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Optimal shapes for lattice point counting

Abstract:

What shape of domain minimizes the n—th eigenvalue of the Dirichlet Laplacian, for large n? (Here we normalize the area to equal 1.) Does the minimizer approach a disk as n tends to infinity? Supporting this idea is the discovery by Antunes and Freitas that among rectangles, the minimizer approaches a square in the limit. Their result for rectangles relies on lattice point counting in ellipses. In joint work with Shiya Liu (University of Illinois), we extend to more general lattice counting problems, proving again that the "most balanced" situation is optimal in the limit. Our work in progress aims to connect these insights back to spectral problems.