PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
Facultad de Física

Teorías de Gauge
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Problema 1.

Considere un campo escalar complejo, acoplado minimalmente al campo electromagnético A_μ. El Lagrangiano es:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi^* \phi \tag{1}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2}$$

$$D_\mu = \partial_\mu + i\, e\, A_\mu \tag{3}$$

(a) Encuentre las reglas de Feynman en espacio de momentos, EXCEPTO EL PROPAGADOR DEL FOTON QUE SE DA POR CONOCIDO $D_{\mu\nu}(k) = -\frac{i}{k^2 + i\varepsilon}\eta_{\mu\nu}$.

Sol:

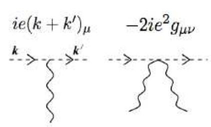


Figura 1. $A\phi^* \phi$ vértice y $AA\phi^* \phi$ vértice

$$\Delta_F(p) = \frac{i}{p^2 - m^2 + i\epsilon}$$

(b) Calcule el grado de divergencia de los diagramas 1PI. Para cual valor de d=dr el modelo sería renormalizable?.

Sol:

$$D(G) = d\,L - 2N_B + V_3 \qquad , N_B = N_f + N_e$$

$$L = N_B - V + 1, \qquad V = V_3 + V_4$$

$$2V_3 + 2V_4 = E_e + 2N_e, \qquad \text{conservación de líneas del escalar}$$

$$V_3 + 2V_4 = E_f + 2N_f, \qquad \text{conservación de líneas del fotón}$$

$$D(G) = d\,(N_B - V + 1) - 2N_B + V_3 = \qquad (d-2)(N_f + N_e) - d(V_3 + V_4) + d + V_3 =$$

$$N_f + N_e = \frac{3}{2}V_3 + 2V_4 - \frac{E_e + E_f}{2}$$

$$D(G) = (d-2)\left(\frac{3}{2}V_3 + 2V_4 - \frac{E_e + E_f}{2}\right) - d(V_3 + V_4) + d + V_3 = V_3\left(\frac{d}{2} - 2\right) + V_4(d-4) - \frac{E_e + E_f}{2}(d-2) + d$$

$$d_r = 4$$

$$D(G) = 4 - E_e - E_f$$

(c) Dibuje y escriba la contribución a un loop para los 1PI divergentes en espacio de momentos, incluyendo los factores combinatorios. Especifique el grado de divergencia de cada uno. Calcule las integrales usando RD.

E_e	E_f	D	
0	1	3	0 por Lorentz
1	0	3	0 regularización dimensional
1	1	2	no existe
2	0	2	
2	1	1	no existe
0	2	2	
0	3	1	0 invarianza de gauge
3	0	1	no existe
4	0	0	0
0	4	0	0, invarianza de gauge

(d) En d=dr,calcule los contratérminos a un "loop" usando regularización dimensional y substracción minimal. Es renormalizable el modelo con la acción original?.

Sol: No. Es necesario agregar al lagrangiano $-\lambda(\phi^*\ \phi)^2$ correspondiente al vértice propio con 4 patas escalares.

(e) Calcule la contribución del campo escalar a la polarización del vacío del fotón, usando regularización dimensional. Note que hay dos diagramas. Para escribir la respuesta en la forma transversal:

$$\Pi^{\mu\nu}(q^2) = (g^{\mu\nu}\, q^2 - q^\mu\, q^\nu)\, \Pi(q^2)$$

es conveniente sumar los dos diagramas al comienzo, poniéndolos sobre un denominador común antes de introducir parámetros de Feynman. Además use la simetría del integrando bajo el cambio de variables $x->1-x$. Escriba explícitamente la parte divergente y la parte finita. Explique claramente de donde viene la dependencia en μ.

