Radiación de Cherenkov

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$$\left(-\nabla^2 + \epsilon(\omega)\frac{\partial^2}{c^2\partial t^2}\right)\phi(\mathbf{x}, t) = \frac{4\pi}{\epsilon(\omega)}\rho(\mathbf{x}, t)$$

$$\left(-\nabla^2 + \epsilon(\omega)\frac{\partial^2}{c^2\partial t^2}\right)\mathbf{A}(\mathbf{x}, t) = \frac{4\pi}{c}\mathbf{J}(\mathbf{x}, t)$$

$$\rho(\mathbf{x}, t) = ze\delta(\mathbf{x} - \mathbf{v}t)$$

$$\mathbf{J}(\mathbf{x}, t) = \mathbf{v}\rho(\mathbf{x}, t)$$

$$F(\mathbf{x}, t) = \frac{1}{(2\pi)^2} \int d^3k \int d\omega \, F(\mathbf{k}, w) e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}$$

$$\left(\mathbf{k}^2 - \epsilon(\omega)\frac{\omega^2}{c^2}\right) \phi(\mathbf{k}, \omega) = \frac{4\pi}{\epsilon(\omega)} \rho(\mathbf{k}, w)$$

$$\left(\mathbf{k}^2 - \epsilon(\omega)\frac{w^2}{c^2}\right) \mathbf{A}(\mathbf{k}, \omega) = \frac{4\pi}{c} \mathbf{J}(\mathbf{k}, \omega)$$

$$\rho(\mathbf{k}, \omega) = \frac{ze}{2\pi} \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

$$\mathbf{J}(\mathbf{k}, \omega) = \mathbf{v}\rho(\mathbf{k}, \omega)$$

$$\phi(\mathbf{k},\omega) = \frac{2ze}{\epsilon(\omega)} \frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v})}{\mathbf{k}^2 - \epsilon(\omega)\frac{w^2}{c^2}}$$

$$\mathbf{A}(\mathbf{k},\omega) = \epsilon(\omega)\frac{\mathbf{v}}{c}\phi(\mathbf{k},\omega)$$

$$\mathbf{E}(\mathbf{k},\omega) = i\left[\epsilon(\omega)\omega\frac{\mathbf{v}}{c^2} - \mathbf{k}\right]\phi(\mathbf{k},\omega)$$

$$\mathbf{B}(\mathbf{k},\omega) = i\epsilon(\omega)\mathbf{k}\times\frac{\mathbf{v}}{c}\phi(\mathbf{k},\omega)$$

- La partícula cargada se mueve a lo largo del eje x

- Se observan los campos a una distancia b perpendicular al movimiento de la partícula

$$\mathbf{E}(\omega) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \, \mathbf{E}(\mathbf{k}, \omega) e^{ibk_2}$$

$$E_1(\omega) = \frac{2ize}{\epsilon(\omega)(2\pi)^{\frac{3}{2}}} \int d^3k \, e^{ibk_2} \left[\frac{\epsilon(\omega)\omega v}{c^2} - k_1 \right] \frac{\delta(\omega - vk_1)}{\mathbf{k}^2 - \epsilon(\omega)\frac{w^2}{c^2}}$$
$$= -\frac{2ize\omega}{v^2(2\pi)^{\frac{3}{2}}} \left[\frac{1}{\epsilon(\omega)} - \beta^2 \right] \int dk_2 \, e^{ibk_2} \int \frac{dk_3}{k_2^2 + k_3^2 + \lambda^2}$$

Donde

$$\lambda^2 = \frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}\epsilon(\omega) = \frac{\omega^2}{v^2}[1 - \beta^2\epsilon(\omega)]$$

$$E_1(\omega) = -\frac{iz e\omega}{v^2(2\pi)^{\frac{1}{2}}}\left[\frac{1}{\epsilon(\omega)} - \beta^2\right]\int \frac{e^{ibk_2}dk_2}{(k_2^2+\lambda^2)^{\frac{1}{2}}}$$
$$= -\frac{iz e\omega}{v^2}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left[\frac{1}{\epsilon(\omega)} - \beta^2\right]K_0(\lambda b)$$

Análogamente

$$E_2(\omega) = \frac{ze\omega}{v}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\frac{\lambda}{\epsilon(\omega)}K_1(\lambda b)$$

$$B_3(\omega) = \epsilon(\omega)\beta E_2(\omega)$$

$$
\begin{aligned}
\left(\frac{dE}{dx}\right)_{b>a} &= \frac{1}{v}\frac{dE}{dt} \\
&= -\frac{c}{4\pi v}\int_{-\infty}^{\infty} 2\pi a B_3 E_1 \, dx \\
&= -\frac{ca}{2}\int_{-\infty}^{\infty} B_3(t)E_1(t)\, dt \\
&= -ca\,\mathrm{Re}\int_0^{\infty} B_3^*(\omega)E_1(\omega)\, d\omega
\end{aligned}
$$

