group: Flow

ERG flow

Analytically approximated flow



Analytical approximation (8, 9)

Existing black hole studies

Take classical Schwarzschild solution

$$ds^2 = f(r)dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega_{d+2} \tag{10}$$

$$\text{with} \quad f(r) = 1 - \frac{2GM}{r^1} \; .$$

Remember, that coupling is scale dependent $G = G_k$

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$$\text{with} \quad f(r) = 1 - \frac{2G_k M}{r^1} \; .$$

Remember, that coupling is scale dependent $G = G_k$

Scale setting
intuition → something like 1/ distance

$$k(r) = \frac{\xi}{d(r)} \quad (12)$$

Something with r, M, G_0, usually [*]

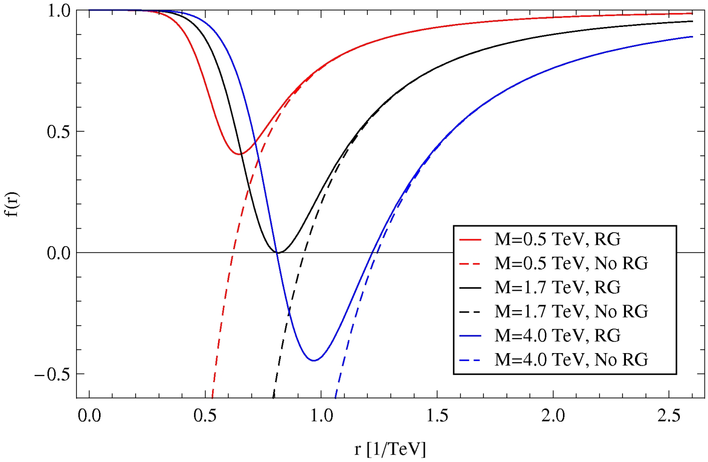
$$d_{(2)}(r) = \int_{\mathcal{C}_r} \sqrt{|ds^2|} \approx |_{UV} \frac{1}{R_H^{\frac{1}{2}}} \frac{2}{3} r^{\frac{3}{2}} \quad (13)$$

Put this into $f(r)$

[*] A. Bonanno and M. Reuter, Phys. Rev. D **62**, 043008 (2000) [arXiv:hep-th/0002196];

Black holes, improved solutions

ERG black holes



f(r) for different values of M [*]

Nice: no singularity, stable remnant, but no solution!

[*] T. Burschil, B. Koch, JETP Lett. 92 (2010) 193-199 [arXiv:0912.4517 [hep-ph]]

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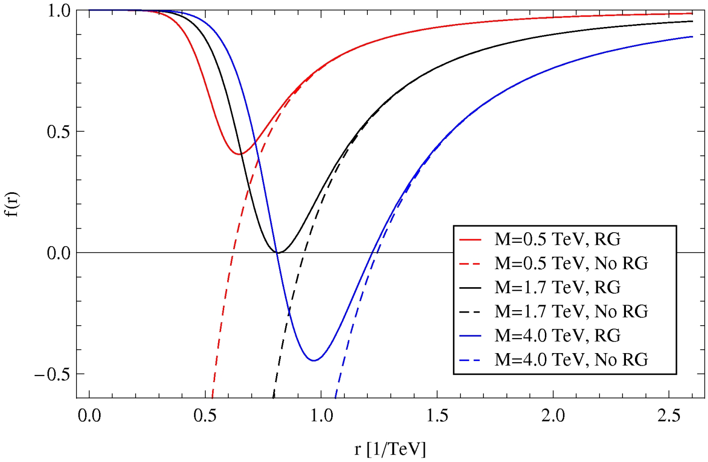

