QUANTUM GRAVITY PHENOMENOLOGY

Jorge Alfaro
Pontificia Universidad Católica de Chile
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Summary

Quantum gravity issues

-Cosmic Inflation

During a brief period of time, the size of the universe growth exponentially.

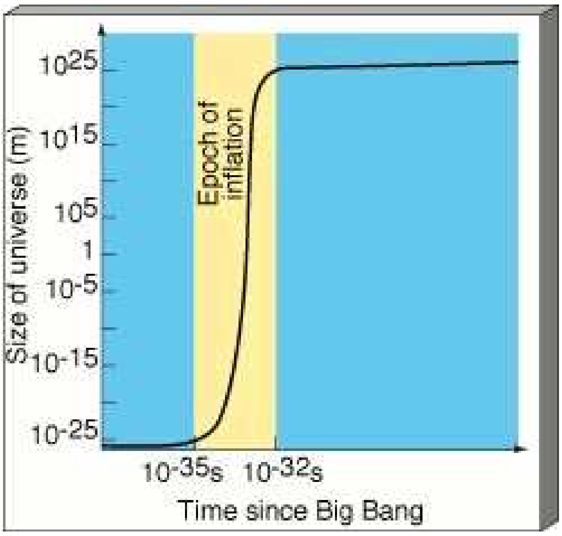


Figure 1.

Inflation predicts that the Universe is flat. WMAP satellite data on the CBMR support it.

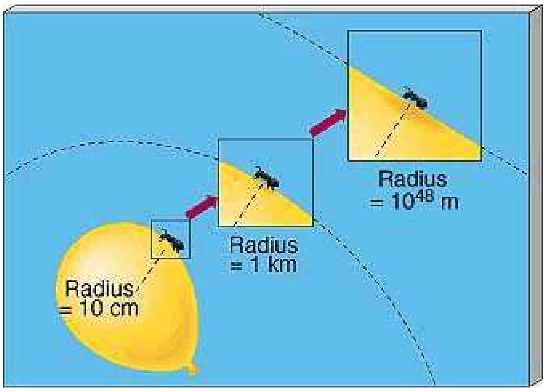


Figure 2.

QUANTUM GRAVITY?

- Gravity can be neglected compared to electroweak or strong forces among elementary particles, BUT at an energy of $M_P \sim 10^{19}$Gev becomes the dominant interaction.Proton mass$\sim$1Gev.

-The Universe at times $t < 10^{-35}$s after Big Bang, had an energy per particle $\sim M_P$.

-Gravity determines the initial conditions for the evolution of the Universe.

-At those early times, the Universe has atomic size, so Quantum Mechanics must be used to describe it.

- General Relativity is not RENORMALIZABLE

$$S = \int d^d x \sqrt{-g} \left(\frac{1}{2\kappa} R + \mathcal{L}_M \right)$$

Here $\kappa = \frac{8\pi G}{c^4}$ has dimension of M^{-2} rendering the perturbative series non-renormalizable.

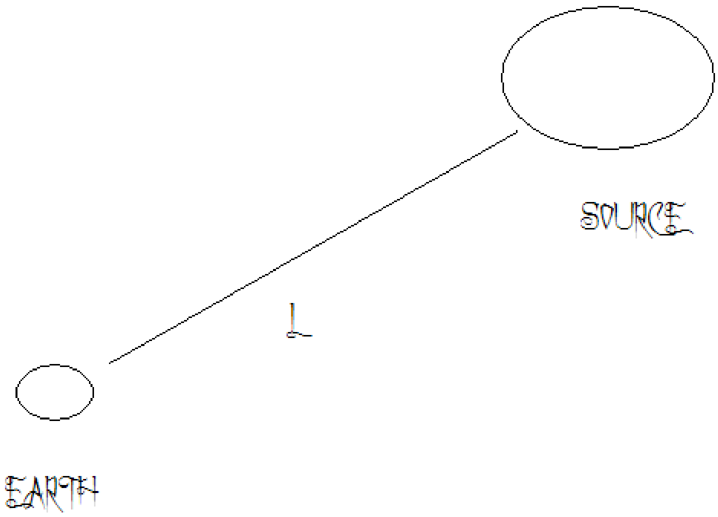
-Two roads to Quantum Gravity:

i) String Theory. Basic objects are not point particles but one dimensional structures(strings). The theory is finite, has a pletora of vacua. Unify all forces d=10,26.

ii) Loop Quantum Gravity. Predicts space is discrete, black hole entropy follows from the Quantum Geometry. Continuum limit difficult.

