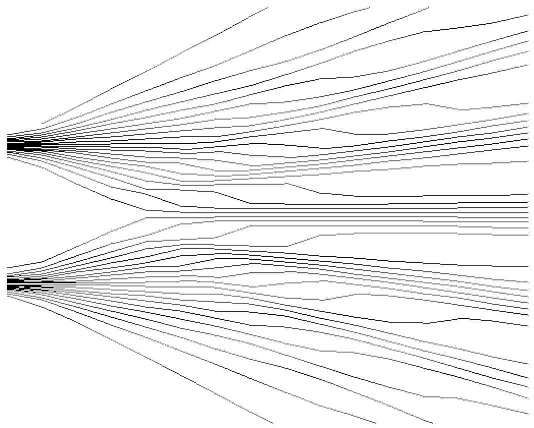
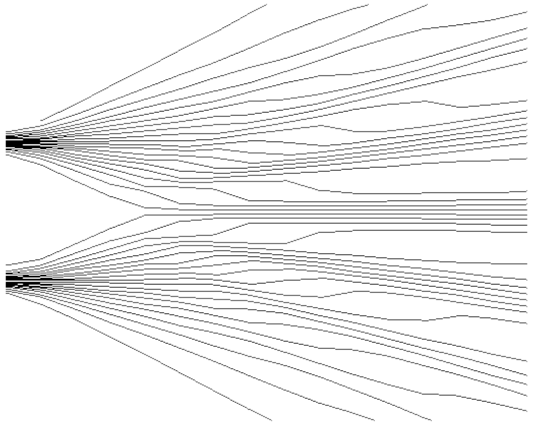


A Geometrical Dual to the Quantum-Klein-Gordon Equation

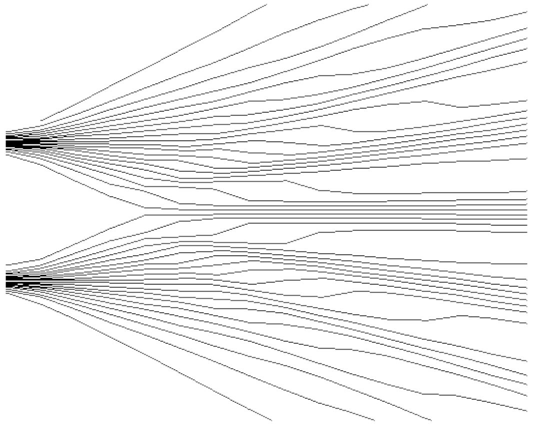
QTRF 5
Växjö

Benjamin Koch



Outline

- Motivation
- The de Broglie-Bohm Interpretation (A)
- The Geometrical Toy Model (B)
- Matching A & B
- Generalizations (n-particles & Interactions)
- Summary & Outlook



Literature

This Talk:

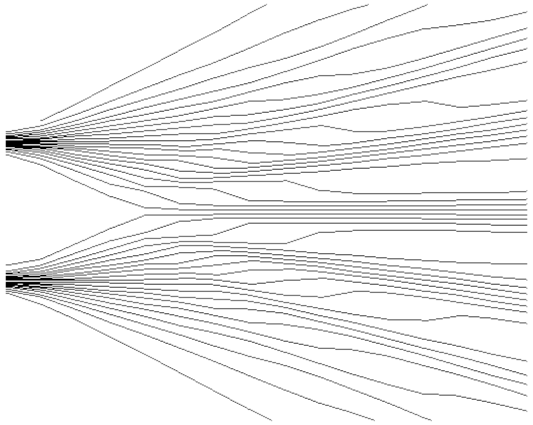
- B. Koch, arXiv:0901.4106;
- B. Koch, arXiv:0810.2786;

Related Ideas:

- F.&A.Shojai, Int.J.Mod.Phys.A15,1859 (2000)
- R. Carroll, arXiv:gr-qc/0406004
- J.M. Isidro et al., arXiv:0808.2351

Motivation

Motivation QM double slit



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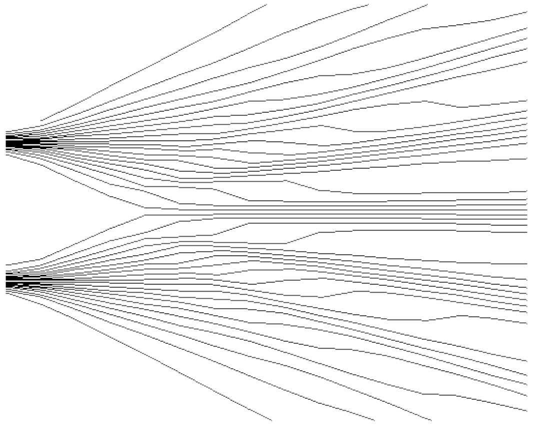

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Observe one particle

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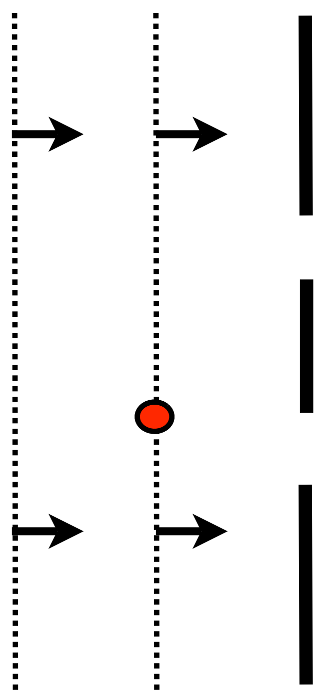
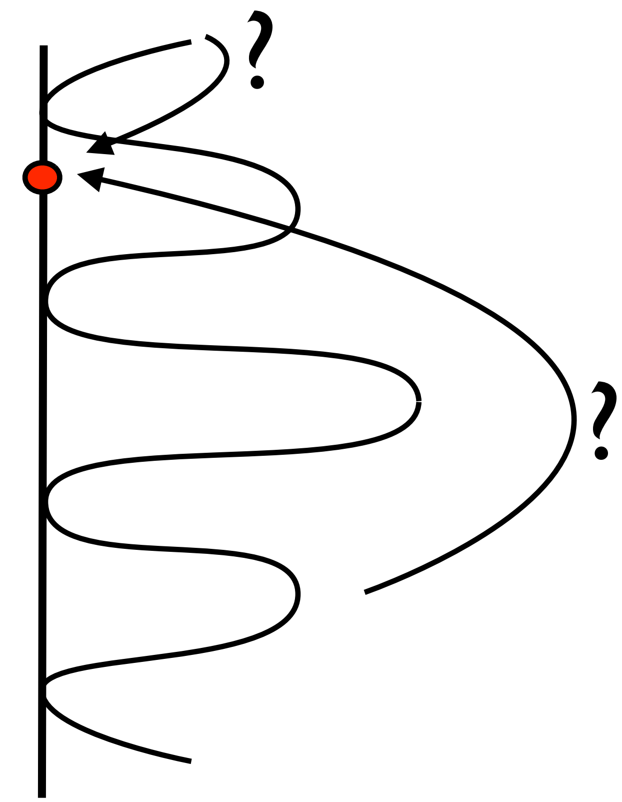


Collapse of the wave function?



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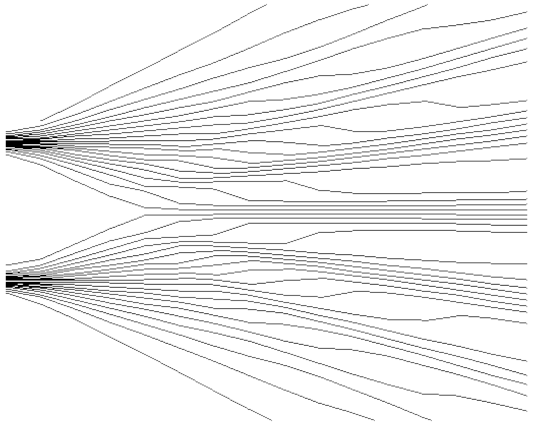

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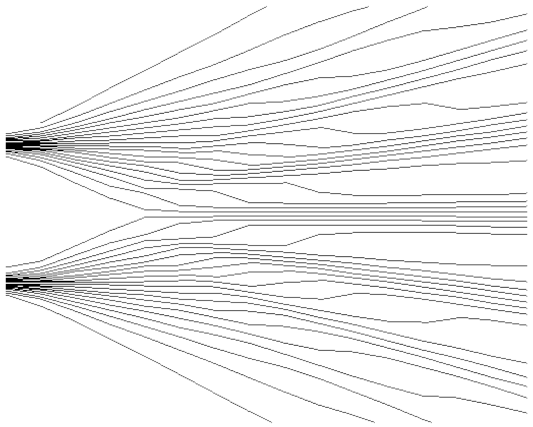
Problem of undefined measurement process
in standard (Copenhagen) interpretation of QM.



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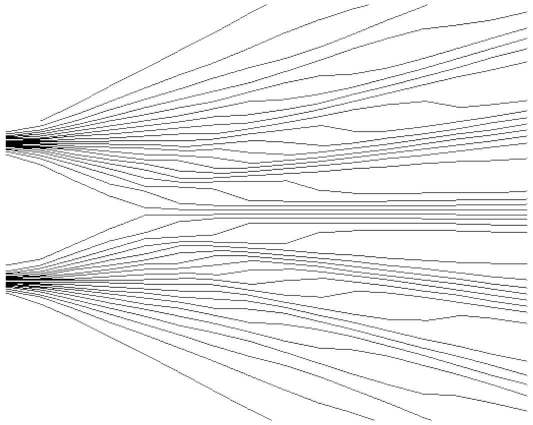

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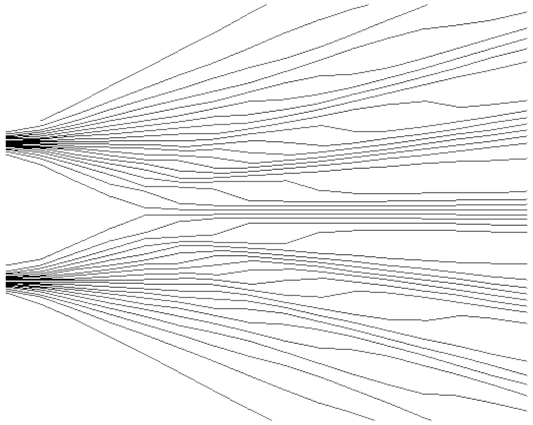

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Alternative interpretation
- Many worlds
- dBB interpretation
- ...

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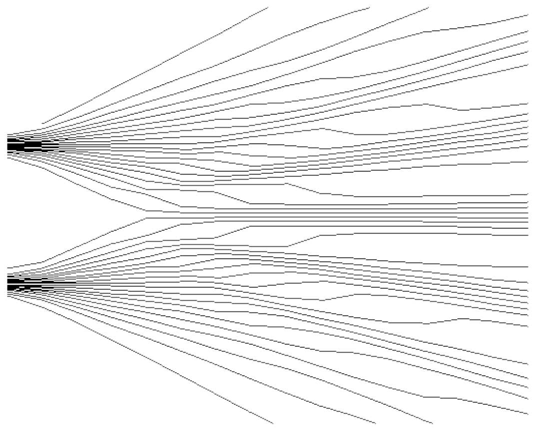

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The de Broglie Bohm Interpretation

