SPECIAL SESSION ON SPECTRAL THEORY AND MATHEMATICAL PHYSICS (CODE SS 9A)

Organizers

Bruno Nachtergaele, University of California, Davis
Rafael Tiedra, Pontificia Universidad Católica de Chile

First Joint Meeting
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Multiple Commutator Estimates for Unitary Operators

Olivier Bourget *

Abstract

The spectral theory of unitary operators is related to the study of periodic time-dependent quantum systems. In this talk, we build over a previous adaptation of Mourre theory to unitary operators and show how multiple commutator estimates lead to C^k smoothness of the corresponding resolvent. Applications are given.

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Analysis of the Boltzmann collision kernel via an N particle stochastic system

Eric Carlen *

Abstract

In 1956, Mark Kac proposed a novel approach to the study of the Boltzmann equation via the large N limit of a stochastic system of N particle undergoing binary collisions. In the 1960's, Henry McKean and his students made many significant contributions to this program, particularly with regard to the problem of propagation of chaos. However, analysis of the rate of equilibration for this model remained an open problem for many years, and progress on this front was much more recent, and until now, had been made only for "Maxwellian molecules". Recent work of myself, Carvalho and Loss extends this progress to the physically significant hard-sphere case, as will be explained in this lecture.

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Some simple examples of exactly soluble spin-1/2 and spin-1 Bose systems

Marco Hector Corgini Videla *

Abstract

For simple Bose atom systems whose energy operators are diagonal in the so-called number operators and their ground state have internal two or three level structure with negative energies, exact expressions for the limit free canonical energy and pressure are obtained. The existence of non-conventional Bose-Einstein condensation is also proved.

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Critical Points for Sojourn Time

Víctor H. Cortés and M.A. Astaburuaga *

Abstract

Consider a self-adjoint operator H on a separable Hilbert space $\mathcal{H}$ with non trivial absolutely continuous component. We study general properties of the real valued functional $\tau_H(\psi) = \int_{\mathbb{R}} |(e^{-itH}\psi, \psi)|^2 dt$, which in Quantum Mechanics represents the sojourn time (or life time) of an initial state $\psi \in \mathcal{H}$.

We characterize the critical points of the sojourn time τ_X, of the operator multiplication by x in $L^2(\mathbb{R})$, and prove that it attains a global maximum in the unit sphere of the Sobolev Space $\mathcal{W}^{1,2}(\mathbb{R})$.

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Representations of affine Hecke algebras related to Macdonald spherical functions

Jan Felipe van Diejen *

Abstract

For any reduced crystallographic root system, we introduce a unitary representation of the (extended) affine Hecke algebra in a Hilbert space of complex functions on the weight lattice. We describe the spectral decomposition of the center in terms of Macdonald's spherical functions. This talk is based on joint work with Erdal Emsiz.

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A nonlinear ODE associated to the quantum sojourn time

Claudio Fernández *

Abstract

We study the nonlinear ordinary differential equation on the half line,

$$-\psi'' + \psi - \lambda \frac{\psi^3}{x} = 0$$

in $[0, \infty)$, with Dirichlet boundary condition at the origin. This equation arises when studying the local maxima of the sojourn time for a free quantum particle whose states belong to an adequate subspace of the unit sphere of the corresponding Hilbert space. We establish several results concerning the existence and asymptotic behavior of the solutions.

These results allow us to discuss a notion of the different type of spectra for the corresponding nonlinear operator $H\psi = -\psi'' + \psi - \frac{\psi^3}{x}$.

Above is a joint work with R. Benguria and P. Duclos (Q.E.P.D.)

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2D Schrödinger with a strong magnetic field: dynamics and spectral asymptotics near boundary

Victor Ivrii *

Abstract

We consider Magnetic Schrödinger operator and study its spectral asymptotics near boundary. Corresponding classical dynamics associated with operator in question inside of domain is a cyclotron movement combined with slow drift movement along level lines of potential. However near boundary dynamics consists of hops along it; this hop dynamics could be torn away from the boundary and become an inner dynamics and v.v. This classical dynamics has profound implications for spectral asymptotics (with rather sharp remainder estimate). We consider also the case of superstrong magnetic field when classical dynamics is at least applicable but the difference between Dirichlet and Neumann boundary conditions are the most drastic.