Sol:

$$\Pi 1(k)_{\mu\nu} = (-ie)^2 \int \frac{d^dp}{(2\pi)^d}\ \ (2p+k)_\mu\ \ (2p+k)_\nu\ \ \frac{i}{(p+k)^2 - m^2 + i\epsilon}\frac{i}{p^2 - m^2 + i\epsilon}$$

$$\Pi 2(k)_{\mu\nu} = 2ie^2 \int \frac{d^dp}{(2\pi)^d}\frac{i}{p^2 - m^2 + i\epsilon}\, g_{\mu\nu}$$

$$\Pi(k)_{\mu\nu} = e^2 \int \frac{d^dp}{(2\pi)^d} \cdot \frac{(2p+k)_\mu(2p+k)_\nu - 2\, g_{\mu\nu}((p+k)^2 - m^2)}{(p^2 - m^2 + i\epsilon)((p+k)^2 - m^2 + i\epsilon)} =$$
$$e^2 \int_0^1\, dx \int \frac{d^dp}{(2\pi)^d}\frac{(2p+k)_\mu(2p+k)_\nu - 2g_{\mu\nu}((p+k)^2 - m^2)}{[(p^2 - m^2 + i\epsilon)x + (1-x)((p+k)^2 - m^2 + i\epsilon)]^2}$$

$$D = p^2 - m^2 + 2(1-x)pk + (1-x)k^2 = [p + (1-x)k]^2 + k^2x(1-x) - m^2 + i\epsilon$$

$p->p-(1-x)k$:

$$N_{\mu\nu} = (2p + k(-1+2x))_\mu(2p + k(-1+2x))_\nu - 2\, g_{\mu\nu}((p+kx)^2 - m^2) =$$
$$4p_\mu p_\nu + k_\mu k_\nu(1-2x)^2 - 2\, g_{\mu\nu}(p^2 + k^2\, x^2 - m^2)$$

$$\Pi(k)_{\mu\nu} = e^2 \int_0^1\, dx \int\, \frac{d^dp}{(2\pi)^d}\frac{4p_\mu p_\nu + k_\mu k_\nu(1-2x)^2 - 2\, g_{\mu\nu}(p^2 + k^2\, x^2 - m^2)}{(p^2 + k^2x(1-x) - m^2 + i\epsilon)^2} =$$

$$e^2 \frac{i}{(4\pi)^{\frac{d}{2}}}\Gamma(1 - \frac{d}{2}) \int_0^1\, dx\, M^{\frac{d}{2}-2}\Big(-2g_{\mu\nu}(m^2 - k^2x(1-x)) + k_\mu k_\nu(1-2x)^2(1 - \frac{d}{2}) - 2\, g_{\mu\nu}\Big[$$

$$-m^2+k^2(\frac{d}{2}x(1-x)+x^2(1-\frac{d}{2})]$$

Coeff(m^2)=0

$$C(k^2 g_{\mu\nu})=2x(1-x)-2\,(\frac{d}{2}x(1-x)+x^2(1-\frac{d}{2}))=(1-\frac{d}{2})[4x(1-x)-1]$$

$$\Pi(k)_{\mu\nu}=e^2\frac{i}{(4\pi)^{\frac{d}{2}}}\Gamma(2-\frac{d}{2})(k_\mu k_\nu-g_{\mu\nu}k^2)\int_0^1 dx\,M^{\frac{d}{2}-2}(1-2x)^2$$

$\epsilon=2-\frac{d}{2}$

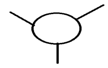
$$2[A]+2-d=0,[A]=1-\epsilon,1=[e]+[A],[e]=\epsilon$$

$$\Pi(k)_{\mu\nu}=e^2\frac{i}{(4\pi)^2}\Gamma(\epsilon)(k_\mu k_\nu-g_{\mu\nu}k^2)\int_0^1 dx(\frac{M}{4\pi\mu^2})^{-\epsilon}(1-2x)^2$$

$$\text{PF}\Pi(k)_{\mu\nu}=-e^2\frac{i}{(4\pi)^2}(k_\mu k_\nu-g_{\mu\nu}k^2)\int_0^1 dx\,ln(\frac{M}{4\pi\mu^2})(1-2x)^2$$

$$\text{PP}\Pi(k)_{\mu\nu}=e^2\frac{i}{(4\pi)^2}\frac{1}{\epsilon}(k_\mu k_\nu-g_{\mu\nu}k^2)\int_0^1 dx(1-2x)^2$$

Problema 2.