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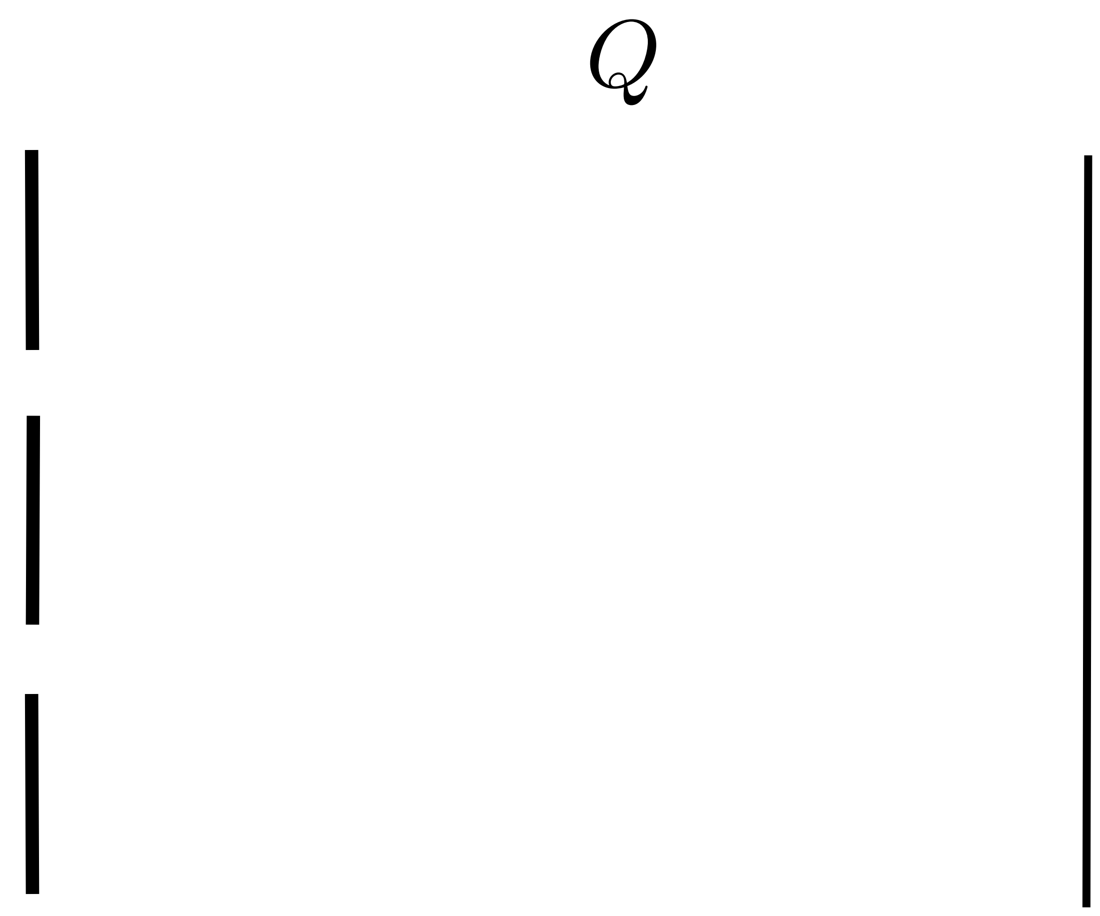

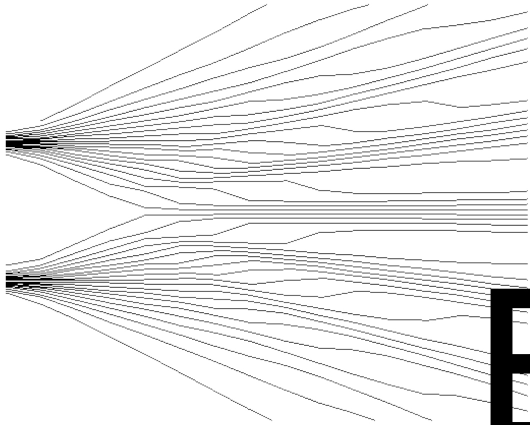

The de Broglie Bohm Interpretation

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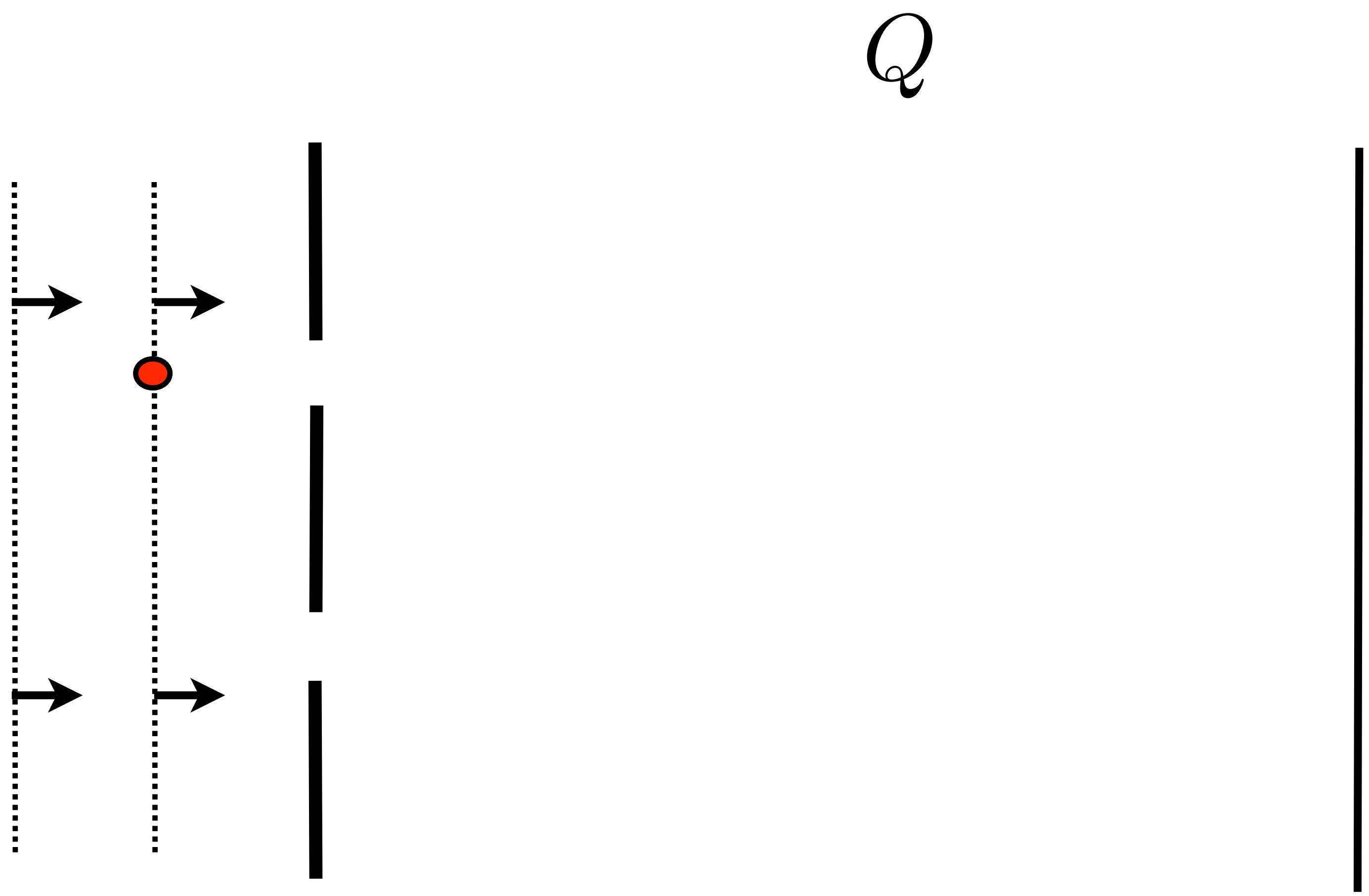

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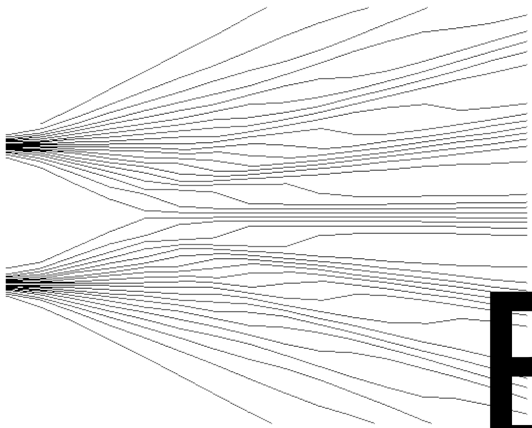

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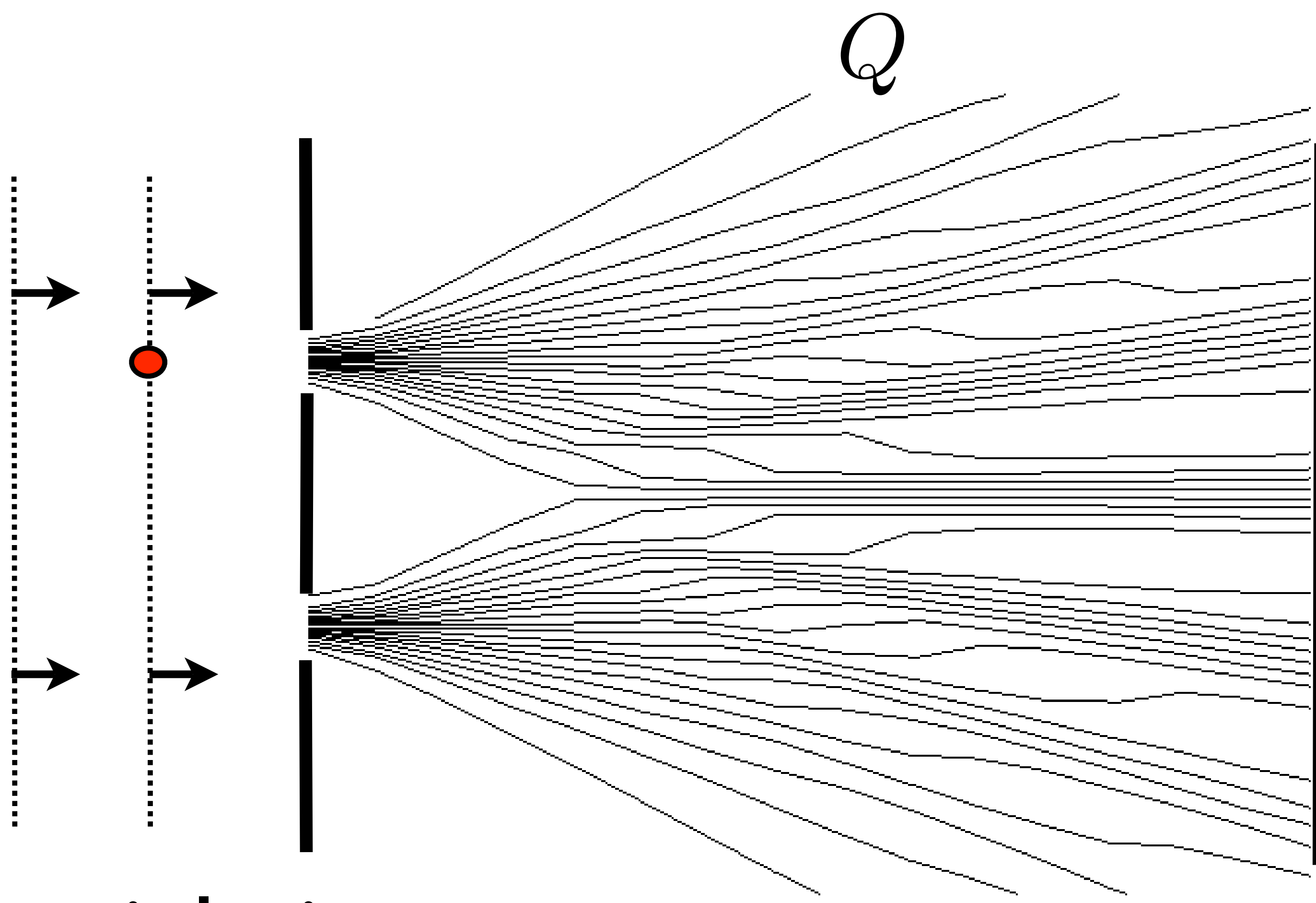

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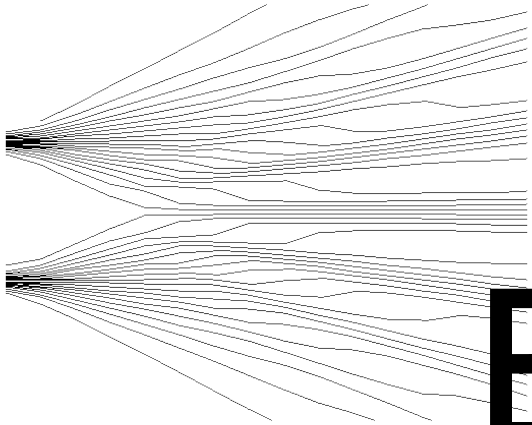