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Quantum diffusion and eigenvector delocalization for random band matrices

Antti Knowles *

Abstract

The general formulation of the universality conjecture for disordered systems states that there are two distinctive regimes depending on the energy and the disorder strength. In the strong disorder regime, the eigenvectors are localized and the local spectral statistics are Poisson. In the weak disorder regime, the eigenvectors are delocalized and the local statistics coincide with those of a Gaussian matrix ensemble. Random band matrices represent natural intermediate models to study the eigenvalue statistics and quantum propagation in disordered systems, as they interpolate between mean-field-type Wigner matrices and random Schrödinger operators. In particular, band matrices provide a means of probing the localization-delocalization transition.

I shall report on recent joint work with L. Erdos. We consider a large class of random band matrices H with band width W, and prove that the quantum time evolution generated by H is diffusive up to time scales of order $W^{d/3}$, where d is the number of spatial dimensions. We also derive an explicit formula for the diffusion constant. As a corollary, we prove that the localization length of an arbitrarily large majority of eigenvectors is larger than a factor $W^{d/6}$ times the band width W.

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Spectral results for Rieffel pseudodifferential operators

Marius Mantoiu *

Abstract

The Rieffel calculus is a general pseudodifferential theory associated to actions of a vector group on spaces or algebras. We use it to state and solve several problems in spectral analysis.

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The Quantum Hall Effect Revisited

Spyridon Michalakis *

Abstract

Maxwell believed that a magnetic field through a conductor affected the conductor alone and not the distribution of its electrons. Edwin Hall, a student at the time at Johns Hopkins, found Maxwell's view peculiar so he set out to test the hypothesis by placing a thin gold plate in a transverse magnetic field. He observed that though a current was supplied in the x-direction of the plate, the magnetic field caused the roaming electrons to drift in the y-direction - now known as the Hall current. A century later, von-Klitzing observed that, under strong magnetic fields, the Hall conductance (the inverse of the resistance the Hall current felt) was quantized in integer multiples of e^2/h (e = electronic charge, h = plank's constant). Five years later he was awarded the Nobel prize in physics. In the meantime, theoretical physicist Robert Laughlin offered an elegant heuristic/explanation for the integer quantum hall effect and was awarded the Nobel prize for his insight into the Fractional Quantum Hall Effect. Since then, an important outstanding problem in Mathematical Physics has been the explanation of the Hall quantization within a realistic framework of interacting electrons without a so-called "flux averaging" assumption. Using recently developed theoretical work on Lieb-Robinson bounds, I will present a proof that the Hall conductance of a tight-binding 2-D model of hopping electrons with finite range interactions is quantized exactly in the thermodynamic limit. More importantly, the proof avoids the "flux averaging" assumption that has plagued all previous attempts. This is joint work with M. Hastings.

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On the infrared problem in nonrelativistic QED

Alessandro Pizzo *

Abstract

The fact that photons are massless particles introduces substantial difficulties into the mathematical analysis of the interactions between nonrelativistic quantum matter and the quantized radiation field. These difficulties are known as the infrared problem in (nonrelativistic) QED. This issue is of particular interest in atomic physics. After a review of the different aspects of the infrared problem, I report on recent progress in tackling some of the open mathematical questions. I describe some novel spectral techniques based on multiscale analysis.

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On the existence of solutions for equations with infinitely many derivatives

P. Górka[*] , H. Prado[**] and E. G. Reyes [†]

Abstract

We study existence, uniqueness and regularity of solutions to equations with infinitely many derivatives. Our main example is the generalized bosonic string equation

$$\Delta e^{-c\Delta} = U(x, \phi), \quad c > 0, \tag{1}$$

on Euclidean space $\mathbb{R}^n$. If $U(x, \phi) = \phi(x) + h(x)$, in which $h \in L^2(\mathbb{R}^n)$, we prove that there exists a unique *real-analytic* solution to the euclidean bosonic string (1) in a Hilbert space $\mathcal{H}^{c,\infty}(\mathbb{R}^n)$ which is embedded into the Sobolev spaces $\mathcal{H}^s(\mathbb{R}^n)$ for each $s > 0$. If U in (1) is nonlinear function of ϕ, we show that, the Euclidean Equation (1) on $\mathbb{R}^n$ admits *real-analytic* solutions under some symmetry and growth assumptions on U. Moreover, the equation (1) also admits real-analytic solutions assuming only that $U(x, \phi)$ is suitably near $U_0(x, \phi) = \phi$.