Gamma Ray Bursts

G. Amelino-Camelia et al., Nature 393(1998)763.



$$L \sim 10^{10} \text{ Light-years}$$

PROPOSITION:

$$\delta v \sim \frac{E}{E_{QG}}$$

produces the structure of GRB with $\Delta t \leq 10^{-3}$ s.
$E_{QG} \sim E_{Planck} = 10^{19}$ Gev.

Dispersion Relation due to Quantum Gravity corrections(Strings)

$$c^2 \vec{p}^{\,2} = E^2 [1 + f(\frac{E}{E_{QG}})]$$

For $E << E_{QG}$, we have

$$c^2 \vec{p}^{\,2} = E^2 [1 + \chi \frac{E}{E_{QG}}], \chi \sim \pm 1$$

Speed of propagation

$$v = \frac{\partial E}{\partial p} \sim c(1 - \chi \frac{E}{E_{QG}})$$

PARTICULAR FEATURES OF THE EFFECT

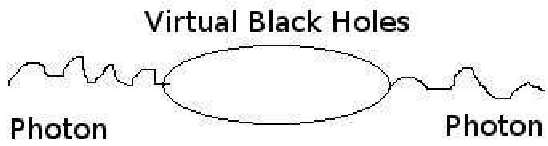
The correction induced by QG effects grows with the energy E. Instead, in a normal medium, it decreases **with E.**

The delay is very small, **EXCEPT** if the wave travels a very large distance L:

$$\Delta t = \chi \frac{L}{c} \frac{E}{E_{QG}}$$

Typical photon energies in GRB: $\sim 0.1 ------ 100 Mev ----> Tev$

With $L \sim 10^{10} light-years$, we get $\Delta t \sim 10^{-3}$ seconds, for $E_{EG} \sim E_P$, which is the right order of magnitude.



Quantum Gravity Vacuum is populated by virtual black holes

--> Non-covariant Dispersion relation as in media at $T \neq 0$.

R. Gambini and J. Pullin, Phys. Rev. D59(1999)124021.

Modified Maxwell Equations

$$\partial_t \vec{E} = -\vec{\nabla} \times \vec{B} + 2\chi l_P \vec{\nabla}^2 \vec{B}$$
$$\partial_t \vec{B} = \vec{\nabla} \times \vec{E} - 2\chi l_P \vec{\nabla}^2 \vec{E}$$
$$|\chi| \sim 1$$

Solutions with definite helicity

$$\vec{E}_\pm = Re((\hat{e}_1 \pm i\hat{e}_2)e^{i(\Omega_\pm t - \vec{k}.\vec{x})}$$
$$\Omega_\pm = \sqrt{k^2 \mp 4\chi l_P k^3}$$
$$\sim k(1 \mp 2\chi l_P k)$$

1. Birefringent Effect: The speed of propagation depends on the helicity.
2. It is diferent from the effect found by Amelino-Camelia et al.
3. The vacuum $|\Delta>$ violates parity.

Neutrinos and Quantum Gravity

J.A., H. Morales-Técotl and L.F. Urrutia, Phys. Rev. Lett. 84(2000)2318.

Modified neutrino Wave equations

$$\left[i\hbar\frac{\partial}{\partial t} - i\hbar\hat{A}\vec{\sigma}\cdot\nabla + \frac{\hat{C}}{2\mathcal{L}} \right]\xi(t,\vec{x}) + m(\alpha - \beta i\hbar\vec{\sigma}\cdot\nabla)i\sigma_2\xi^*(t,\vec{x}) = 0,$$

The dispersion relation corresponding to it is:

$$E^2_\pm(p,\mathcal{L}) = (A^2 + m^2\beta^2)p^2 + m^2\alpha^2 + \left(\frac{C}{2\mathcal{L}}\right)^2 \pm Bp, \qquad B = A\left(\frac{C}{\mathcal{L}} + 2\alpha\beta m^2\right), \tag{1}$$

where A, B, C have been expressed in momentum space and depend on $\mathcal{L}$. The $\pm$ in Eq. (1) stand for the two neutrino helicities. Let us emphasize that the solution $\xi(t,\vec{x})$ to Eq.(3) is given by an appropriate linear combination of plane waves and helicity eigenstates, given that the neutrinos considered are massive.

