Problem 1. An amount of charge q is uniformly spread out in a layer on the surface of a disc of radius a.

(a) Use elementary methods based on the azimuthal symmetry of the charge distribution to find the potential at any point on the axis of symmetry.

(b) With the aid of (a) find an expression for the potential at any point $\vec{r}\,(|\vec{r}\,| > a)$ as an expansion in angular harmonics.

Problem 2. The cube in Fig. 1.7 has 5 sides grounded. The sixth side, insulated from the others, is held at a potential 40. What is the potential at the center of the cube and why?
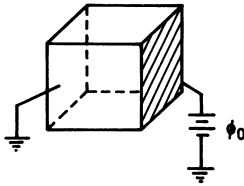

Fig. 1.7

Problem 3. A dielectric sphere of radius a and dielectric constant ε_1 is placed in a dielectric liquid of infinite extent and dielectric constant ε_2 A uiform electric field E was originally present in the liquid. Find the resultant electric field inside and outside the sphere.

Problem 4. A sphere of dielectric constant ε is placed in a uniform electric field E_o. Show that the induced surface charge density is

$$\sigma(\theta) = \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} 3\varepsilon_0 E_0 \cos\theta$$

where θ is measured from the E_o direction. If the sphere is rotated at an angular velocity w about the direction of E_o, will a magnetic field be produced? If not, explain why no magnetic field is produced. If so, sketch the magnetic field lines.

Problem 5. A perfectly conducting sphere is placed in a uniform electric field pointing in the z-direction.

(a) What is the surface charge density on the sphere?

(b) What is the induced dipole moment of the sphere?

Problem 6. A surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$ is glued to the surface of a spherical shell of radius R (σ_0 is a constant and θ is the polar angle). There is a vacuum, with no charges, both inside and outside of the shell. Calculate the electrostatic potential and the electric field both inside and outside of the spherical shell.

Problem 7. Consider a sphere of radius R centered at the origin. Suppose a point charge q is put at the origin and that this is the only charge inside or outside the sphere. Furthermore, the potential is $\Phi = V_0 \cos\theta$ on the surface of the sphere. What is the electric potential both inside and outside the sphere?

Problem 8. If the potential of a spherical shell of zero thickness depends only on the polar angle θ and is given by $V(\theta)$, inside and outside the sphere there being empty space,

(a) show how to obtain expressions for the potential $V(r, \theta)$ inside and outside the sphere and how to obtain an expression for the electric sources on the sphere.

(b) Solve with $V(\theta) = V_0 \cos^2\theta$.

1

Problem 9. Take a very long cylinder of radius r made of insulating material. Spray a cloud of electrons on the surface. They are free to move about the surface so that, at first, they spread out evenly with a charge per unit area σ_0. Then put the cylinder in a uniform applied electric field perpendicular to the axis of the cylinder. You are asked to think about the charge density on the surface of the cylinder, $\sigma(\theta)$, as a function of the applied electric field E_a. In doing this you may neglect the electric polarizability of the insulating cylinder.

(a) In what way is this problem different from a standard electrostatic problem in which we have a charged conducting cylinder? When are the solutions to the two problems the same? (Answer in words.)

(b) Calculate the solution for a(0) in the case of a conducting cylinder and state the range of value of E_a for which this solution is applicable to the case described here.

Problem 10. Two semi-infinite plane grounded aluminium sheets make an angle of 60°. A single point charge $+q$ is placed as shown in Fig. 1.33. Make a large drawing indicating clearly the position, size of all image charges. In two or three sentences explain your reasoning.
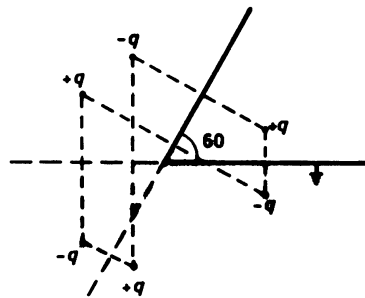
Fig. 1.33

Problem 11. Suppose that the region z > 0 in three-dimensional space is filled with a linear dielectric material characterized by a dielectric constant ε_1, while the region z < 0 has a dielectric material ε_2. Fix a charge -q at (x, y, z) = (0,0, a) and a charge q at (0,0, -a). What is the force one must exert on the negative charge to keep it at rest?

Problem 12. When a cloud passes over a certain spot on the surface of the earth a vertical electric field of E = 100 volts/meter is recorded here. The bottom of the cloud is a height d = 300 meters above the earth and the top of the cloud is a height d = 300 meters above the bottom. Assume that the cloud is electrically neutral but has charge +q at its top and charge -q at its bottom. Estimate the magnitude of the charge q and the external electrical force (direction and magnitude) on the cloud. You may assume that there are no charges in the atmosphere other than those on the cloud.

Problem 13. Two similar charges are placed at a distance 2b apart. Find, approximately,the minimum radius a of a grounded conducting sphere placed midway between them that would neutralize their mutual repulsion.