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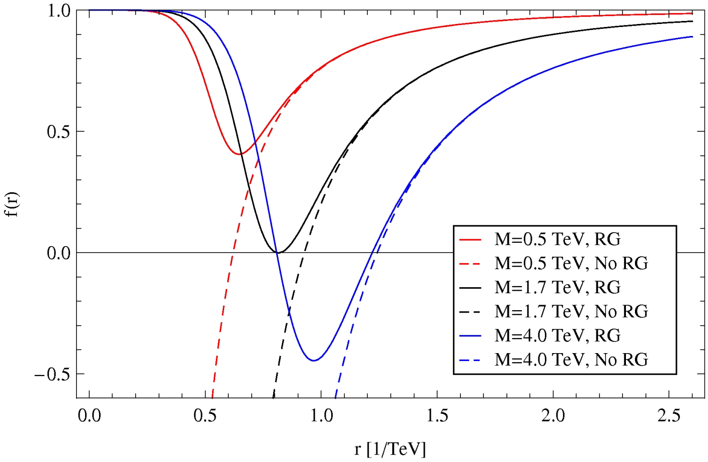

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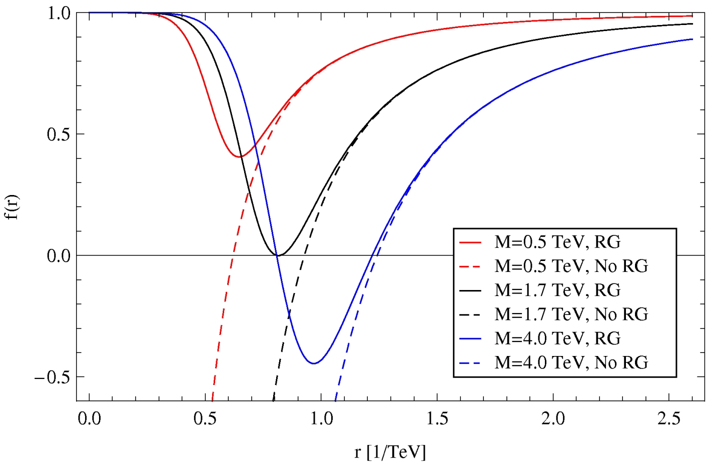

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No solution

Plug improved solution $f(r)$ into Einstein equations

$$G_{\mu\nu} \neq -g_{\mu\nu}\Lambda_k + 8\pi G_k T_{\mu\nu} \tag{14}$$

Why?
Because $G \to G_k \to G(r)$
Need take into account variable $G(r)$ when deriving the eoms,

$$\begin{aligned} G_{\mu\nu} &\neq -g_{\mu\nu}\Lambda_k + 8\pi G_k T_{\mu\nu} - \Delta t_{\mu\nu} \quad \text{with,} \\ \Delta t_{\mu\nu} &= G_k \left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu \right) \frac{1}{G_k} \end{aligned} \tag{15}$$

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New strategy

Black hope physics

(Andrey Zelnikov)

Black holes, beyond improved solutions

ERG black holes

New strategy

Black **hope** physics

(Andrey Zelnikov)

New strategy

Find improved black hole solution, that really is a solution

$$
\begin{aligned}
G_{\mu\nu} &= -g_{\mu\nu}\Lambda_r + 8\pi G_r T_{\mu\nu} - \Delta t_{\mu\nu} \quad \text{with,} \qquad (16)\\
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\end{aligned}
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A priory nothing to do with ERG

But: Can compare this $g(r)$, $\lambda(r)$ to ERG g_k, λ_k

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Equations of motion

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Ansatz

$$ds^2 = -f(r)dt^2 + 1/f(r)dr^2 + r^2 d\theta^2 + r^2 \sin(\theta) d\phi^2 \qquad (18)$$

where

$$f(r) = (1 - 2\frac{MG(r)}{r} - \frac{l(r)}{3} r^2). \qquad (19)$$

Solution

$$\begin{aligned} G(r) &= -\frac{16\pi c_2}{r-2c_1} \\ \Lambda(r) &= \frac{-1}{12r(r-2c_1)^2c_1^3}\left\{\left(c_1^2\left(12c_1^2+384\Sigma\pi c_2+c_3\right)+24r^3c_1^3c_4+\dots\right\}\right. \\ l(r) &= c_4+\frac{1}{48c_1^4}\left\{\frac{576\Sigma\pi c_1 c_2}{r-2c_1}+\frac{8c_1^3\left(12c_1^2+96\Sigma\pi c_2+c_3\right)}{r^3}+\dots\right\} \end{aligned} \quad (20)$$

Four constants of integration c_1, c_2, c_3, c_4 & time rescaling $t->qt$ to "play" with