Ultra High Energy Cosmic Rays

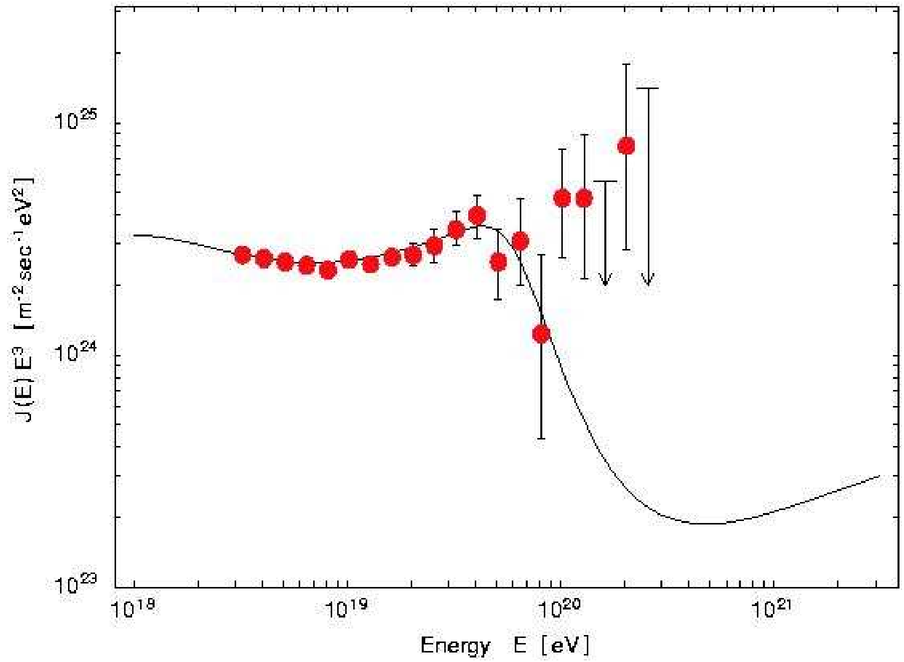
In this talk we are concerned with the observation of ultra high energy cosmic rays (UHECR), i.e. those cosmic rays with energies greater than $\sim 4 \times 10^{18}$ eV.

- Although not completely clear, it has been suggested that these high energy particles are possibly heavy nuclei (we will assume here that they are protons).
- By virtue of the isotropic distribution with which they arrive to us, they originate in extragalactic sources.

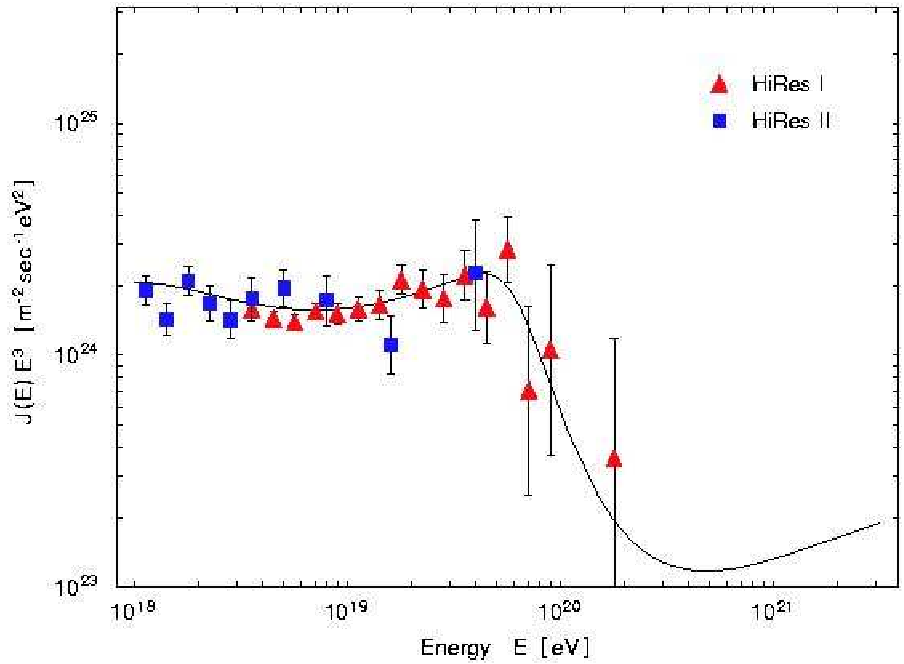
The Greisen-Zatsepin-Kuz'min (GZK) cutoff