Problem 14. A charge q is placed inside a spherical shell conductor of inner radius r1 and outer radius r2. Find the electric force on the charge. If the conductor is isolated and uncharged, what is the potential of its inner surface?

Problem 15. Consider an electric dipole $\vec{P}$. Suppose the dipole is fixed at a distance z_0 along the z-axis and at an orientation θ with respect to that axis (i.e., $\vec{P}.\hat{z} = P\cos\theta$). Suppose the xy plane is a conductor at zero potential.

Give the charge density on the conductor induced by the dipole.

Problem 16. Two large flat conducting plates separated by a distance D are connected by a wire. A point charge Q is placed midway between the two plates, as in Fig. 1.40. Find an expression for the surface charge density induced on the lower plate as a function of D,Q and z (the distance from the center of the plate).
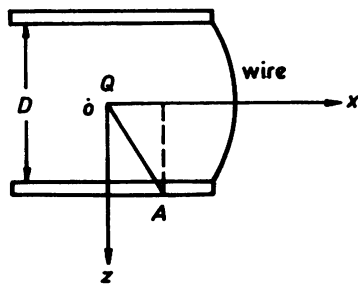
Fig. 1.40

Problem 17. Two large parallel conducting plates are separated by a small distance 4x. The two plates are grounded. Charges Q and -Q are placed at distances x and 3x from one plate as shown in Fig. 1.42.

(a) How much energy is required to remove the two charges from between the plates and infinitely apart?

(b) What is the force (magnitude and direction) on each charge?

(c) What is the force (magnitude and direction) on each plate?

(d) What is the total induced charge on each plate? (Hint: What is the potential on the plane midway between the two charges?)

(e) What is the total induced charge on the inside surface of each plate after the -Q charge has been removed, the +Q remaining at rest?
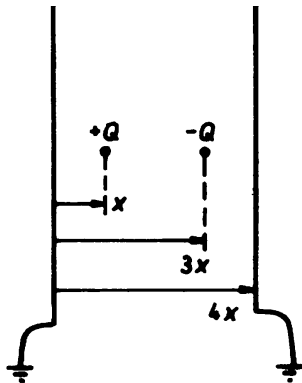
Fig. 1.42

Problem 18. Consider a thin spherical shell of dielectric which has a radius R and rotates with an angular velocity w. A constant surface charge of density σ is placed on the sphere, and this produces a uniform magnetic field which is proportional to w. Suppose that the mass of the shell is negligible.

(a) Find the magnetic field inside and outside the rotating shell.

(b) A constant torque N is applied parallel to w. How long does it take for the shell to stop?

Problem 19. A long hollow right circular cylinder made of iron of permeability μ is placed with its axis perpendicular to an initially uniform magnetic flux density B. Assume that B_0 is small enough so that it does not saturate the iron, and that the permeability μ is a constant in the field range of our interest.

(a) Sketch the magnetic field lines in the entire region before and after the cylinder is placed in the field.

(b) Let the inner and outer radii of the cylinder be b and a respectively. Derive an expression for B inside the cylinder.

Problem 20. A rectangular coil of dimensions a and b and resistance R moves with constant velocity v into a magnetic field B as shown in Fig. 2.36. Derive an expression for the vector force on the coil in terms of the given parameters.
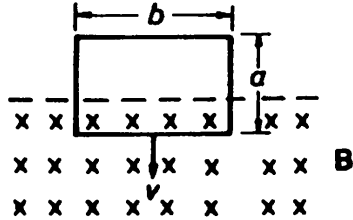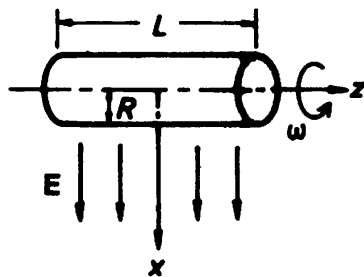

Fig. 2.36

Problem 21. A very long insulating cylinder (dielectric constant ε) of length L and radius R(L>>R) has a charge Q uniformly distributed over its outside surface. An external uniform electric field is applied perpendicular to the cylinder's axis: $\vec{E} = Eo\hat{z}$, (see Fig. 2.44). Ignore edge effects.

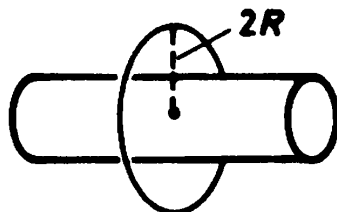(a) Calculate the electric potential everywhere (i.e. inside and outside the cylinder).

Now the electric field E_0 is removed and the cylinder is made to rotate with angular velocity w .

(b) Find the magnetic field (magnitude and direction) inside the cylinder.

(c) A single-turn coil of radius 2R and resistance ρ is wrapped around the cylinder as shown in Fig. 2.45, and the rotation of the cylinder is slowed down linearly $(w(t) = w_0(1 - t/t_o))$ as a function of time. What is the magnitude of the induced current in the coil? In what direction does the current flow?

Fig. 2.44

Fig. 2.45