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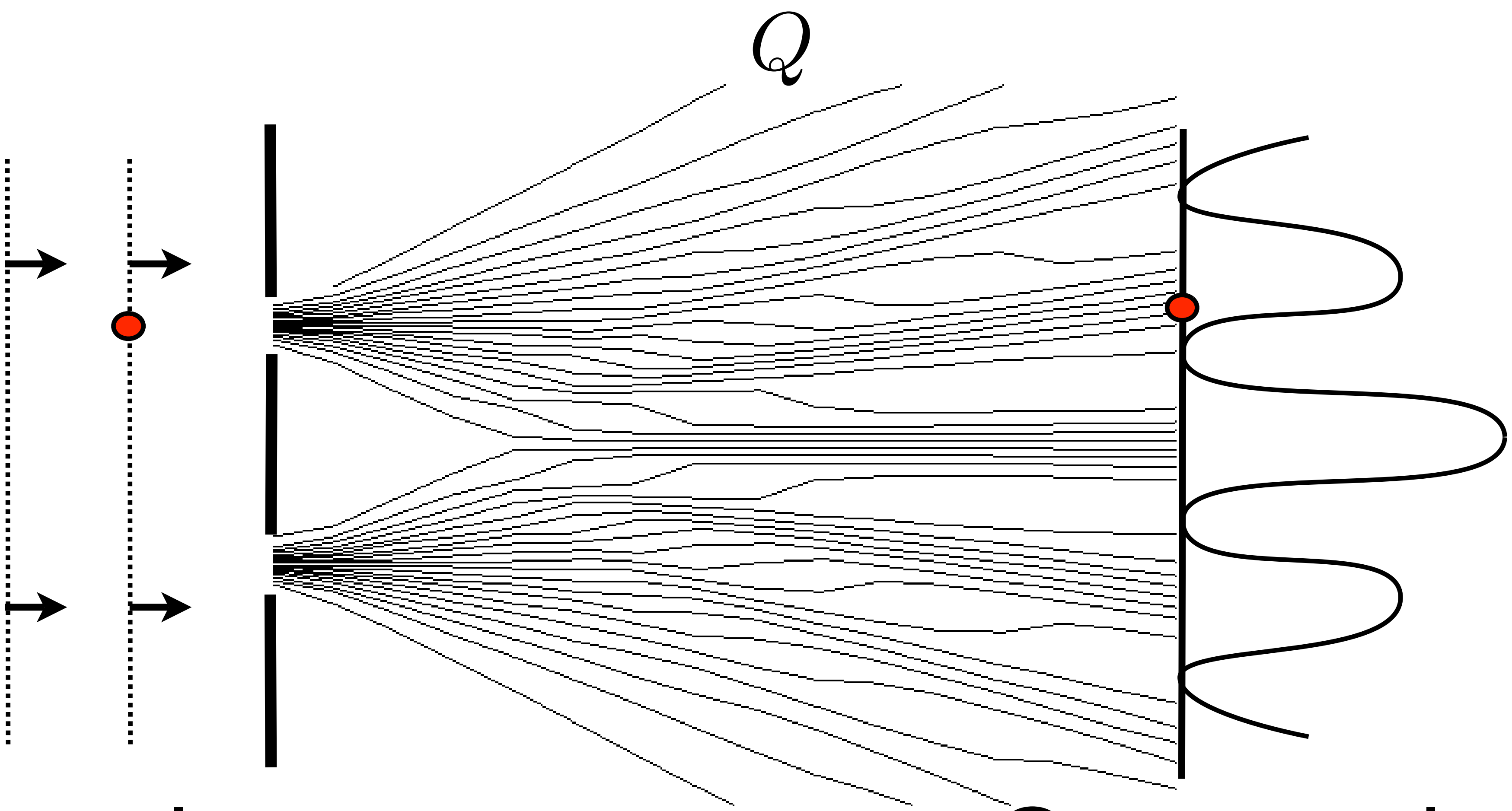
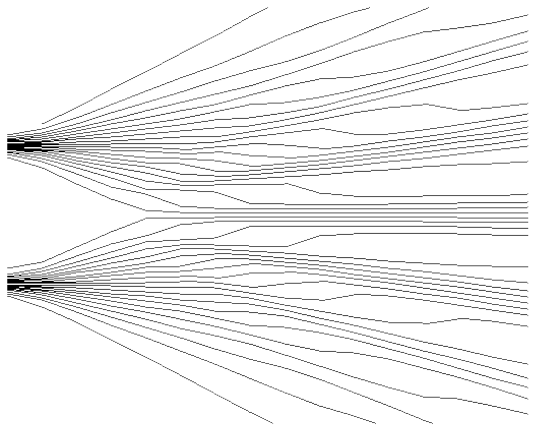


One particle in

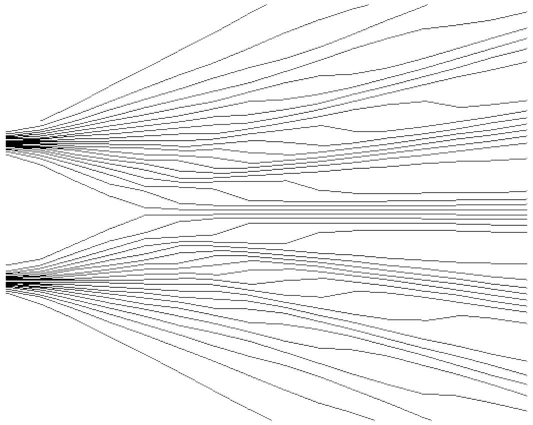
One particle out



dBB

Klein Gordon equation:

$$\left(\partial^m\partial_m + \frac{M^2}{\hbar^2}\right)\Phi(x) = 0$$



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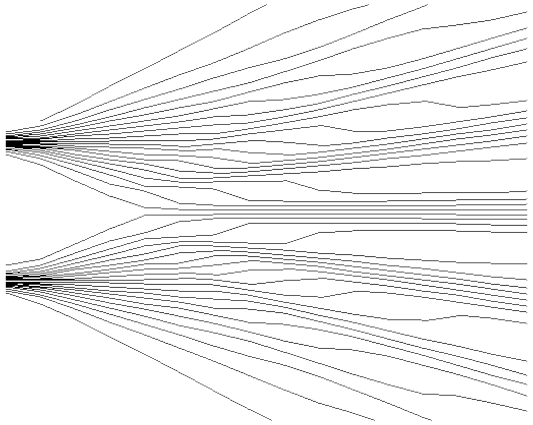

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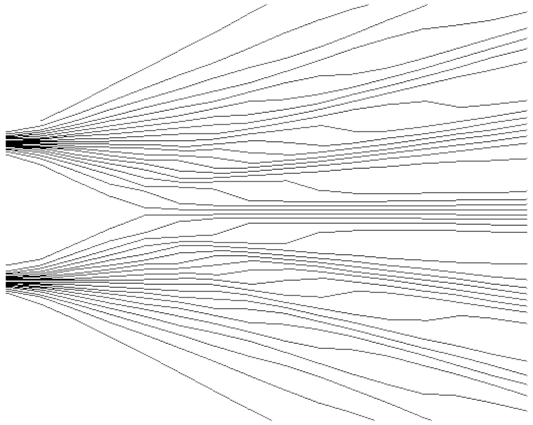
continuity



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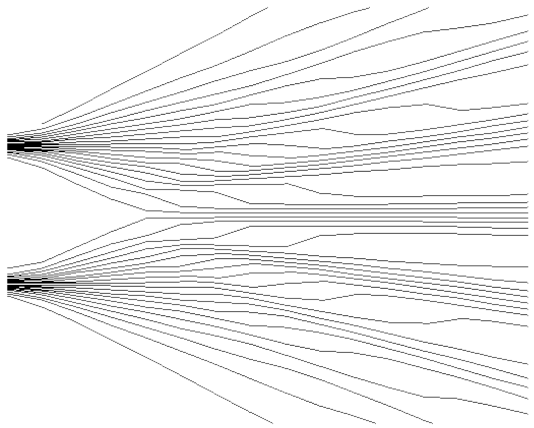
$$2MQ = (\partial^m S_Q)(\partial_m S_Q) - M^2$$ Hamilton-Jacobi

„quantum potential“ $Q = \frac{\hbar}{2M}\frac{\partial^m \partial_m \sqrt{\rho}}{\sqrt{\rho}}$



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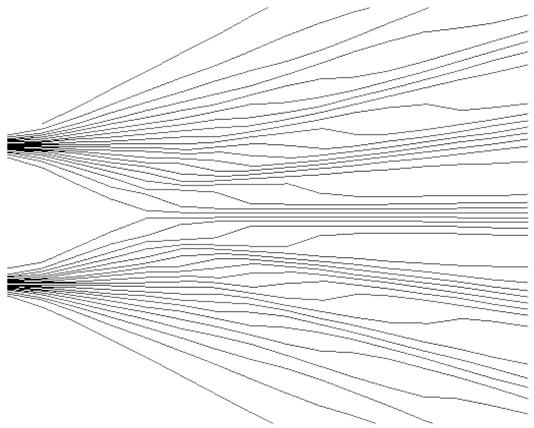

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$$p^m = M\frac{dx^m}{ds} = -\partial^m S_Q$$

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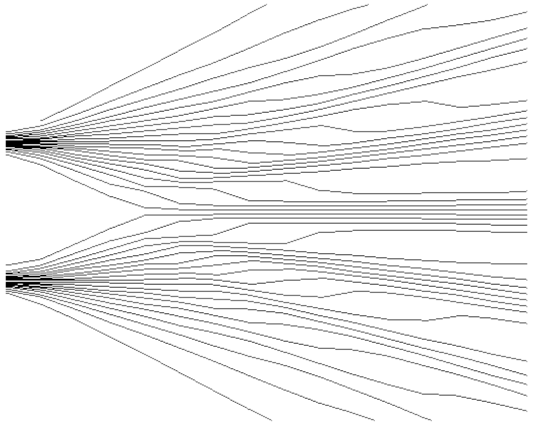

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$$M\frac{d^2x^m}{ds^2} = \partial^m Q$$

*H. Nikolic, Found. Phys. Lett. 17, 363;
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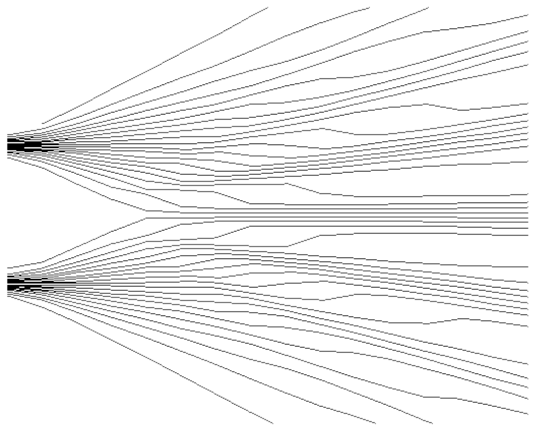

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remember!

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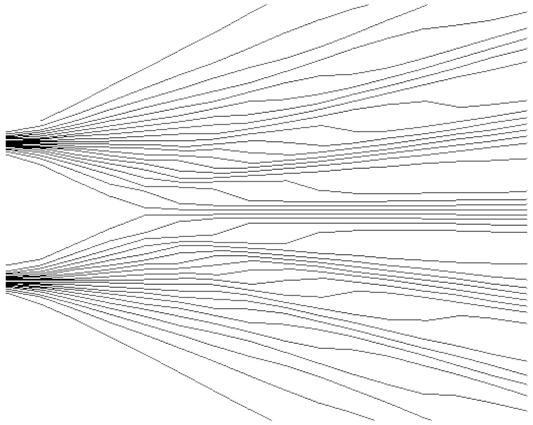

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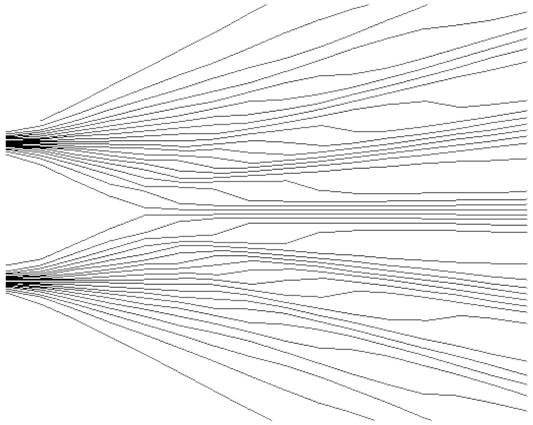
Geometrical Toy Model



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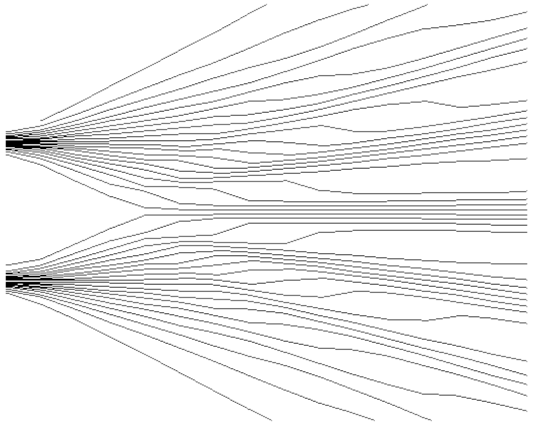