1) Encuentre el factor combinatorio del gráfico: 

Sol: 3 vértices cúbicos 9.6.3.4.2/(6^3.3!)=1

2) Calcule:

$\int \frac{d^dp}{(2\pi)^d}\frac{p_\mu p_\nu}{(p^2+2p.k-m^2+i\varepsilon)^2}$ y encuentre el polo y la parte finita en $\varepsilon=0$. $\varepsilon=2-d$

Sol:

$$\int\frac{d^dp}{(2\pi)^d}\frac{p_\mu p_\nu}{(p^2+2p.k-m^2+i\varepsilon)^2}=\int\frac{d^dp}{(2\pi)^d}\frac{p_\mu p_\nu}{((p+k)^2-k^2-m^2+i\varepsilon)^2}=$$

$$\int\frac{d^dp}{(2\pi)^d}\frac{(p-k)_\mu(p-k)_\nu}{(p^2-k^2-m^2+i\varepsilon)^2}=\int\frac{d^dp}{(2\pi)^d}\frac{p_\mu p_\nu+k_\mu k_\nu}{(p^2-k^2-m^2+i\varepsilon)^2}=$$

$$\frac{1}{d}\eta_{\mu\nu}\int\frac{d^dp}{(2\pi)^d}\frac{p^2}{(p^2-k^2-m^2+i\varepsilon)^2}+\int\frac{d^dp}{(2\pi)^d}\frac{k_\mu k_\nu}{(p^2-k^2-m^2+i\varepsilon)^2}=$$

$$\frac{1}{d}\eta_{\mu\nu}i(-1)\frac{1}{(4\,\pi)^{d/2}}\ \frac{\Gamma\left(1-\frac{d}{2}\right)}{\Gamma(1)}\left(\frac{1}{\Delta-i\varepsilon}\right)^{1-\frac{d}{2}}\ +\ i(-1)^2\frac{1}{(4\,\pi)^{d/2}}\ \frac{\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(2)}\left(\frac{1}{\Delta-i\varepsilon}\right)^{2-\frac{d}{2}}\left(\Delta\frac{1}{d}\eta_{\mu\nu}\ +\ k_\mu k_\nu\right)=$$

$$-\frac{1}{d}\eta_{\mu\nu}i\frac{1}{(4\,\pi)^{d/2}}\frac{\Gamma\left(1-\frac{d}{2}\right)}{\Gamma(1)}\left(\frac{1}{\Delta-i\varepsilon}\right)^{1-\frac{d}{2}}+i\frac{1}{(4\,\pi)^{d/2}}\frac{\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(2)}\left(\frac{1}{\Delta-i\varepsilon}\right)^{2-\frac{d}{2}}\left(\Delta\frac{1}{d}\eta_{\mu\nu}+k_\mu k_\nu\right)=$$

$$-\frac{1}{2-\varepsilon}\eta_{\mu\nu}i\frac{1}{(4\,\pi)^{1-\varepsilon/2}}\frac{\Gamma\left(\frac{\varepsilon}{2}\right)}{1}\left(\frac{1}{\Delta-i\varepsilon}\right)^{\frac{\varepsilon}{2}}+i\frac{1}{(4\,\pi)}\left(\frac{1}{\Delta-i\varepsilon}\right)\left(\Delta\frac{1}{2}\eta_{\mu\nu}+k_\mu k_\nu\right)=$$

$$-\frac{1}{2}\left(1+\frac{\varepsilon}{2}\right)\eta_{\mu\nu}i\frac{1}{(4\,\pi)}\Gamma\left(\frac{\varepsilon}{2}\right)\left(1-\frac{\varepsilon}{2}\ln\left(\frac{\Delta}{4\pi}\right)\right)+\ldots.$$

$$-\frac{i}{8\pi}\eta_{\mu\nu}\left(\frac{\Gamma\left(1+\frac{\varepsilon}{2}\right)}{\frac{\varepsilon}{2}}+\Gamma\left(1+\frac{\varepsilon}{2}\right)\left(1-\ln\left(\frac{\Delta}{4\pi}\right)\right)\right)+\ldots=$$

$$-\frac{i}{8\pi}\eta_{\mu\nu}\left(\frac{1+\frac{\varepsilon}{2}\Gamma'(1)}{\frac{\varepsilon}{2}}+\Gamma\left(1+\frac{\varepsilon}{2}\right)\left(1-\ln\left(\frac{\Delta}{4\pi}\right)\right)\right)+\ldots\doteq$$

$$-\frac{i}{4\pi}\frac{\eta_{\mu\nu}}{\varepsilon}-\frac{i}{8\pi}\eta_{\mu\nu}\left(\Gamma'(1)+1-\ln\left(\frac{\Delta}{4\pi}\right)\right)+i\frac{1}{(4\,\pi)}\left(\frac{1}{\Delta-i\varepsilon}\right)\left(\Delta\frac{1}{2}\eta_{\mu\nu}+k_\mu k_\nu\right)$$