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Quantization of edge currents along magnetic barriers and magnetic guides

Gueorgui Raykov *

Abstract

We investigate the edge conductance of particles submitted to an Iwatsuka magnetic field, playing the role of a purely magnetic barrier. We also consider magnetic guides generated by generalized Iwatsuka potentials. In both cases we prove quantization of the edge conductance. Next, we consider magnetic perturbations of such magnetic barriers or guides, and prove stability of the quantized value of the edge conductance. Further, we establish a sum rule for edge conductances. Regularization within the context of disordered systems is discussed as well.

This is a joint work with François Germinet (Cergy-Pontoise) and Nicolas Dombrowski (Santiago).

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Diffusion of waves in a random environment: problems and results

Jeffrey Schenker *

Abstract

I will discuss the problem of proving diffusion of waves in a random environment in the context of the lattice Schroedinger equation. A major difficulty that arises is recurrence – return of portions of the wave packet to regions previously visited. I will show that, if recurrence is eliminated by making the environment evolve randomly in time, then diffusion results in an elementary way.

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On Universially Typical Sets

Ruedi Seiler *

Abstract

Universially typical sets play an interesting role in compression of data. In this talk I discuss and present results on the question of lifting know results on ergodic information sources from the 1-dimensional setting to the multidimensional setting. Although classical, the results are prerequesits for an analogous analysis of quantum ergodic sources. The question was motivated by the attempt to find a multidimensional Lempel-Ziv algorithm.

This is joint work with Tyll Krüger, Guido Montafur and Rainer Siegmund-Schultze.

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Stability and Absence of Binding for Multi-Polaron Systems

Robert Seiringer *

Abstract

The polaron is a model of an electron interacting with the quantized polarization field of a dielectric crystal. We prove stability of matter for such systems, i.e., we show that the energy per particle is bounded, independently of the number of particles. We also show that if the coupling constant of the electron-phonon interaction is sufficiently small, polarons do not bind.

(This is joint work with R. Frank, E. Lieb and L. Thomas.)

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Lieb-Robinson Bounds in Many Body Physics

Robert Sims *

Abstract

We review some recent progress on quasi-locality results for the dynamics of quantum many body systems. Briefly, estimates, known as Lieb-Robinson bounds, demonstrate that the time evolution of an observable initially supported in a particular region of space essentially depends only on those degrees of freedom located at a distance $d < vt$ away from the initial support. The number $v > 0$ is called the Lieb-Robinson velocity of the system under consideration. Such bounds have proven useful in a number of recent applications.

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Low Energy Spectrum of the Quantum Heisenberg Ferromagnet

Shannon Starr *

Abstract

The quantum Heisenberg ferromagnet remains a challenge for mathematicians. On the basis of linear spin wave theory, it is widely accepted that the model spontaneously magnetizes at sufficiently low positive temperatures in dimensions three and higher. But that has not been proved mathematically, although the corresponding result for the antiferromagnet has been proved by Dyson, Lieb and Simon. With Conomos, a student, we had proved that the spectral gap of the ferromagnet is correctly predicted by linear spin wave theory on boxes. In particular the gap is $O(1/L^2)$ for a box of sidelength L. With Nachtergaele and Spitzer we can extend this to prove that all energies on the order $O(1/L^2)$ are well approximated by linear spin wave theory. To prove a phase transition, one would want to raise the energy scale to $O(1)$. I'll describe our work.

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Limit Theorems for the Asymmetric Simple Exclusion Process

Craig A. Tracy* and Harold Widom **

Abstract

The asymmetric simple exclusion process (ASEP) is a continuous time Markov process of interacting particles on the integer lattice $\mathbb{Z}$ with state space $\{0,1\}^{\mathbb{Z}}$. The dynamics for this process are given asfollows: Each particle has an exponential "alarm clock" which rings at rate one. When the alarm goes off the particle flips a coin and with probability p attempts tojump one step to the right and with probability $q = 1 - p$ attempts to jump one step to the left. If there is a particle at the destinationat that instant, the jump is suppressed and the alarm is reset. All clocks are independent. A rigorous construction of this process was given by Liggett.

In this talk we report on exact formulas for the distribution of the current (equivalently, the height function) in ASEP as well as various limit theorems. We consider both step initial condition and step Bernoulli initial condition. We conclude with a discussion of the relevance of these results to the Kardar-Parisi-Zhang equation in the theory of stochastic growth processes as well as the recent experimental results of Takeuchi and Sano.

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