-Their propagation in open space is affected by the cosmic microwave background radiation (CMBR), producing a friction on UHECR making them release energy in the form of secondary particles and affecting their possibility to reach great distances.
- Cosmic rays with energies above 1×10^{20} eV should not travel more than ~ 100 Mpc.

The Greisen-Zatsepin-Kuz'min (GZK) cutoff. Data



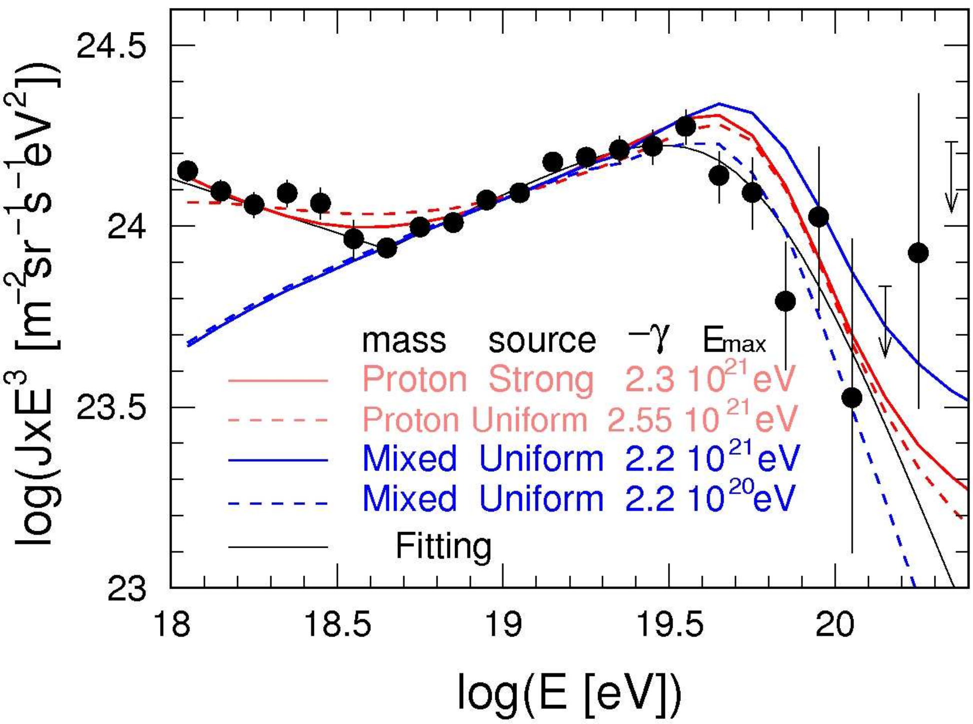
UHECR spectrum and AGASA observations. The figure shows the UHECR spectrum $J(E)$ multiplied by E^3, for uniform distributed sources, without evolution, and with a maximum generation energy $E_{\max} = \infty$.



UHECR spectrum and HiRes observations. The figure shows the UHECR spectrum $J(E)$ multiplied by E^3.

The Auger Observatory has recently reported his observations on the highest energy cosmic rays.

They see the GZK cutoff in the flux. But still some of the cosmic rays have a trans GZK energy. This means that Lorentz invariance violation may be necessary to explain their presence, if nearby sources of such cosmic rays are not found.



The combined energy spectrum multiplied by E^3, and the predictions of three astrophysical models. The input assumptions of the models (mass composition at the sources, the source distribution, spectral index and exponential cutoff energy per charge at the acceleration site) are indicated in the figure.