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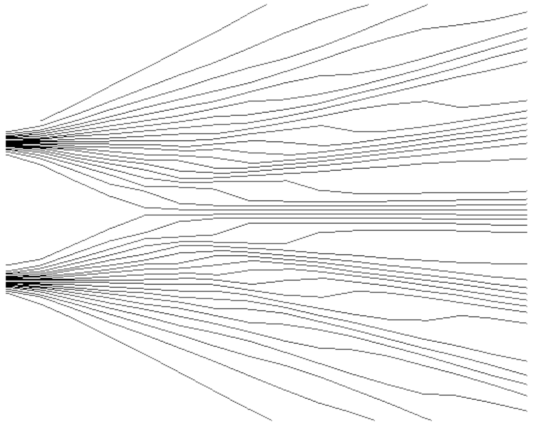
Action:

$$S = \int d^4 x \sqrt{|\hat{g}|}(\hat{R} + \kappa \hat{\mathcal{L}}_M)$$

Describes matter in a curved space-time

Geometrical Toy Model



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- Ricci scalar $\hat{R}$

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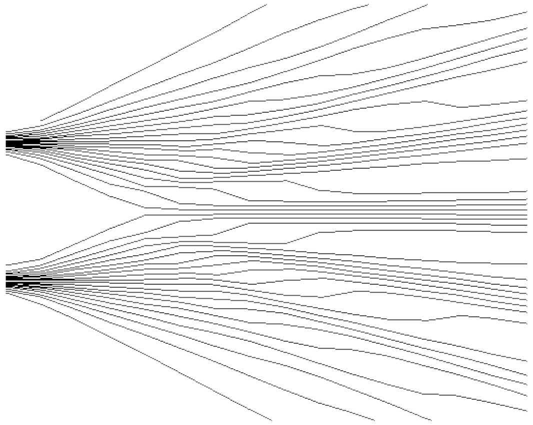

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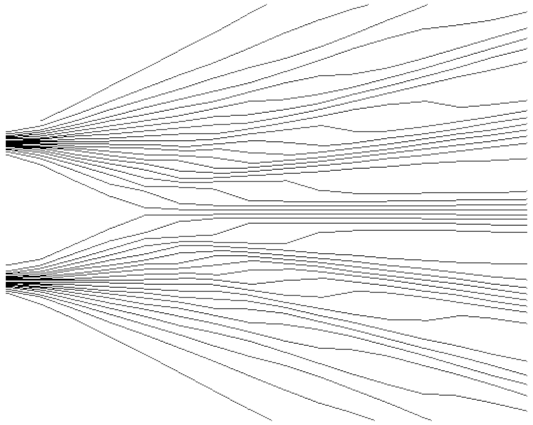
Describes matter in a curved space-time

Curvature:	- Metric	$\hat{g}$
	- Ricci scalar	$\hat{R}$
Matter:	- Coupling	κ
	- Lagrangian	$\hat{\mathcal{L}}_M$

Geometrical Toy Model



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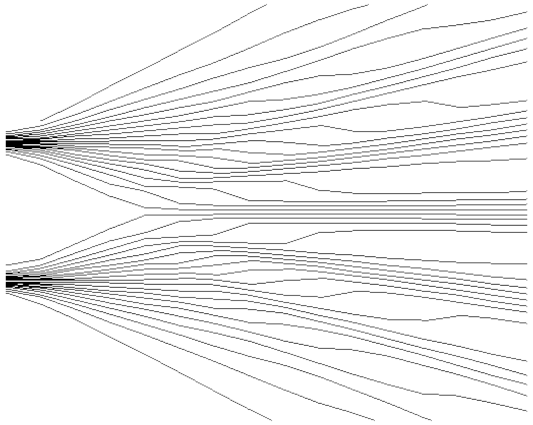

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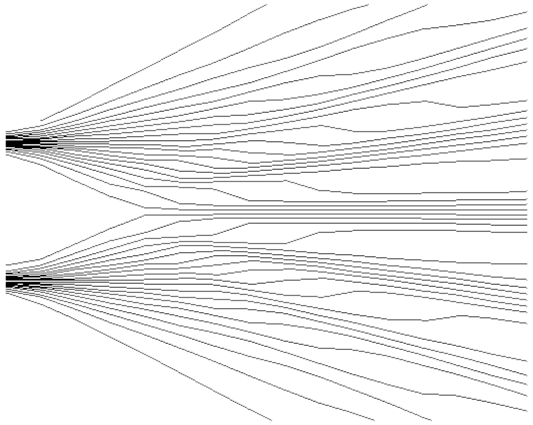

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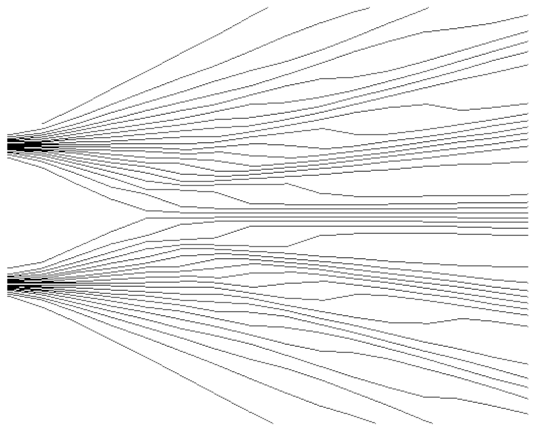

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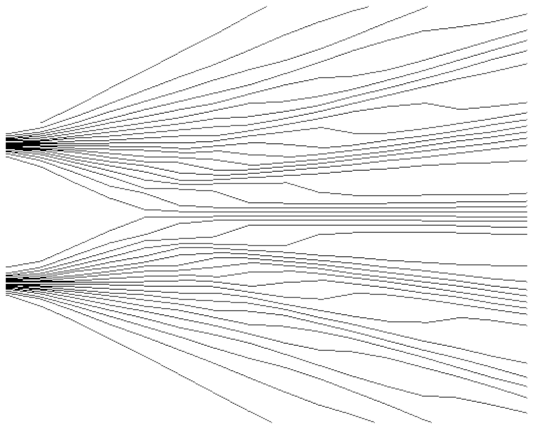

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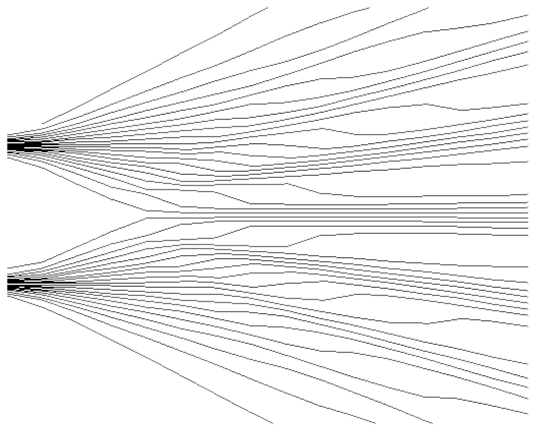
Geometrical Toy Model



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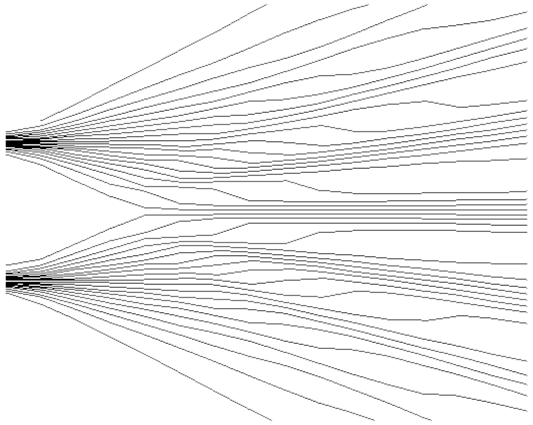
$$\mathcal{L}_M = p^m p_m - M_G^2$$

Geometrical Toy Model



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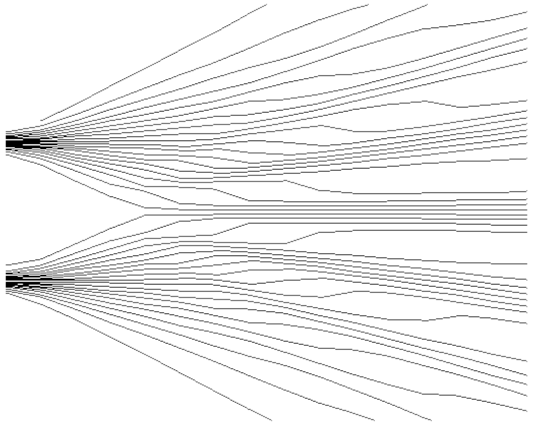
Geometrical Toy Model



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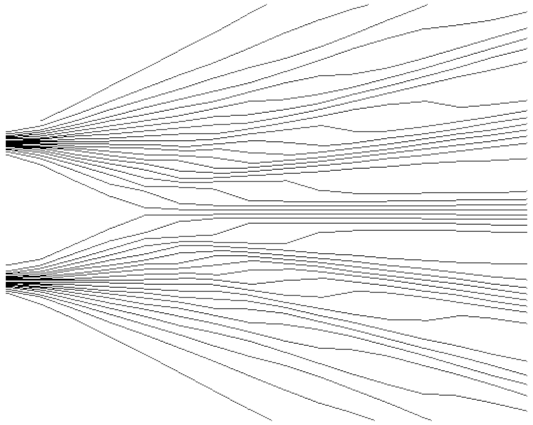

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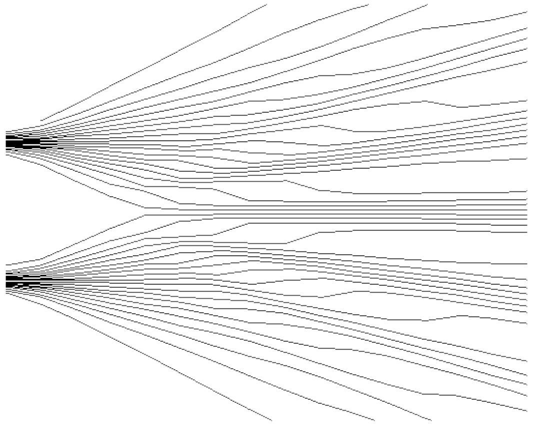

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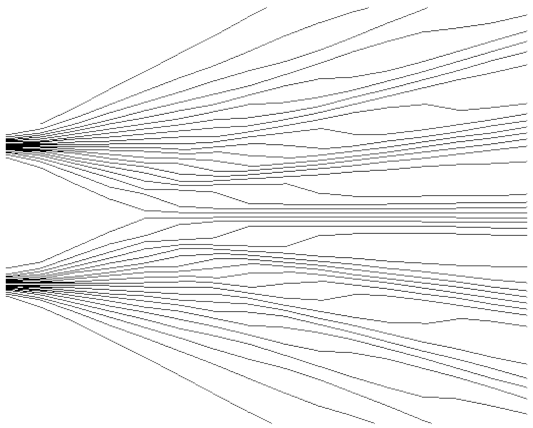

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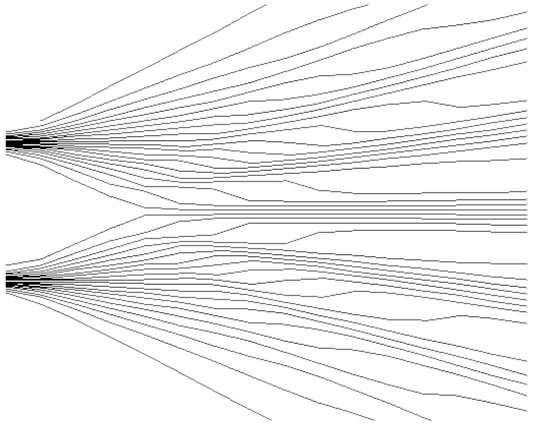

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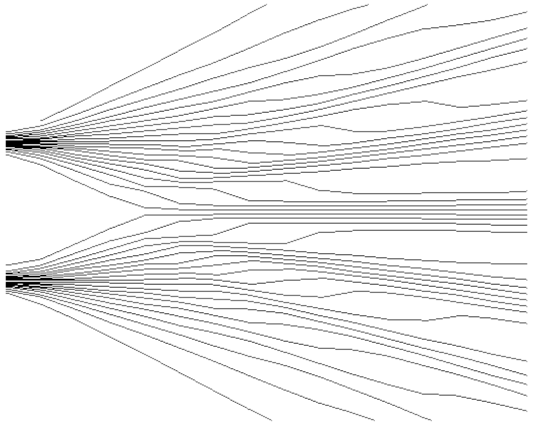

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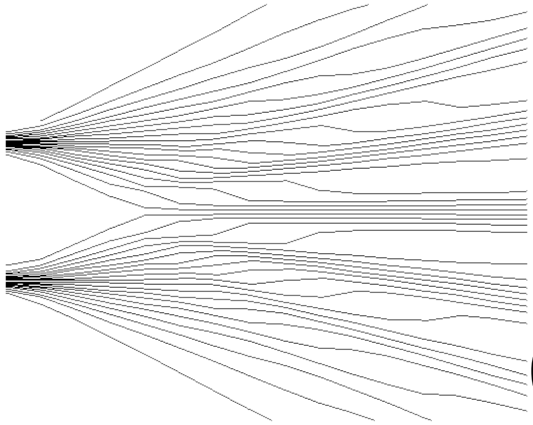