Calculate invariant quantity

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{144c_1^4 + 9216\Sigma\pi c_1^2 c_2 + 147456\Sigma^2\pi^2 c_2^2 + 24c_1^2 c_3 + 768\Sigma\pi c_2 c_3 + c_3^2}{27c_1^2 r^6} + \mathcal{O}(r^{-5}) \qquad (21)$$

Singularity persists like Schwarzschild, but can choose

$$\hat{c}_2 = -\frac{12c_1^2 + c_3}{384\Sigma\pi} \qquad (22)$$

Singularity improves

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}|_{\hat{c}_2} = \frac{2}{c_1^2 r^2} + \mathcal{O}(r^{-1}) \qquad (23)$$

Black holes, beyond improved solutions

Singularity

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}|_{\hat{c}_2} = \frac{2}{c_1^2 r^2} + \mathcal{O}(r^{-1}) \tag{24}$$

further choose

$$\hat{c}_1 = c_1 \to \infty \tag{25}$$

Finite tensor density

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}|_{\hat{c}_2,\hat{c}_1} = \frac{8}{3}c_4^2 \tag{26}$$

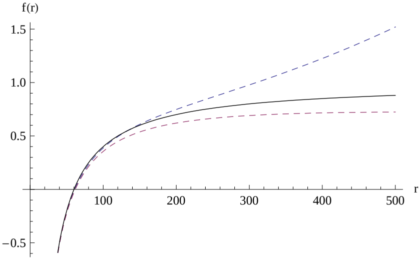
but simple metric

$$f(r)|_{\hat{c}_2,\hat{c}_1} = 1 - \frac{c_4}{3}r^2 \tag{27}$$

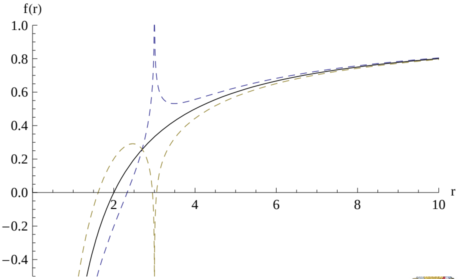
$\Rightarrow$ Either boring or singularity persists

At least learned that c_4 something to do with Λ_0

Reproduce Newtons law in certain regimes



g_{00} reproducing Newton for small r



g_{00} reproducing Newton for large r

This is for g_{00} rescaled $t- > B \cdot t$ but g_{11} is problematic

Hubu ... we have a problem

Problem with lensing

g_{11} problem:
Ignore Λ should get Brans Dicke metric *

$$ds^2 = (1 - 2\frac{MG}{r} + \frac{3M^2G^2}{2r^2} + \dots)dt^2 - (1 + \frac{MG}{r} + \dots)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{28}$$

Ignore in our solution c_4 and expand in $1/r$

$$ds^2 = (1 - 2\frac{MG}{r} + \frac{3M^2G^2}{2r^2} + \dots)dt^2 - (2 + 2\frac{MG}{r} + \dots)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{29}$$

Bad for geodesics and gravitational lensing of relativistic trajectories

* Weinberg, Gravitation and Cosmology

Try to find $c_4 \neq 0$ such that get approximately classically (confirmed) solution

$$f_s(r) = 1 - 2\frac{G_0 M_0}{r} - r^2 \frac{\Lambda_0}{3} \tag{30}$$

Demand for new solution

$$f(r_m) = 1 \quad \text{and} \quad f'|_{r_m} = 0 \tag{31}$$

and appromxiate horizons

$$r_0 \approx 2 G_0 M_0 \quad \text{and} \quad r_1 \approx \sqrt{\frac{3}{\Lambda_0}} \tag{32}$$

with $G(r) \approx G_0$ for $r_1 \gg r$

Hubu ... we have a problem

Solve Problem with lensing

four conditions, allow to fix four constants

$$c_{4,s} = \frac{12c_1^2 + \frac{4\sqrt{3}\tilde{c}_3 c_1}{\sqrt{\tilde{c}_3+12c_1^2}} + \frac{16\sqrt{3}c_1^3}{\sqrt{\tilde{c}_3+12c_1^2}} - \tilde{c}_3 \ln[3] - \tilde{c}_3 \ln\left[\tilde{c}_3 + 12c_1^2\right] + 2\tilde{c}_3 \ln\left[-6c_1 + \sqrt{3}\sqrt{\tilde{c}_3 + 12c_1^2}\right]}{32c_1^4} \tag{33}$$

where $\tilde{c}_3 = c_3 + 382\pi\Sigma c_2$.