LIV of the integration measure: JA, Phys. Rev.Lett. 94,221302(2005)

- **The main effect of QG is to deform the measure of integration of Feynman graphs at large four momenta by a tiny LIV. The classical lagrangian is unchanged.**
  Equivalently, we can say that QG deforms the metric of space-time, introducing a tiny LIV proportional to (d-4)α, d being the dimension of space time in Dimensional Regularization and α is the only arbitrary parameter in the model.
- Such small LIV could be due to quantum fluctuations of the metric of space-time produced by QG:virtual black holes , D-branes, compactification of extra-dimensions or spin-foam anisotropies. A precise derivation of α will have to wait for additional progress in the available theories of QG.An intriguing possibility may be provided by the anisotropy between spatial and temporal directions found necessary to recover our universe at macroscopic scales in a recent numerical simulation of Quantum Gravity (Ambjorn).
- Within the Standard Model, such LIV implies several remarkable effects, which are wholly determined up to one arbitrary parameter (α).The main effects are:
- The maximal attainable velocity for particles is not the speed of light, but depends on the specific couplings of the particles within the Standard Model.
  Also birrefringence occurs for charged leptons, but not for gauge bosons. In particular, photons and neutrinos have different maximum attainable velocities. This could be tested in the next generation of neutrino detectors such as NUBE.
- Vertices in the SM will pick up a finite LIV.

Cutoff regulator:

- To see what are the implications of the asymmetry in the measure for renormalizable theories, we will mimic the Lorentz asymmetry of the measure by the replacement
$$\int d^dk -> \int d^dk\, R(\frac{k^2+\alpha k_0^2}{\Lambda^2})$$
- Here R is an arbitrary function, Λ is a cutoff with mass dimensions, that will go to infinity at the end of the calculation. We normalize $R(0) = 1$ to recover the original integral. $R(\infty) = 0$ to regulate the integral. α is a real parameter. Notice that we are assuming that rotational invariance in space is preserved. More general possibilities such as violation of rotational symmetry in space can be easily incorporated in our formalism.
- This regulator has the property that for logarithmically divergent integrals, the divergent term is Lorentz invariant whereas when the cutoff goes to infinity a finite LIV part proportional to α remains.

One Loop:Bosons

- Let D be the naive degree of divergence of a One Particle Irreducible (1PI) graph. The change in the measure induces modifications to the primitively log divergent integrals(D=0) In this case, the correction amounts to a finite LIV.

The finite part of 1PI Green functions will not be affected. Therefore, Standard Model predictions are intact, except for the maximum attainable velocity for particles, which receives a finite wholly determined contribution from Quantum Gravity.

- Let us analyze the primitivily divergent 1PI graphs for bosons first.
  Self energy: $\chi(p) = \chi(0) + A^{\mu\nu} p_\mu p_\nu + c\ o\ n\ v\ e\ r\ g\ e\ n\ t$, $A^{\mu\nu} = \frac{1}{2}\partial_\mu\partial_\nu\chi(0)$. We have:

  $$A^{\mu\nu} = c_2\eta^{\mu\nu} + a^{\mu\nu}$$

  c_2 is the log divergent wave function renormalization counterterm; $a^{\mu\nu}$ is a finite LIV. The on-shell condition is:

  $$p^2 - m^2 - a^{\mu\nu} p_\mu p_\nu = 0$$

  If spatial rotational invariance is preserved, the nonzero components of the matrix a are:

  $$a^{00} = a_0; \quad a^{ii} = -a_1$$

  So the maximum attainable velocity for this particle will be:

  $$v_m = \sqrt{\frac{1-a_1}{1-a_0}} \sim 1 - (a_1 - a_0)/2 \tag{2}$$

One Loop:Fermions

For fermions, we have the self energy graph

$$\Sigma(p) = \Sigma(0) + s_{\mu\nu}\gamma_\nu p_\mu$$

$s_{\mu\nu}\gamma_\nu = \partial_\mu\Sigma(0)$. Moreover

$$s_{\mu\nu} = s\eta_{\mu\nu} + a_{\mu\nu}/2$$

s is a log divergent wave function renormalization counterterm; $a_{\mu\nu}$ is a finite LIV. The maximum attainable velocity of this particle will be given again by equation (3).