Sol2:

$$\int\frac{d^dp}{(2\pi)^d}\frac{p_\mu p_\nu}{(p^2+2p.k-m^2+i\varepsilon)^2}=\frac{1}{4(n+1)n}\partial_\mu\partial_\nu\int\frac{d^dp}{(2\pi)^d}\frac{1}{(p^2+2p.k-m^2+i\varepsilon)^n}|_{n=0}=$$

$$\frac{1}{4(n+1)n}\partial_\mu\partial_\nu i(-1)^n\frac{1}{(4\,\pi)^{d/2}}\frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)}\left(\frac{1}{\Delta-i\varepsilon}\right)^{n-\frac{d}{2}},\Delta=k^2+m^2$$

$$=i(-1)^n\frac{1}{(4\,\pi)^{d/2}}\frac{1}{4(n+1)n}\frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)}\partial_\mu\left[-\left(n-\frac{d}{2}\right)2k_\nu\left(\frac{1}{\Delta-i\varepsilon}\right)^{n-\frac{d}{2}+1}\right]=$$

$$-i(-1)^n\frac{1}{(4\,\pi)^{d/2}}\frac{1}{2}\frac{\Gamma\left(n-\frac{d}{2}+1\right)}{\Gamma(n+2)}\left[\eta_{\mu\nu}\left(\frac{1}{\Delta-i\varepsilon}\right)^{n-\frac{d}{2}+1}-2k_\mu k_\nu\left(n-\frac{d}{2}+1\right)\left(\frac{1}{\Delta-i\varepsilon}\right)^{n-\frac{d}{2}+2}\right]|_{n=0}=$$

$$-i\frac{1}{(4\,\pi)^{d/2}}\frac{\Gamma\left(-\frac{d}{2}+1\right)}{2}\left[\eta_{\mu\nu}\left(\frac{1}{\Delta-i\varepsilon}\right)^{-\frac{d}{2}+1}-2k_\mu k_\nu\left(-\frac{d}{2}+1\right)\left(\frac{1}{\Delta-i\varepsilon}\right)^{-\frac{d}{2}+2}\right]$$

3) En d dimensiones calcule:

$$\mathrm{Tr}(\not{k}_1\not{k}_2\not{k}_3\not{k}_4),\not{k}=k_\mu\gamma^\mu$$

Sol:

$$\begin{aligned}
\mathrm{tr}(\gamma^a\gamma^b\gamma^c\gamma^d) &= A\eta^{ab}\eta^{cd} + B\eta^{ac}\eta^{bd} + C\eta^{ad}\eta^{bc} \quad = A\eta^{da}\eta^{bc} + B\eta^{db}\eta^{ac} + C\eta^{dc}\eta^{ab}, \quad A = C \\
\mathrm{tr}(\gamma^a\gamma^b\gamma^c\gamma_c) &= (A\,d + B + A)\eta^{ab} = d\,\mathrm{tr}(1)\eta^{ab}, \qquad \mathrm{tr}(1) = 4 \qquad A(d+1) + B = 4d \\
\mu = \nu = \alpha = \beta &= 0, 4 = 2A + B \qquad A = 4, B = -4 \\
\mathrm{Tr}(\not{k}_1\not{k}_2\not{k}_3\not{k}_4) &= 4(k_1.k_2 - k_1.k_3 + k_1.k_4)
\end{aligned}$$

Recuerde que: $\int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2 - \Delta + i\varepsilon)^n} = i(-1)^n \frac{1}{(4\pi)^{d/2}} \frac{\Gamma\left(n - \frac{d}{2}\right)}{\Gamma(n)} \left(\frac{1}{\Delta - i\varepsilon}\right)^{n - \frac{d}{2}}$

Tiempo: 2:30 horas
BUENA SUERTE!