$$c_{1,s} = \frac{3^{2/3}}{4(2G_0M_0\Lambda_0^2)^{1/3}} \tag{34}$$

$$c_{3,s} = \frac{12\cdot 6^{2/3}G_0(G_0M_0)^{2/3}(-4\Sigma + 3M_0)\Lambda_0^{4/3} - 9\cdot 6^{1/3}\left(G_0M_0\Lambda_0^2\right)^{1/3}}{8G_0M_0\Lambda_0^2} \tag{35}$$

$$c_{2,s} = \frac{G_0}{32\pi(2G_0M_0\Lambda_0^2/9)^{1/3}} \tag{36}$$

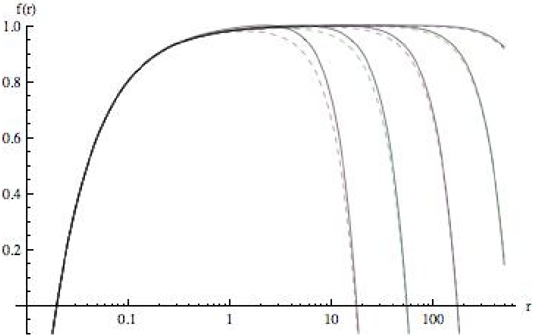
Hubu ... we have a problem
Solve Problem with lensing

This gives

$$f(r) = \quad ugly\ldots \tag{37}$$

but approximately

$$f(r) = 1 - 2\frac{G_0 M_0}{r} + \mathcal{O}\left(\Lambda_0^{2/3}\right) \tag{38}$$



Shows, existence of parameter choices that are in agreement with all experiments that confirm classical tests.

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$$\Rightarrow$$

Confidence to continue with studying couplings

Induced coupling flow

ERG black holes

Have dimensionfull couplings G, Λ and integration constants c_i
Want dimensionless expressions

First redefine 4 dimensionless integration constants

$$\left.\begin{array}{lcl} c_1 & = & -\frac{g_I}{2g_U^*\Sigma} \\ c_2 & = & -\frac{g_I}{16\Sigma^3\pi} \\ c_3 & = & \frac{3g_I(8g_U^{*3}-g_I g_U^*+2g_I^2\lambda_U^*)}{g_U^{*3}\Sigma^2} \\ c_4 & = & -\frac{\Sigma^2 l_I}{2} \end{array}\right\} \leftrightarrow \left\{\begin{array}{lcl} \lambda_U^* & = & -\frac{12c_1^2+c_3+384c_2\Sigma\pi}{48c_1^3\Sigma} \\ l_I & = & -\frac{2c_4}{\Sigma^2} \\ g_U^* & = & \frac{8c_2\Sigma^2\pi}{c_1} \\ g_I & = & -16c_2\Sigma^3\pi \end{array}\right.$$

Metric reads

$$f(r) = \frac{1}{6g_I^2 g_U^{*2}\Sigma r}\Big\{g_I\left(-6g_U^{*3}\Sigma^2 r^2+4g_I^3\lambda_U^*-6g_I^2 g_U^*\Sigma r\lambda_U^*+g_I g_U^{*2}\Sigma r\left(6+\Sigma^2 r^2 l_I+12\Sigma r\lambda_U^*\right)\right)$$
$$+6g_U^{*3}\Sigma^3 r^3(g_U^*-2g_I\lambda_U^*)\mathrm{Log}\left[\frac{g_I}{g_U^*\Sigma r}+1\right]\Big\}$$

Note: Now only r and Σ have scale dimension

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Want dimensionless expressions