One Loop:Gauge bosons

Consider the most general quadratic Lagrangian which is gauge invariant, but could permit LIV's [1]

$$L = c^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$c^{\mu\nu\alpha\beta}$ is antisymmetric in $\mu\nu$ and $\alpha\beta$ and symmetric by $(\alpha,\beta) <-> (\mu,\nu)$ It implies that the most general expression for the self-energy of the gauge boson will be

$$\Pi^{\nu\beta}(p) = c^{\mu\nu\alpha\beta} p_\alpha p_\mu \Pi(p)$$

We see that

$$p_\nu \Pi^{\nu\beta}(p) = 0$$

$c^{\mu\nu\alpha\beta}$ is given by a logarithmically divergent integral.
We get:

$$c^{\mu\nu\alpha\beta} = c_2(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\beta}\eta^{\nu\alpha}) + a^{\mu\nu\alpha\beta}$$

c_2 is a Lorentz invariant counterterm and $a^{\mu\nu\alpha\beta}$ is a LIV.

1. A Chern-Simons term is absent due to the symmetry $k_\mu -> -k_\mu$, which is preserved by the regulator.

It is clear that the same argument applies to massive gauge bosons that got their mass by spontaneous gauge symmetry breaking as well as to the graviton in linearized gravity.

LIV Dimensional Regularization

We generalize dimensional regularization to a d dimensional space with an arbitrary constant metric $g_{\mu\nu}$. We work with a positive definite metric first and then Wick rotate. We will illustrate the procedure with an example. Here $g = det(g_{\mu\nu})$ and $\Delta > 0$.

$$\frac{1}{\sqrt{g}}\int \frac{d^dk}{(2\pi)^d}\frac{k_\mu\, k_\nu}{(k^2+\Delta)^n} =$$

$$\frac{1}{\sqrt{g}\Gamma(n)}\int_0^\infty \mathrm{d}t\, t^{n-1}\int \frac{d^dk}{(2\pi)^d}k_\mu\, k_\nu\, e^{-t(g^{\alpha\beta}k_\alpha\, k_\beta+\Delta)} =$$

$$\frac{1}{(4\pi)^{d/2}}\frac{g_{\mu\nu}}{2}\frac{\Gamma(n-1-d/2)}{\Gamma(n)}\frac{1}{\Delta^{n-1-d/2}}$$

In the same manner, after Wick rotation, we obtain a generalization of dimensional regularization suitable for an arbitrary constant metric.

These definitions preserve gauge invariance, because the integration measure is invariant under shifts. To get a LIV measure, we assume that $g^{\mu\nu} = \eta^{\mu\nu} + (4\pi)^2\alpha\eta^{\mu 0}\eta^{\nu 0}\epsilon$ where $\epsilon = 2 - \frac{d}{2}$. A formerly divergent integral will have a pole at $\epsilon = 0$, so when we take the physical limit, $\epsilon -> 0$, the answer will contain a LIV term.

To define the counterterms, we used the minimal substraction scheme(MSS); that is we substract the poles in ϵ from the 1PI graphs.

LIV Dimensional Regularization reinforces our claim that these tiny LIV's originates in Quantum Gravity. In fact the sole change of the metric of space time is a correction of order ϵ and this is the source of the effects studied above. Quantum Gravity is the strongest candidate to produce such effects because the gravitational field is precisely the metric of space-time and tiny LIV modifications to the flat Minkowsky metric may be produced by quantum fluctuations.

Explicit One loop computations:

We use LIV Dimensional Regularization.

Photons The LIV photon self-energy in the SM is:
$L\Pi^{\mu\nu}(q) = -\frac{23}{3}e^2\alpha\, q_\alpha\, q_\beta(\eta^{\alpha\beta}\delta_0^\mu\delta_0^\nu + \eta^{\mu\nu}\delta_0^\alpha\delta_0^\beta - \eta^{\nu\beta}\delta_0^\mu\delta_0^\alpha - \eta^{\mu\alpha}\delta_0^\nu\delta_0^\beta)$

It follows that the maximal attainable velocity is

$$v_\gamma = 1 - \frac{23}{6}e^2\alpha$$

We have included coupling to quarks and charged leptons as well as 3 generations and color.