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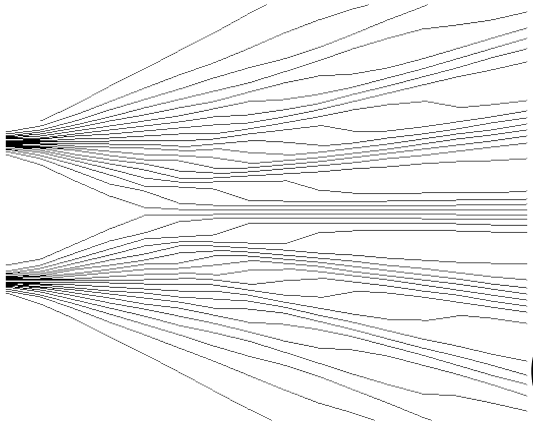

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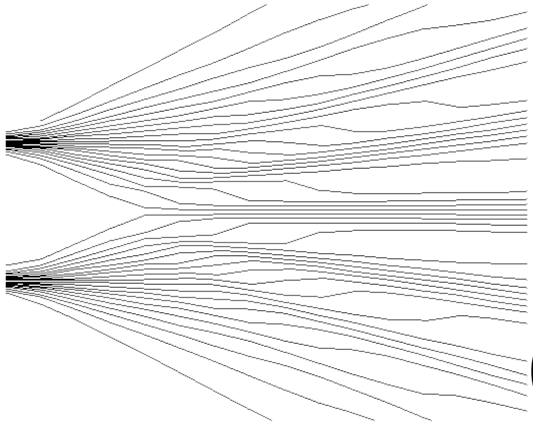

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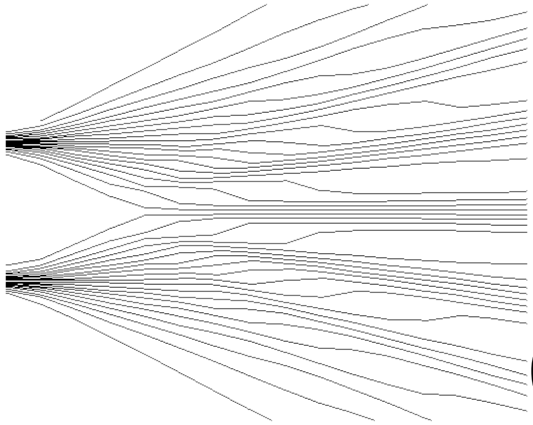

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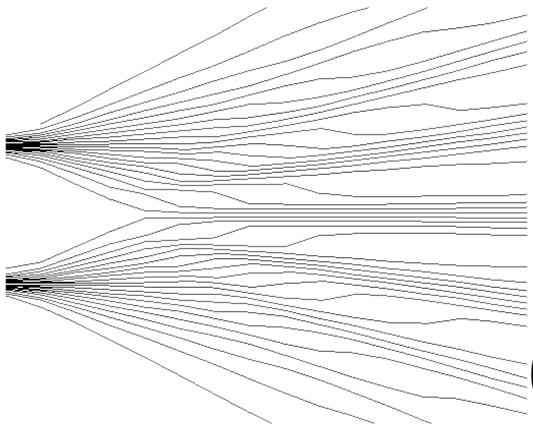

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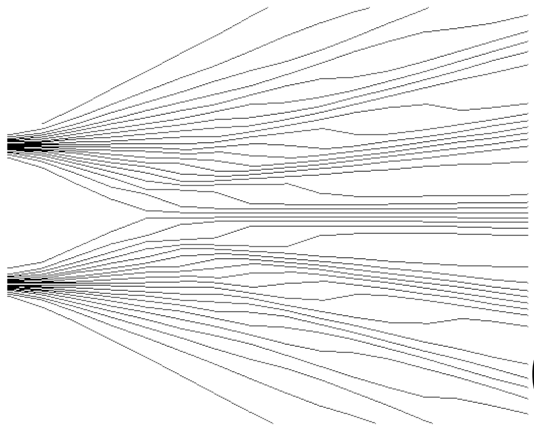

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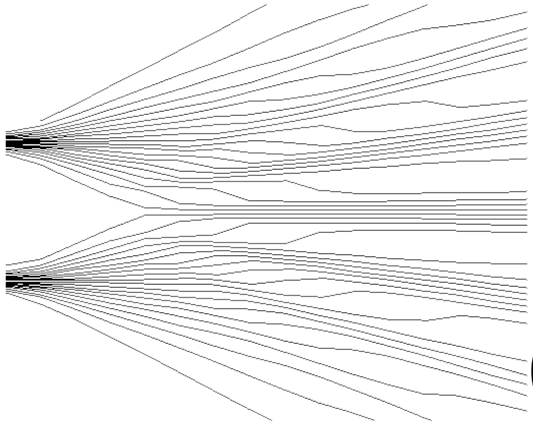

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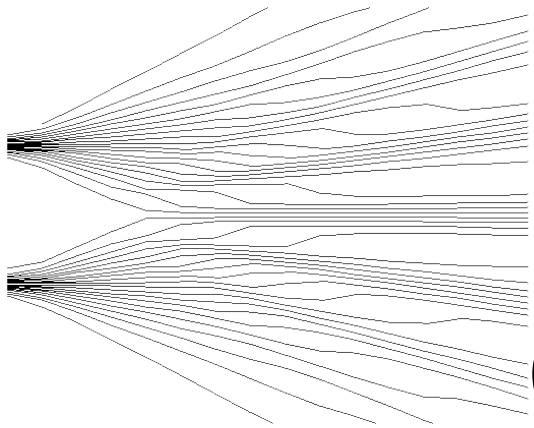

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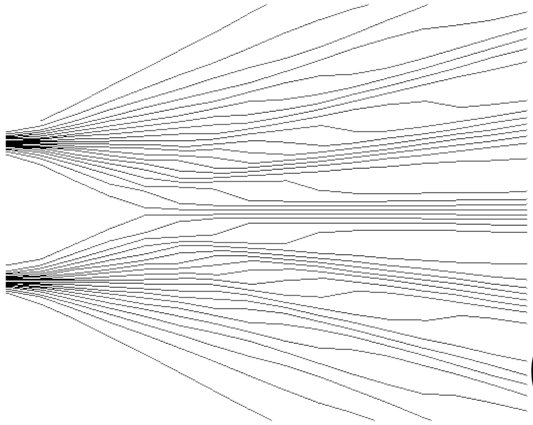

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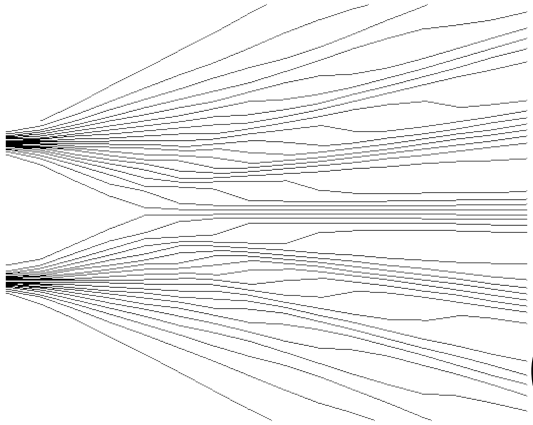

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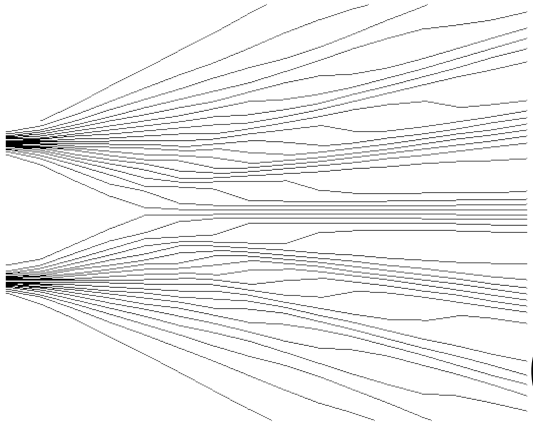

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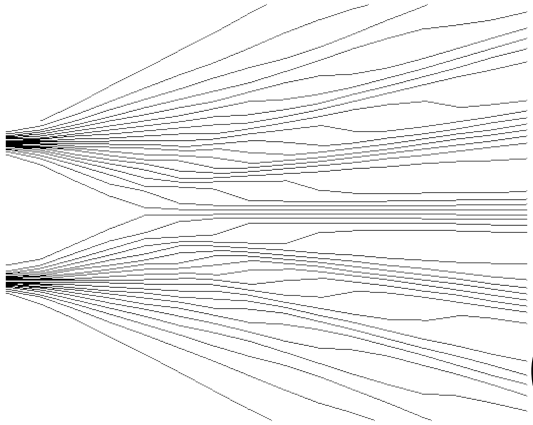

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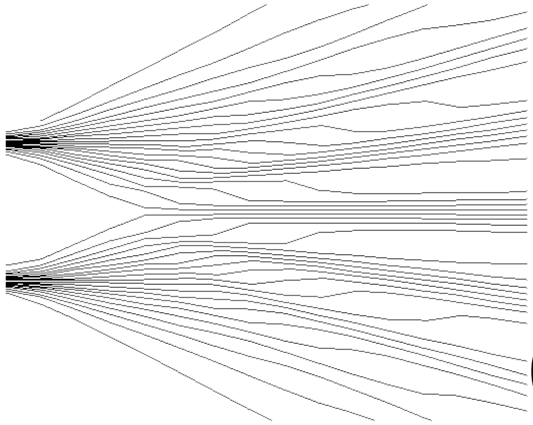

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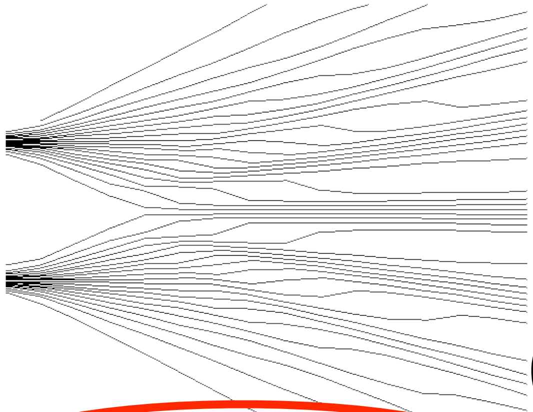

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Matching
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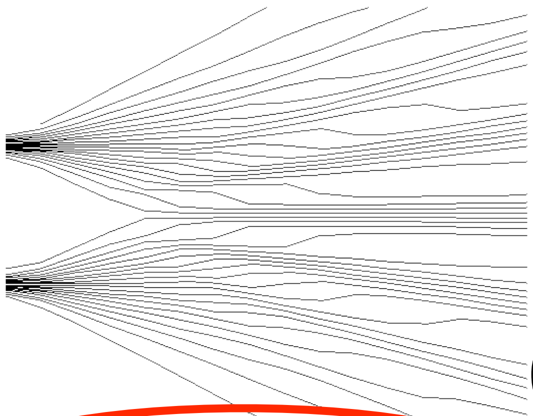

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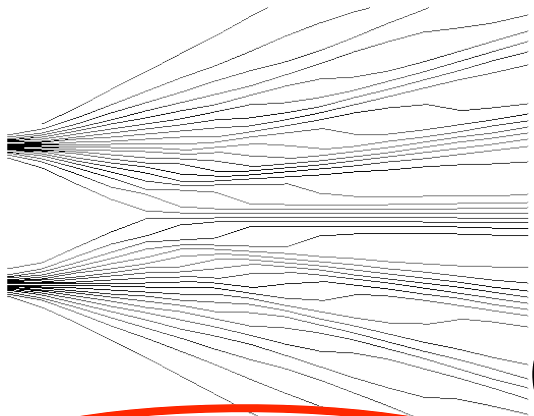

Matching dBB & Toy Model

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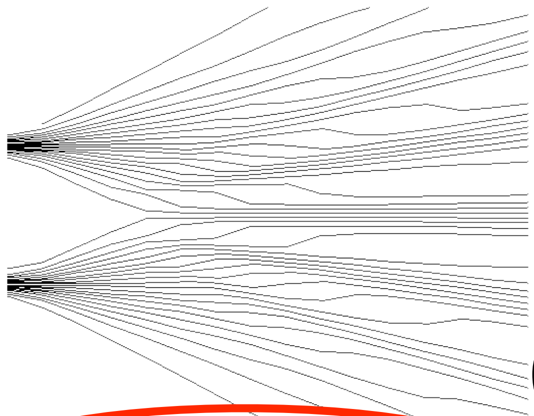

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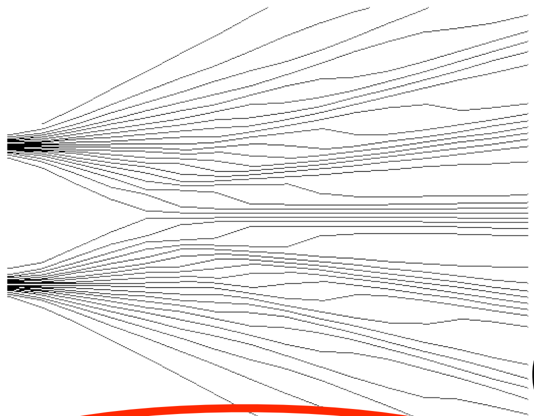