First redefine 4 dimensionless integration constants

$$\left.\begin{array}{lcc} c_1 & = & -\frac{g_I}{2 g_U^* \Sigma} \\ c_2 & = & -\frac{g_I}{16 \Sigma^3 \pi} \\ c_3 & = & \frac{3 g_I (8 g_U^{*3} - g_I g_U^* + 2 g_I^2 \lambda_U^*)}{g_U^{*3} \Sigma^2} \\ c_4 & = & -\frac{\Sigma^2 l_I}{2} \end{array}\right\} \leftrightarrow \left\{\begin{array}{lcc} \lambda_U^* & = & -\frac{12 c_1^2 + c_3 + 384 c_2 \Sigma \pi}{48 c_1^3 \Sigma} \\ l_I & = & -\frac{2 c_4}{\Sigma^2} \\ g_U^* & = & \frac{8 c_2 \Sigma^2 \pi}{c_1} \\ g_I & = & -16 c_2 \Sigma^3 \pi \end{array}\right.$$

Metric reads

$$\begin{aligned} f(r) \quad = \quad & \frac{1}{6 g_I^2 g_U^{*2} \Sigma r} \left\{ g_I \left(-6 g_U^{*3} \Sigma^2 r^2 + 4 g_I^3 \lambda_U^* - 6 g_I^2 g_U^* \Sigma r \lambda_U^* + g_I g_U^{*2} \Sigma r \left(6 + \Sigma^2 r^2 l_I + 12 \Sigma r \lambda_U^* \right) \right) \right. \\ & \left. + 6 g_U^{*3} \Sigma^3 r^3 (g_U^* - 2 g_I \lambda_U^*) \mathrm{Log}\left[\frac{g_I}{g_U^* \Sigma r} + 1 \right] \right\} \end{aligned} \quad (39)$$

Note: Now only r and Σ have scale dimension

Induced coupling flow
ERG black holes

Have dimensionfull couplings G and Λ
want dimensionless couplings

$$g(r) = k^2 G(r); \quad \lambda(r) = \frac{\Lambda(r)}{k^2} \tag{40}$$

Problem: k^2 could be any adequate combination of physical scales r, Σ?
Parametrize this freedom by parameters a, c

$$g(r) = G(r)\frac{\Sigma^2}{(\Sigma r)^a} \tag{41}$$

$$\lambda(r) = \Lambda(r)\frac{(\Sigma r)^c}{\Sigma^2}$$

Interested in non-trivial UV fixed points $\Rightarrow$ choose a, c

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Induced coupling flow

ERG black holes

One finds a non-trivial UV fixed point only if $a = 0$ $c = 1$:

$$\begin{aligned} g_U(r \to 0) &= g_U^* \\ \lambda_U(r \to 0) &= \lambda_U^* \end{aligned}$$

Justifies notation of the previously chosen dimensionless parameters.

Induced coupling flow

ERG black holes

Dimensionless couplings after redefinition:

$$g_U(r) = \frac{g_I}{\left(\frac{g_I}{g_U^*} + \Sigma r\right)} \tag{42}$$

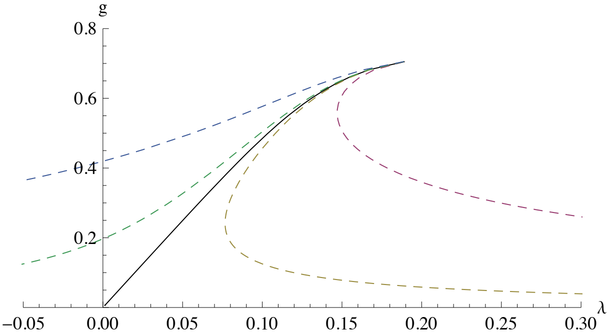
$$\begin{aligned} \lambda_U(r) &= \frac{1}{2g_I^2(g_I + g_U^*\Sigma r)^2} \Big\{ g_I \left(g_I^3(\Sigma r l_I + 2\lambda_U^*) - 12 g_U^{*3}\Sigma^2 r^2 + 3 g_I^2 g_U^* \Sigma r (\Sigma r l_I + 8\lambda_U^*) \right) \\ &+ g_I^2 g_U^{*2} \Sigma r \left(2\Sigma^2 r^2 l_I - 11 + 24 \Sigma r \lambda_U^* \right) + 6 g_U^* \Sigma r \left(g_I^2 + 3 g_I g_U^* \Sigma r + 2 g_U^{*2} \Sigma^2 r^2 \right) (g_U^* - 2 g_I \lambda_U^*) \ln\left[\frac{g_I}{g_U^* \Sigma r} + 1 \right] \Big\} \end{aligned} \tag{43}$$