Neutrinos: The maximal attainable velocity is

$$v_\nu = 1 - (3 + \tan^2\theta_w)\,\frac{g^2\,\alpha}{8}$$

In this scenario, we predict that neutrinos emitted simultaneously with photons in gamma ray bursts will not arrive simultaneously to Earth . The time delay during a flight from a source situated at a distance D will be of the order of $(10^{-22} - 10^{-23})D/c \sim 10^{-5} - 10^{-6}$ s, assuming $D = 10^{10}$ light-years. No dependence of the time delay on the energy of high energy photons or neutrinos should be observed. Photons will arrive earlier(later) if $\alpha < 0(\alpha > 0)$. These predictions could be tested in the next generation of neutrino detectors such as NUBE.

Using R_ξ-gauges we have checked that the LIV is gauge invariant. The gauge parameter affects the Lorentz invariant part only.

Electron self-energy in the Weinberg-Salam model. Birrefringence:

Define: $e_L = \frac{1-\gamma^5}{2}e$, $e_R = \frac{1+\gamma^5}{2}e$, where e is the electron field. We get $v_L = 1 - (\frac{g^2}{\cos^2\theta_w}(\sin^2\theta_w - 1/2)^2 + e^2 + g^2/2)\frac{\alpha}{2}$;

$v_R = 1 - (e^2 + \frac{g^2\sin^4\theta_w}{\cos^2\theta_w})\frac{\alpha}{2}$

The difference in maximal speed for the left and right helicities is $\sim (10^{-23} - 10^{-24})$.

We see that ratios of LIVťs are α-independent. For instance:

$$\frac{v_L - v_\gamma}{v_R - v_\gamma} = \frac{\frac{10}{3}e^2 - (\frac{g^2}{\cos^2\theta_w}(\sin^2\theta_w - 1/2)^2 + g^2/2)\frac{1}{2}}{\frac{10}{3}e^2 - (\frac{g^2\sin^4\theta_w}{\cos^2\theta_w})\frac{1}{2}}$$

Mesons and Baryons, JA, PRD72:024027,2005

In order to apply our results to the computation of the UHECR spectrum and other phenomena, we must calculate the maximal attainable velocity(MAV) of hadrons. As we mentioned before, the problem is hadronization. One way to get an estimation of the effect is using effective lagrangians.

We use the results of (ecker and fearing) for the wave function renormalization of pions and nucleons in the chiral lagrangian and Heavy Baryon Chiral Perturbation Theory. They get:

$$Z_\pi^{-1} = 1 - \frac{4m_\pi^2}{3(4\pi)^2F^2}\frac{1}{\epsilon} + \text{finite}$$

$$Z_N^{-1} = 1 - \frac{9\,g_A^2\,m_\pi^2}{4(4\pi)^2F^2}\frac{1}{\epsilon} + \text{finite}$$

Here, m_π is the renormalized pion mass, F is the renormalized decay constant of pions and g_A is the axial vector coupling constant, in the chiral limit.

Using the LIV metric, we can read off the MAV for pions and nucleons:

$$c_\pi = 1 + \frac{2m_\pi^2\alpha}{3F^2}$$

$$c_N = 1 + \frac{9m_\pi^2 g_A^2\alpha}{8F^2}$$

Bounds on α

We can get bounds on α, studying the threshold conditions for:

Pair Creation $\gamma + p \to p + e^+ + e^-$, which dominates the spectrum up to an energy $\sim 4 \times 10^{19}$ eV.;

Photo-Pion Production $\gamma + p \to p + \pi$, which determines the spectrum for $E > 8 \times 10^{19}$.

Combining the two reactions and the standard values, $m_\pi = 139Me\,v$, $g_A = 1.26$, $F = 92.4Me\,v$, we get a more stringent bound from recent Auger data:

$-\alpha < 1.3 \times 10^{-24}$

This implies that photons are the fastest particles and they arrive before neutrinos coming from the same source of GRB. Moreover, photons become unstable. They decay in a electron positron pair above an energy E_0. See below.