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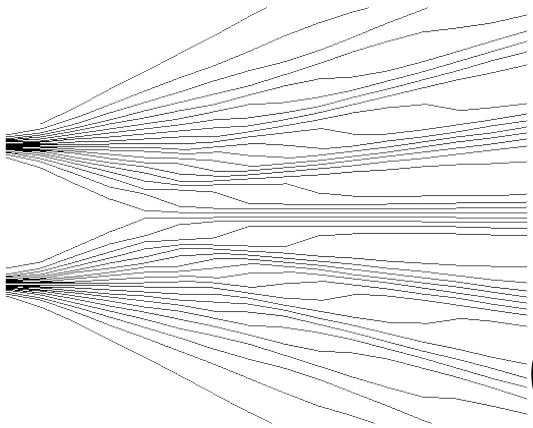

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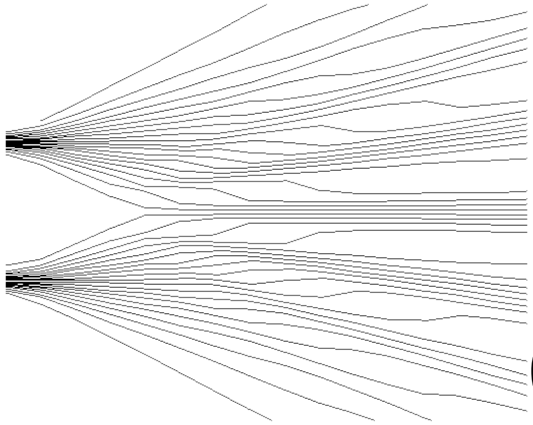

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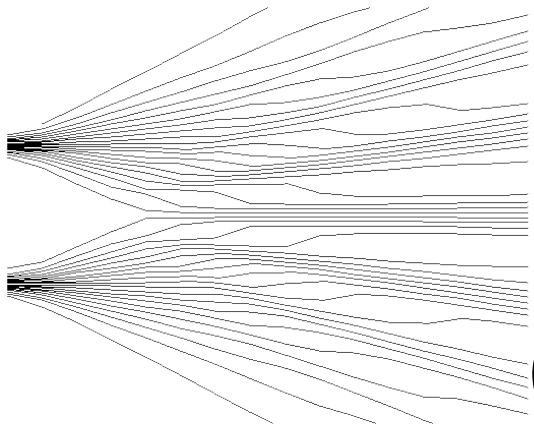

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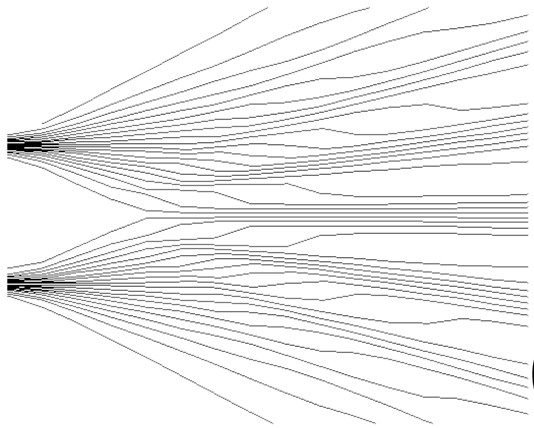

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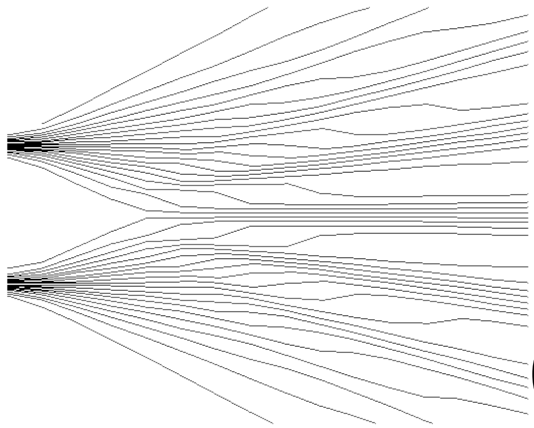

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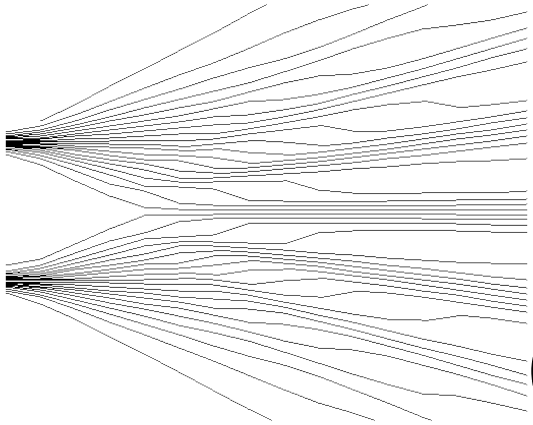

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- Short: Use the 4-✓ relations
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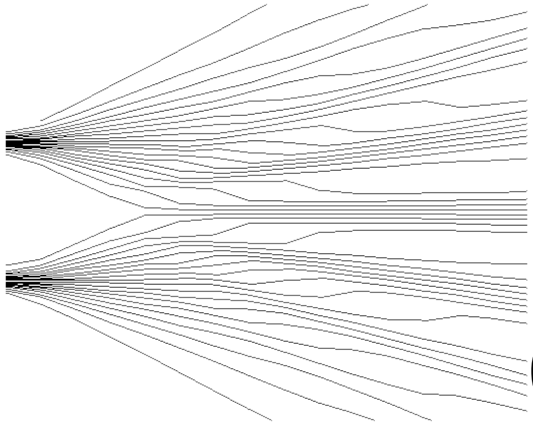

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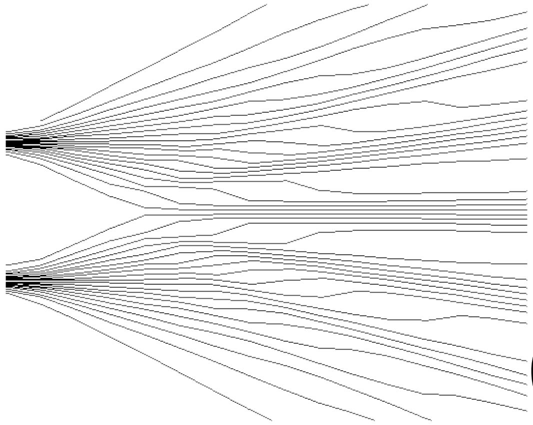

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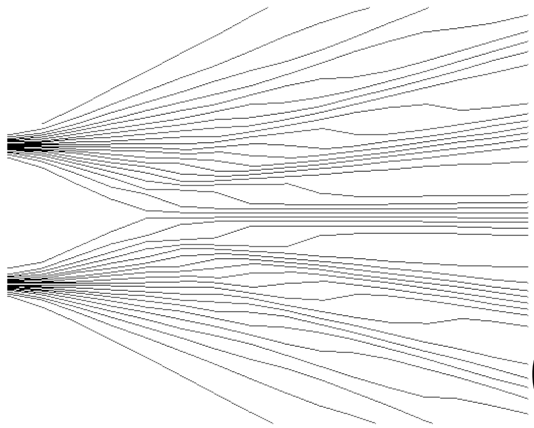

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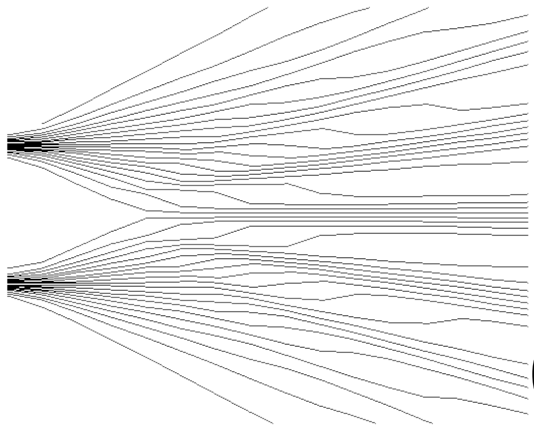

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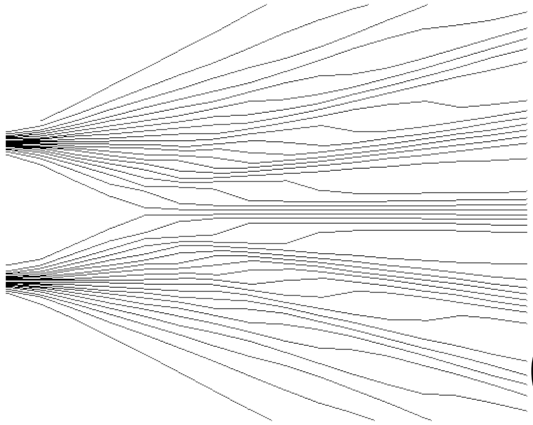

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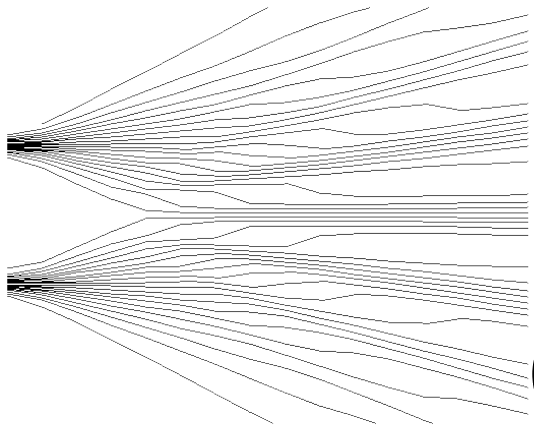

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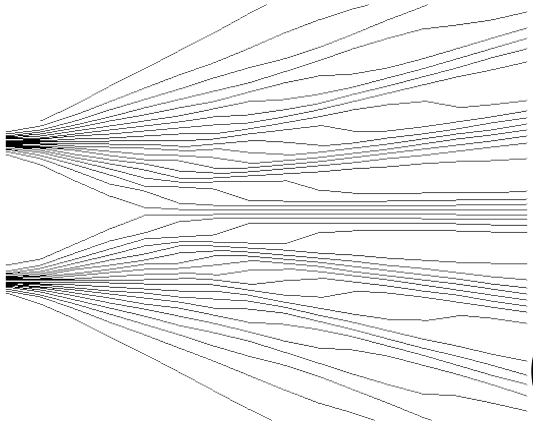

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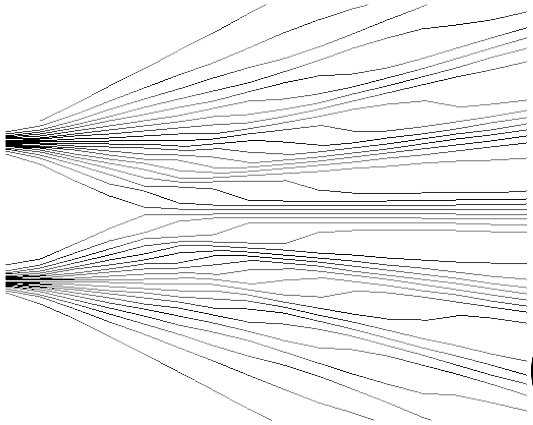

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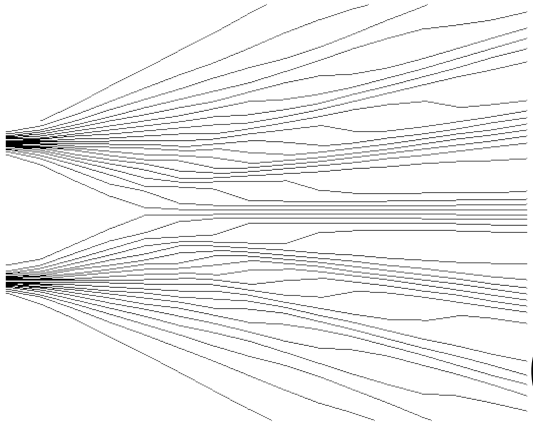