Parametric plot of these functions

Induced coupling flow

ERG black holes

Obtain flow for an UV fixed point



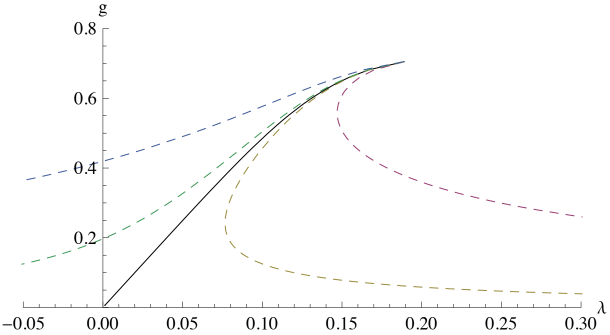
BH induced flow

Wow, that looks familiar ⇒ compare to ERG

Induced coupling flow

ERG black holes

Obtain flow for an UV fixed point



BH induced flow

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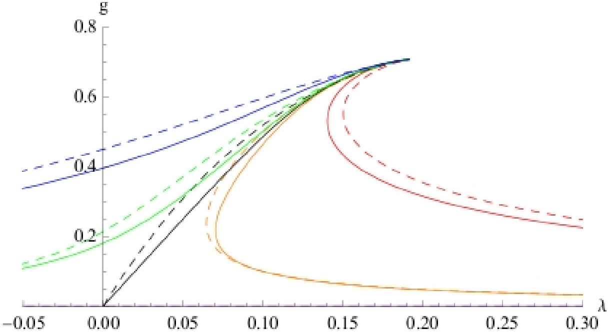
Induced coupling flow

ERG and BH induced comparison

Compare flow from BH solution and from ERG approach



BH induced: solid line, ERG: dashed line

Looks so good, compare analytically

First gravitational coupling

ERG:

$$g_{ERG}(k) = \frac{k^2 G_0 g_U^*}{g_U^* + G_0 k^2} \tag{44}$$

BH induced:

$$g_U(r) = \frac{g_I}{\left(\frac{g_I}{g_U^*} + \Sigma r\right)} \tag{45}$$

Perfect match for scale setting

$$r \equiv \frac{g_I}{k^2 G_0 \Sigma} \tag{46}$$

First gravitational coupling

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Perfect match for scale setting

$$r \equiv \frac{g_I}{k^2 G_0 \Sigma} \quad (46)$$

Induced coupling flow

ERG and BH induced comparison

Cosmological constant (using scale setting):

ERG:

$$\lambda(k)_{ERG} = \lambda_U^* + \frac{1}{k^2}\Lambda_0 - \frac{g_U^*\lambda_U^*}{G_0 k^2}\text{Log}\left[\left(1 + G_0\frac{k^2}{g_U^*}\right)\right] \quad (47)$$

BH induced and for small fixed points $\lambda_U^*, g_U^* \ll 1$:

$$\lambda_U(k)|_{UV} \approx \lambda_U^* + \frac{1}{k^2}\frac{g_I l_I}{2G_0} \quad (48)$$

$$-\frac{g_U^*\lambda_U^*}{G_0 k^2}\frac{(6g_I - 3\frac{g_U^*}{\lambda_U^*})}{g_I}\text{Log}\left[\left(1 + G_0\frac{k^2}{g_U^*}\right)\right]$$

Also complete match for $g_I = 3g_U^*/(5\lambda_U^*)$ and $l_I = \Lambda_0 G_0/g_I$

Induced coupling flow

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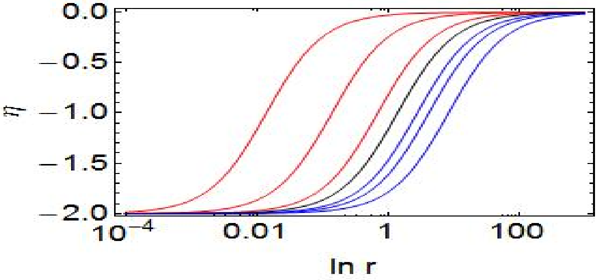
Also complete match for $g_I = 3g_U^*/(5\lambda_U^*)$ and $l_I = \Lambda_0 G_0/g_I$