Since $c_{photon} > c_{proton}$, Proton is stable under Cerenkov radiation in vacuum, which is highly suppressed. So $\alpha < 0$ is prefered for this reason also.

A MORE PRECISE BOUND

Lorentz Invariance Violation and the Observed Spectrum of Ultrahigh Energy Cosmic Rays. S.T. Scully, F.W. Stecker, Published in Astropart.Phys.31:220-225,2009. e-Print: arXiv:0811.2230 [astro-ph]

We present the results of a detailed calculation of the modification of the UHECR spectrum caused by LIV using the formalism of Coleman and Glashow. We then compare these results with the experimental UHECR data from Auger and HiRes. Based on these data, we find a best fit amount of LIV of $4.5^{+1.5}_{-4.5} \times 10^{-23}$,consistent with an upper limit of 6×10^{-23}. This possible amount of LIV can lead to a recovery of the cosmic ray spectrum at higher energies than presently observed. Such an LIV recovery effect can be tested observationally using future detectors.
This means:

$$\delta_{\pi p} < 6 \times 10^{-23}$$

$$\delta_{\pi p} = c_\pi - c_p = \frac{2m_\pi^2\alpha}{3F^2} - \frac{9m_\pi^2 g_A^2\alpha}{8F^2} = 2.53(-\alpha), -\alpha < 2.37 \times 10^{-23}$$

In what follows we used $-\alpha = 10^{-24}$

Photon decay

It has been pointed out in (Coleman,Glashow) that if $c_{photon} > c_{electron}$ then the process $\gamma \to e^+ + e^-$ is allowed above an energy E_0: $E_0 = m_e\sqrt{\frac{2}{\delta c}}$ where $\delta c = c_\gamma - c_e$. In our case, we have:
$\delta c_L = -\alpha(\frac{23}{6}e^2 - (\frac{g^2}{\cos^2\theta_w}(\sin^2\theta_w - 1/2)^2 + e^2 + g^2/2)/2)$
$\delta c_R = -\alpha(\frac{23}{6}e^2 - (e^2 + \frac{g^2\sin^4\theta_w}{\cos^2\theta_w})/2)$
Therefore,

$$E_{L_0} = 1.5 \times 10^9 \,\mathrm{Gev}$$
$$E_{R_0} = 1.3 \times 10^9 \,\mathrm{Gev}$$

We should not detect photons with energies above $1.5 \times 10^9 Gev$

Neutral pion Stability

We study the main decay process of neutral pion $\pi_0 \to \gamma + \gamma$ .This becomes forbidden if $c_\gamma > c_\pi$ and above an energy
$E_\pi = \frac{m_\pi}{\sqrt{2(c_\gamma - c_\pi)}}$
Using the bound $c_\gamma - c_\pi < 10^{-22}$ obtained in (antonov), we get $-\alpha < 5.4 \times 10^{-23}$
Better bound from recent Auger data:
$-\alpha < 1.3 \times 10^{-24}$
In our numerical estimates we have chosen $\alpha = -1 \times 10^{-24}$.
We get $E_\pi = 7 \times 10^{19} eV$. Therefore we expect that neutral pions above this energy are stable, so they could be a primary component of UHECR.

Conclusions

The Standard Model in the LIV background metric studied here implies:

- If the coupling constants are small as in the Electroweak theory, the dominant LIV is the one loop contribution. This is true also for QCD due to asymptotic freedom, but extrapolation to lower energies is not simple due to hadronization.
- We have computed the LIV induced by Quantum Gravity on Baryons and Mesons, using the Chiral Lagrangian approach. This permitted to fix that $\alpha < 0$.
- Studying several available processes, we found bounds on α:
  From pion stability and the most stringent experimental bound found in (antonov): $-\alpha < 5.4 \times 10^{-23}$.
  From recent Auger data: $-\alpha < 1.3 \times 10^{-24}$
  Then, several predictions are obtained:
- Photons are unstable above an energy $1.5 \times 10^{9} Gev$.
- **Neutral pions are stable above an energy** $E_\pi = 7 \times 10^{19} eV$.
  Our results are generic: All particles will have a modified maximum attainable velocity and birrefringence occurs for charged leptons, but not for gauge bosons, due to the chiral nature of the Electroweak couplings.

THANK YOU