Para $|\lambda a| >> 1$, se usas las formas asintóticas de las funciones de *Bessel*

$$E_1(\omega, a) \to i\frac{ze\omega}{c^2}\left[1 - \frac{1}{\beta^2\epsilon(\omega)}\right]\frac{e^{-\lambda a}}{\sqrt{\lambda a}}$$

$$E_2(\omega, a) \to \frac{ze}{v\epsilon(\omega)}\sqrt{\frac{\lambda}{a}}e^{-\lambda a}$$

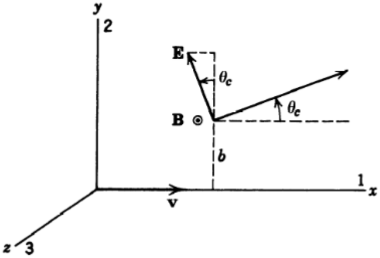
$$B_3(\omega, a) \to \beta\epsilon(\omega)E_2(\omega, a)$$

$$(-caB_3^* E_1) \to \frac{z^2e^2}{c^2}\left(-i\sqrt{\frac{\lambda^*}{\lambda}}\right)\omega\left[1 - \frac{1}{\beta^2\epsilon(\omega)}\right]e^{-(\lambda+\lambda^*)a}$$

Para $\epsilon(\omega)$ Real y $\beta^2\epsilon(\omega) > 1$ entonces λ es imaginario puro

$$v > \frac{c}{\sqrt{\epsilon(\omega)}}$$

$$\left(\frac{dE}{dx}\right)_{\text{rad}} = \frac{z^2e^2}{c^2}\int_{\epsilon(\omega)\beta^2>1}\omega\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)\,d\omega$$

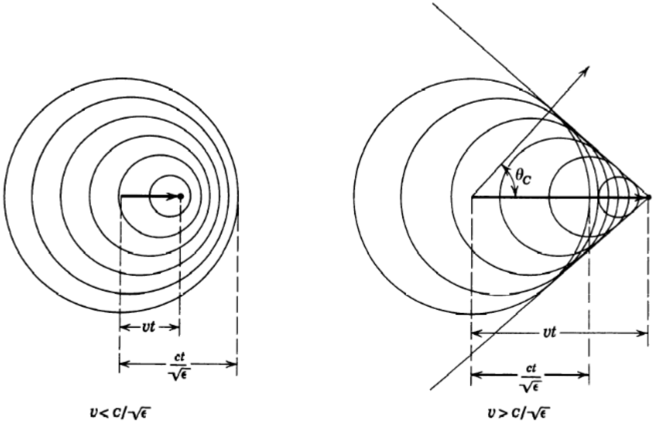


[1]

$$\tan\theta_C = -\frac{E_1}{E_2}$$

$$\cos\theta_C = \frac{1}{\beta\sqrt{\epsilon(\omega)}}$$

[1] Jackson, John David. Classical Electrodynamics. 3rd ed. New York: Wiley, 1999. Print. p639, fig. 13.4



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[2] Jackson, John David. Classical Electrodynamics. 3rd ed. New York: Wiley, 1999. Print. p639, fig. 13.5

$$\mathbf{A}(\mathbf{x},t) = \frac{2ze}{(2\pi)^2}\boldsymbol{\beta}\int d^3k\,\frac{e^{ik_1(x-vt)}e^{i\mathbf{k}_\perp\cdot\boldsymbol{\rho}}}{k_1^2(1-\beta^2\epsilon)+k_\perp^2}$$

$$\begin{aligned}\mathbf{A}(\mathbf{x},t) &= \boldsymbol{\beta}\frac{2ze}{\sqrt{(x-vt)^2-(\beta^2\epsilon-1)\rho^2}}\\ &= \boldsymbol{\beta}\frac{2ze}{\sqrt{(x-vt)^2-\rho^2\cot^2(\frac{\pi}{2}-\theta_C)}}\end{aligned}$$

Aplicaciones

- Permite calcular la velocidad de partículas veloces
- Detector de neutrinos *Super-Kamiokande*
- En reactores nucleares

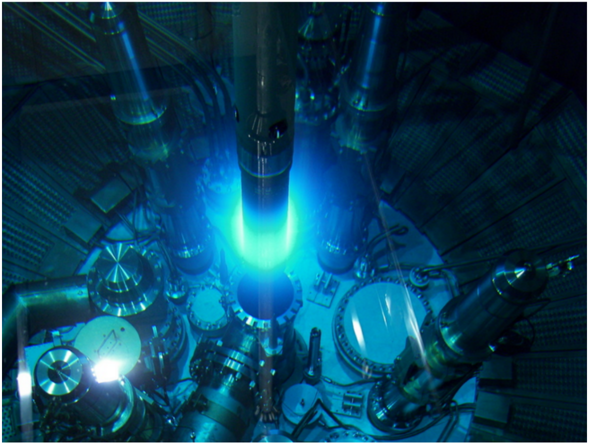

Figura: Cherenkov radiation in the reactor pool of the FRM II

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[3](Picture: Jürgen Neuhaus, FRM II)

Referencias

- Jackson, John David. Classical Electrodynamics. 3rd ed. New York: Wiley, 1999. Print.