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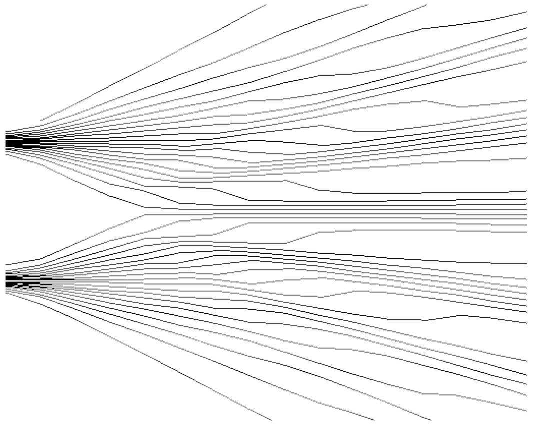
Geometrical toy model

$$\int d^4x\sqrt{\hat{g}}\hat{R} + \kappa\hat{\mathcal{L}}_M$$



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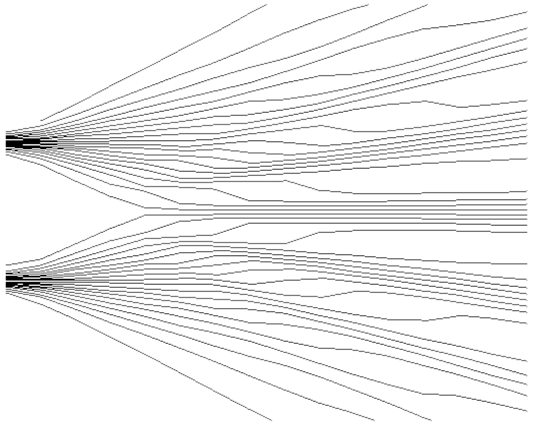
Duality between dBB picture & Geometrical toy model



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- Multiple particles
- Interactions with external em-field
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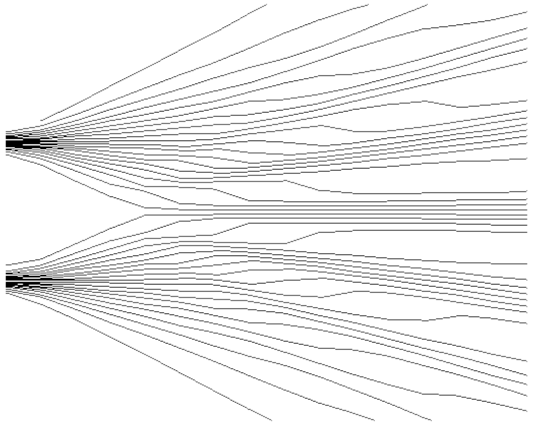

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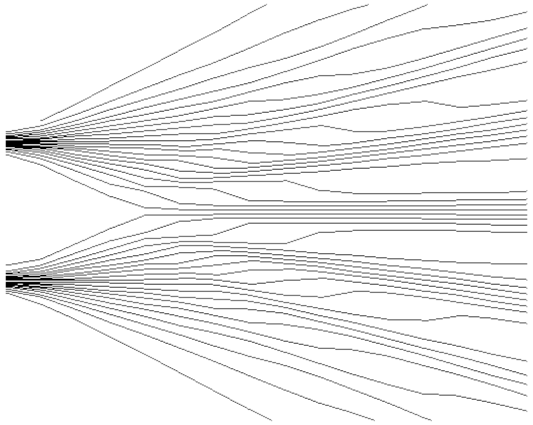

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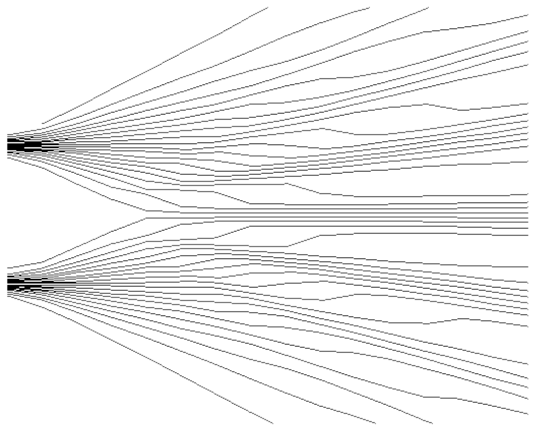

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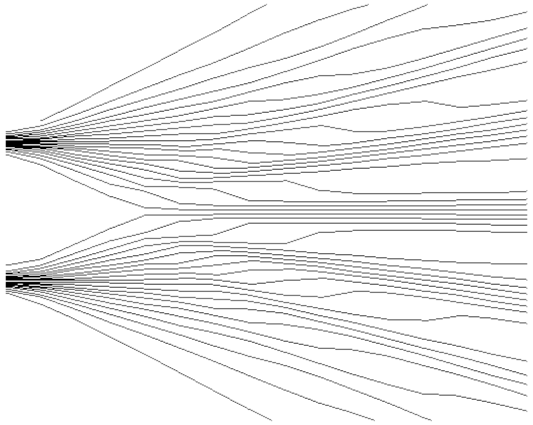

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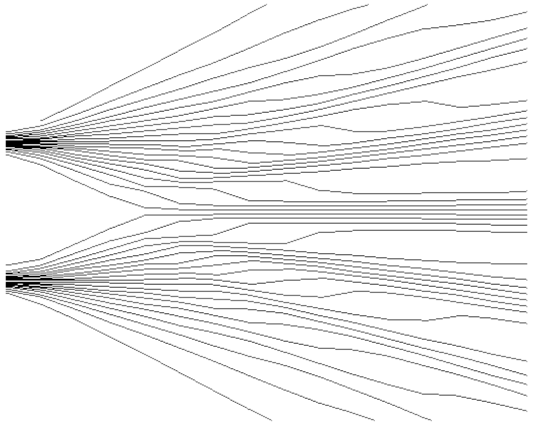

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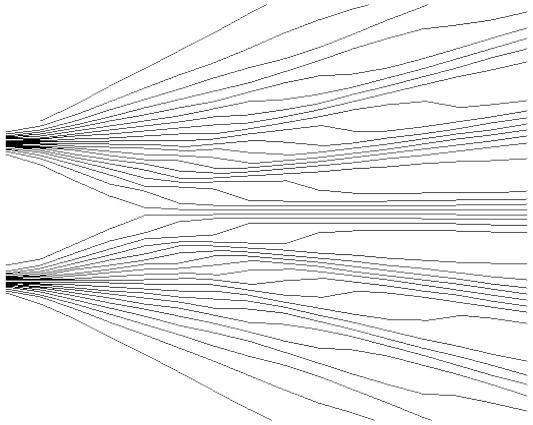

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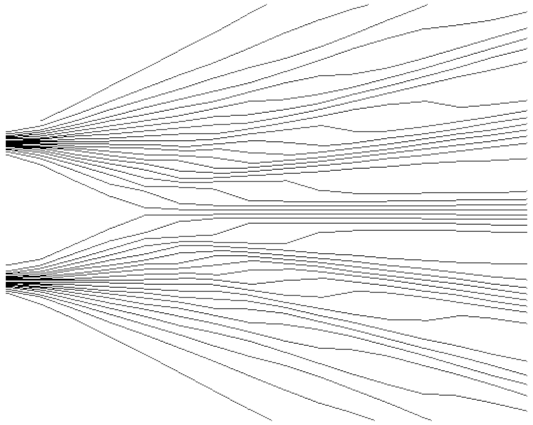
Thank you!



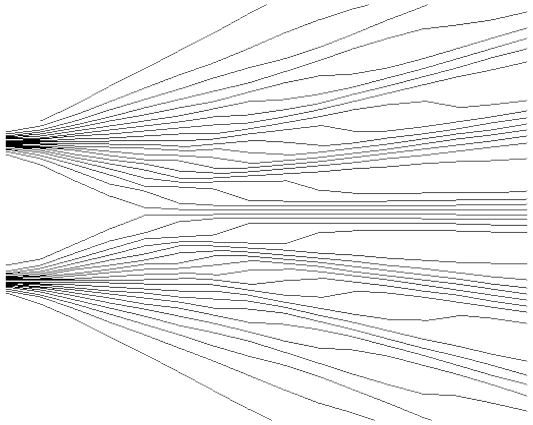
Backup

Interaction with external em-field

Klein-Gordon in dBB picture	Toy model
Minimal coupling	Canonical momentum (classical)
$\partial_m \to \partial_m + ieA_m/\hbar$	$\hat{p}^\mu \to \hat{\pi}^\mu = -(\hat{\partial}^\mu S_Q + e\hat{A}^\mu)$

⇘ ⇙

$$M\frac{d^2x_j^m}{ds^2} = \partial_j^m Q + e\pi_{jn}F^{mn}$$



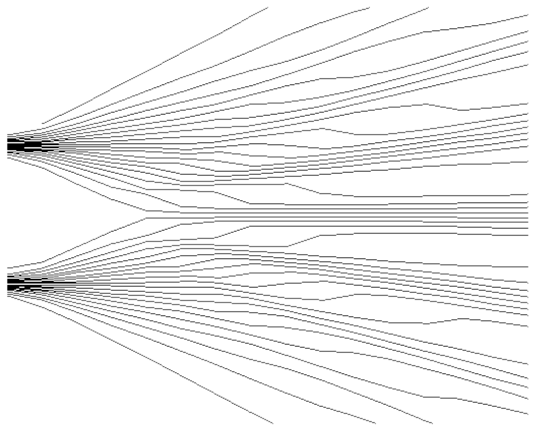
Backup

No go theorems for deterministic QM:

Bell inequalities: Do not apply because dBB is a non-local theory

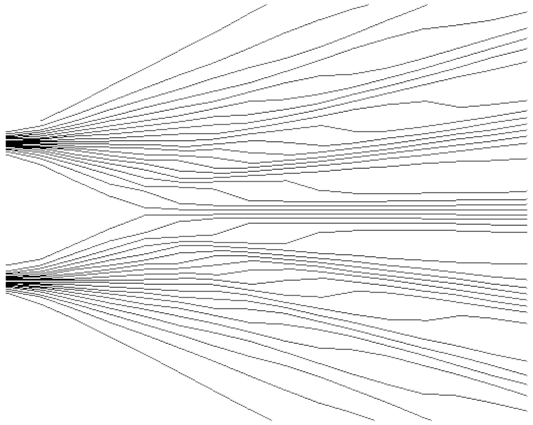
Kochen-Specker theorem:

Does not apply because dBB is a contextual theory



Multi particle KG in dBB

Important because:



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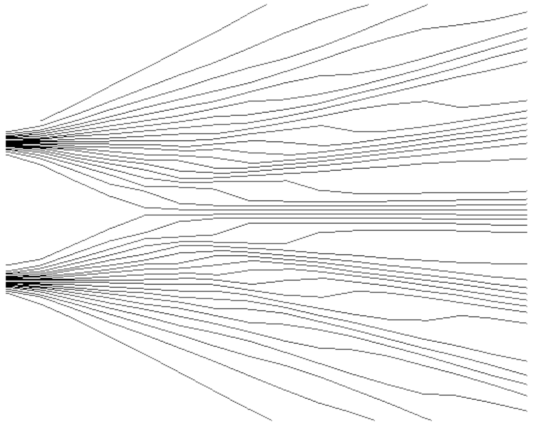
Important because:

-The dBB theory is only consistent with QM if it includes the multi particle case.



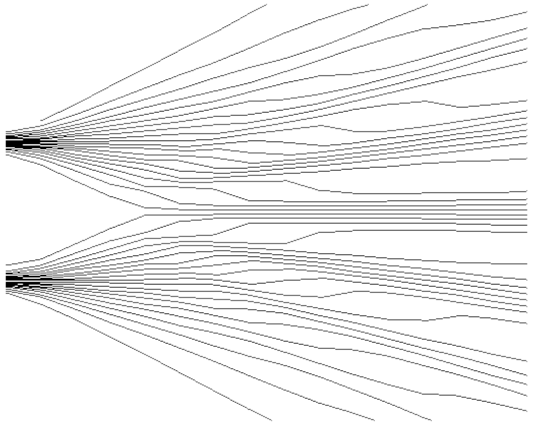
Multi particle KG in dBB

Important because:

- -The dBB theory is only consistent with QM if it includes the multi particle case.
- - Single particle interpretation of KG fails, multi particle description first step towards QFT

Multi particle KG in dBB

*S. Schweber, An Introduction to Relativistic QFT, Dover Publications, 1961



Multi particle KG in dBB

n-particle wave function:

$$\psi(x_1;\ldots;x_n) = \frac{\mathcal{P}_S}{\sqrt{n!}} < 0|\Phi(x_1)\ldots\Phi(x_n)|n >$$

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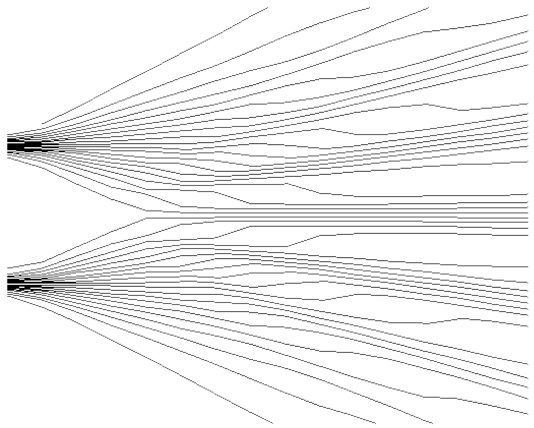