Important “observable” in quantum theories of gravity:
Anomalous Dimension η

$$\partial_t g(k) \quad = \quad \beta_g(\lambda_k, g_k) = [d - 2 + \eta(k)]g(k) \qquad (49)$$

here

$$\eta(r) = -2 + 2\frac{r/g_I}{\frac{1}{g_U^* \Sigma} + r/g_I}. \qquad (50)$$



Induced coupling flow

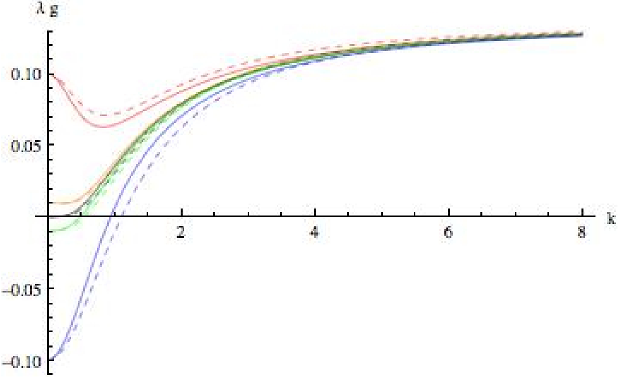
Product of adimensional couplings

In "each" ERG calculation, different values of the fixed points g_U^*, λ_U^*.

Product $g_U^* \cdot \lambda_U^*$ much more stable

$\Rightarrow$

Expect $g(k) \cdot \lambda(k)$ to be more stable



Looks good, but still many issues and open questions

- Experimental restrictions on parameters? (D.& O.)
- Possible dark matter interpretation? (D.& O.)
- Thermodynamics and horizon structure? (...?)
- Calculate energy for arbitrary parameters (...?)
- Is this a coincidence or something deeper?

Black holes are wise guys, so maybe some truth in this result

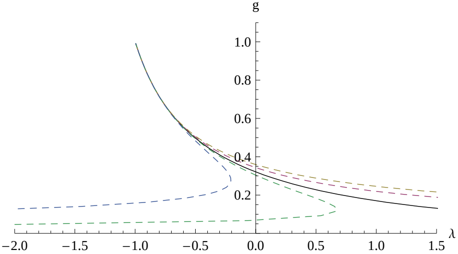
Summary

- ERG and the coupling flow are promising approaches
- It is possible to go beyond "improving" solutions
  " Black hope physics "
- The induced coupling flow and the ERG flow are very similar
- More to explore and more to learn

Obrigado

Backups

Choosing other values for a, c one can get an IR fixed point

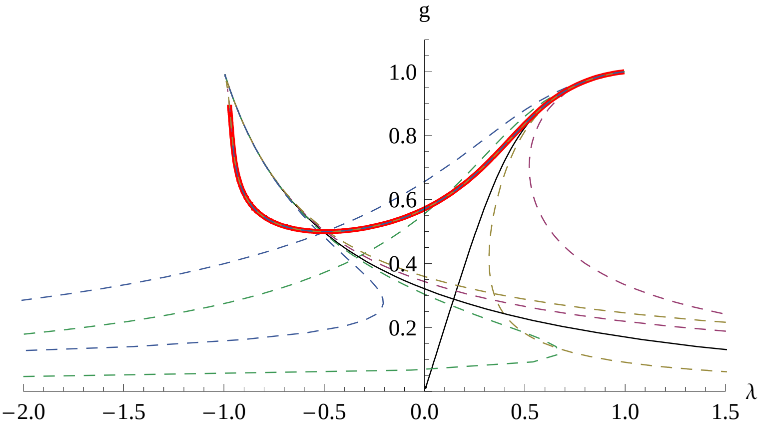


Induced IR flow

Induced coupling flow

ERG black holes

UV- IR connection



Induced UV-IR flow

Only one possible curve, once interpolation and fixed points are fixed.

Backups