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n-particle KG equation*:

$$\left(\sum_i \partial_i^m \partial_{mi} + n\frac{M^2}{\hbar^2}\right)\psi(x_1;\ldots;x_n) = 0$$

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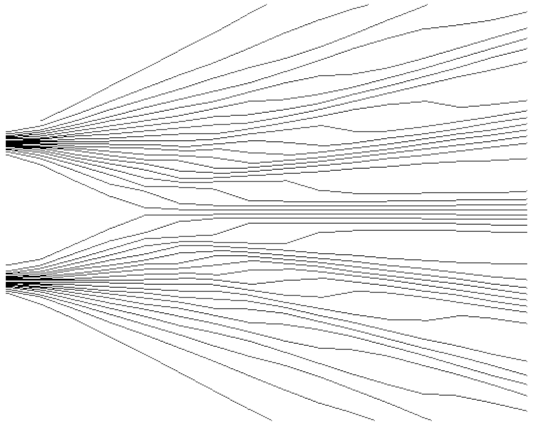

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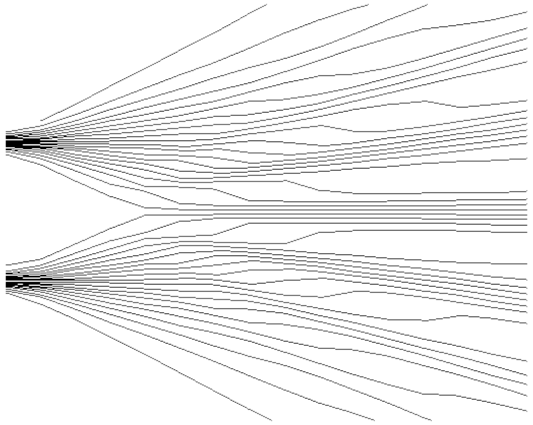

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Same splitting:

$$\psi(x_1;\ldots;x_n) = \sqrt{\rho(x_1;\ldots;x_n)}\,\exp(iS_Q(x_1;\ldots;x_n)/\hbar)$$

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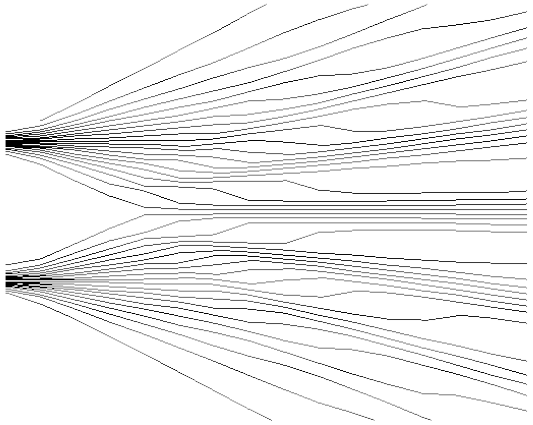

Multi particle KG in dBB

$$\left(\sum_i \partial_i^m \partial_{mi} + n\frac{M^2}{\hbar^2}\right)\psi(x_1;\ldots;x_n)=0$$

Same splitting:

$$\psi(x_1;\ldots;x_n)=\sqrt{\rho(x_1;\ldots;x_n)}\exp(iS_Q(x_1;\ldots;x_n)/\hbar)$$

Same momentum definition*:

$$p_j^m=-\partial_j^m S_Q(x_1;\ldots;x_n)$$

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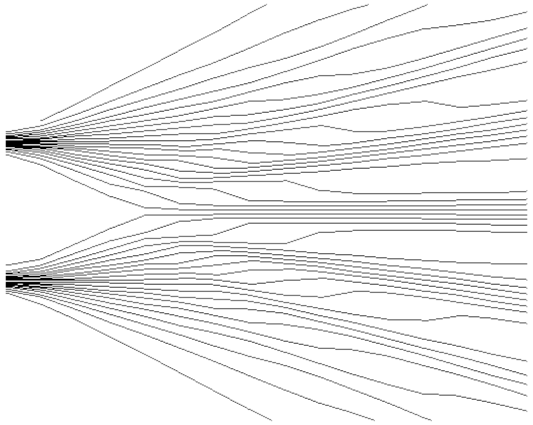

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$$\Rightarrow \begin{aligned} 2MQ &= (\partial^L S)(\partial_L S) - nM^2 \quad \text{with} \\ Q &= \frac{\hbar^2}{2M}\frac{\partial^L\partial_L P}{P} \quad , \\ 0 &= \partial_L\left(P^2(\partial^L S)\right) \quad , \\ p^L &= M\frac{dx^L}{ds} = -\partial^L S \quad , \\ \frac{d^2x^L}{ds^2} &= \frac{(\partial^N S)(\partial^L\partial_N S)}{M^2} \quad \text{with} \\ \frac{d}{ds} &= \frac{dx^L}{ds}\partial_L \quad . \end{aligned}$$

Set of equations

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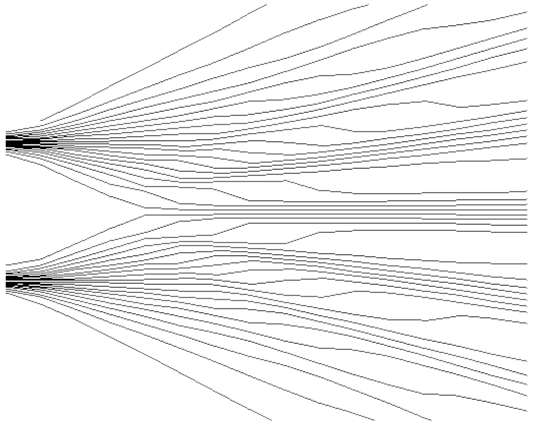
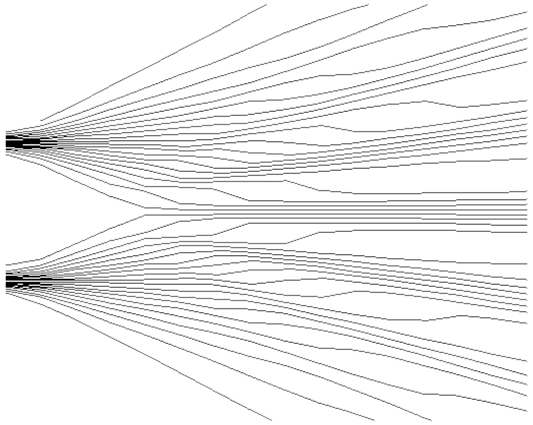
Toy model



Toy model

Action for one particle in 4n-dimensional space-time

$$S(g_{\Lambda\Delta}) = \int dx^{4n} \sqrt{\hat{g}} \{\hat{R} + \kappa \hat{\mathcal{L}}_M\}_{sym}$$



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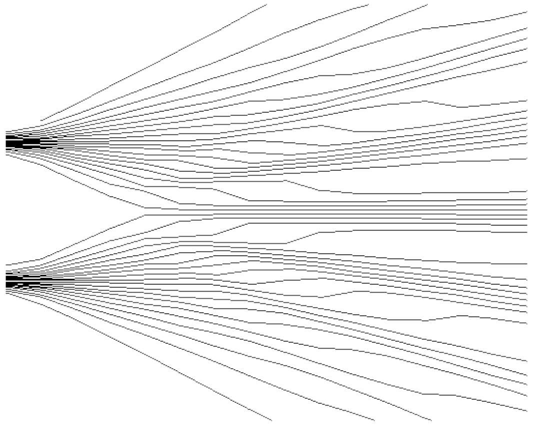

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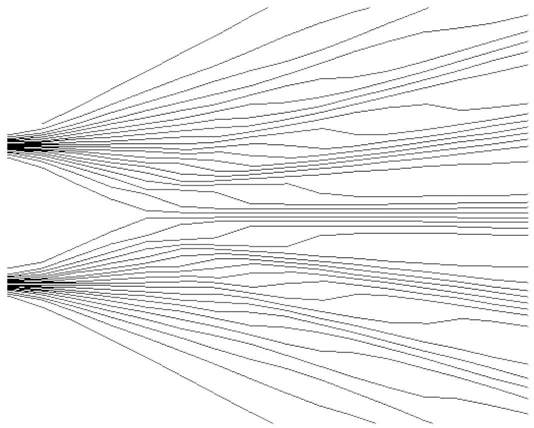
Symmetrization:



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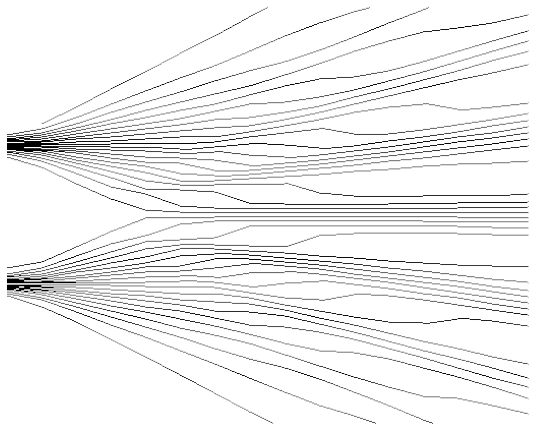
Symmetrization:

Equations of motion + conservation of $\hat{T}^{\Lambda\Delta}$



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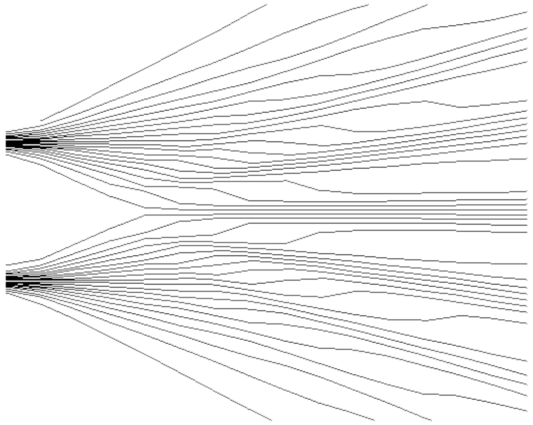
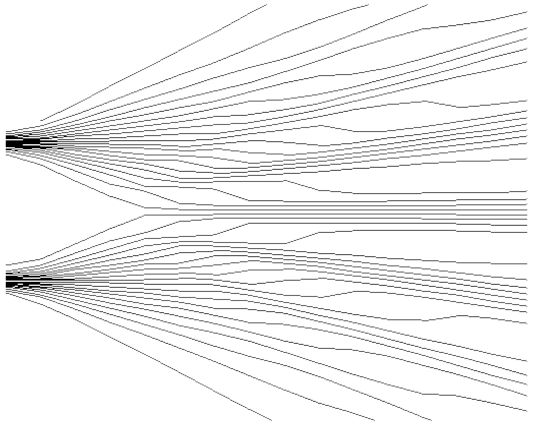


Matching multi particle dBB & Toy model

Klein-Gordon in dBB picture		Geometrical toy model
	$\sqrt{\rho} \equiv \phi$ $S_Q \equiv S_G$ $\hbar^2 \equiv \frac{2(4n-1)}{1-2n}/\kappa$ $M \equiv M_G$	



Matching multi particle dBB & Toy model

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quantum phase	$S_Q \equiv S_G$	H-principal function
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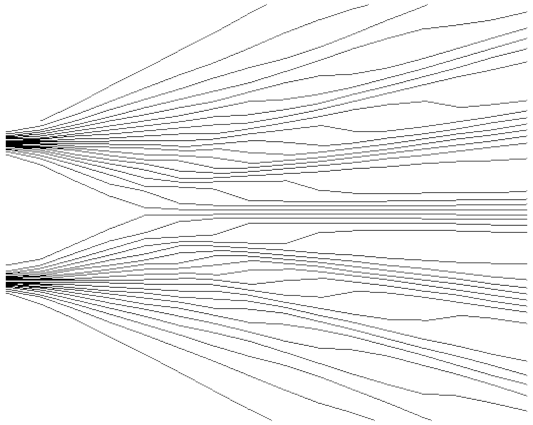
mass	$M \equiv M_G$	mass



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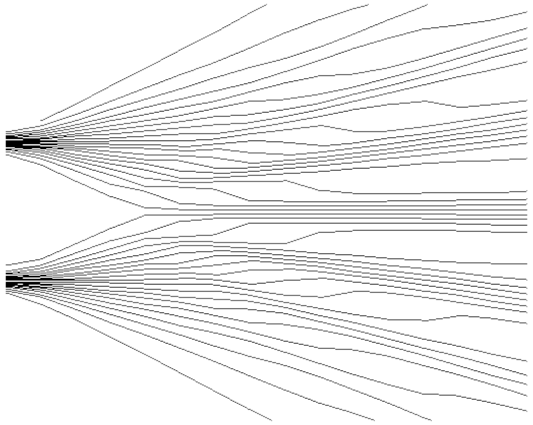

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„Running“ coupling of the geometrical toy model