$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

Reemplazo minimal:

$$D_\mu = \partial_\mu - i e A_\mu$$

$$\mathcal{L} = \bar{\psi}(iD\!\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$

Lagrangiano de la electrodinámica cuántica.

Propagador del electrón

$$\mathcal{L}_0 = \bar{\psi}(i\not{\partial} - m)\psi + \bar{j}\,\psi + \bar{\psi}\,j$$

$$(i\not{\partial} - m)\psi = -j\,, \quad (i\not{\partial} - m)S_F(x-y) = i\delta(x-y)$$

$$S_F(x-y) = \int \frac{d^dp}{(2\pi)^d} S_F(p) e^{-i\,p.(x-y)}, \qquad (\not{p} - m)S_F(p) = i$$

$$S_F(p) = \frac{i}{\not{p} - m} = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$$

$$\partial_\mu A_\mu = B(x)$$

En la integral funcional, promediando sobre B

$$Z = \int dA\, d\bar{\psi}\, d\psi\, dB\, e^{-\frac{1}{2\alpha}B^2} \delta(\partial_\mu A_\mu - B(x)) e^{-S}$$

Esto lleva al lagrangiano de fijación de gauge

$$\mathcal{L}_{\mathrm{GF}} = \frac{1}{2\alpha}(\partial_\mu A_\mu)^2$$

$$\delta A_\mu = \partial_\mu \alpha(x), \qquad \alpha(x) = \lambda C(x)$$
$$\lambda^2 = 0, \quad \lambda C(x) = -C(x)\lambda$$
$$\delta = \lambda \delta_{\mathrm{BRST}}, \quad \delta_{\mathrm{BRST}} A_\mu = \partial_\mu C(x)$$

$C(x)$es el campo fantasma.

Notar:

$$\begin{aligned} \delta(AB) = & \ (\delta A)B + A(\delta B) = \\ (\lambda \delta_{\mathrm{BRST}} A)B + A\lambda\delta_{\mathrm{BRST}} B = & \quad \lambda \delta_{\mathrm{BRST}}(AB) \end{aligned}$$

Si A es un fantasma, se tiene que:

$$\delta_{\mathrm{BRST}} A = -A \delta_{\mathrm{BRST}}$$

Imponiendo nilpotencia:

$$0 = \delta^2_{\mathrm{BRST}} A_\mu = \ \partial_\mu \delta_{\mathrm{BRST}} C(x), \quad \delta_{\mathrm{BRST}} C(x) = 0$$

Para tener los gauges conocidos introducimos:

$$\delta_{\mathrm{BRST}}\bar{C}(x) = B(x)$$
$$\delta_{\mathrm{BRST}}B(x) = 0$$

$\bar{C}(x)$ es el campo antifantasma. $B(x)$ es el campo de Nakanishi-Lautrup.

Para fijar el gauge se agrega en el lagrangiano:$\mathcal{L}_{\mathrm{GF}} = -\delta_{\mathrm{BRST}} F$

F debe tener número de fantasma -1.

Sea $F = \bar{C}(x)(\partial_\mu A_\mu + \alpha B)$,

$$\mathcal{L}_{\mathrm{GF}} = -\alpha B^2 - B\partial_\mu A_\mu + \bar{C}(x)\partial^2 C(x)$$

Integrando sobre B:

$$-2\alpha B_0 - \partial_\mu A_\mu = 0 \quad B_0 = -\frac{\partial_\mu A_\mu}{2\alpha}$$

$$\mathcal{L}_{\mathrm{GF}} = -\alpha B_0^2 - B_0\partial_\mu A_\mu + \bar{C}(x)\partial^2 C(x) = \frac{(\partial_\mu A_\mu)^2}{4\alpha} + \bar{C}(x)\partial^2 C(x)$$

La simetría de $\mathcal{L}_{\mathrm{GF}}$ es:

$$\delta_{\mathrm{BRST}} A_\mu = \partial_\mu C(x) \quad \delta_{\mathrm{BRST}} C(x) = 0 \quad \delta_{\mathrm{BRST}} \bar{C}(x) = -\frac{\partial_\mu A_\mu}{2\alpha}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\alpha}(\partial.A)^2 + j.A,\ \partial_\mu F^{\mu\nu} - \frac{1}{\alpha}\partial^\nu\partial.A + j^\nu = 0,$$

$$\Box A^\nu - \partial^\nu\partial_\mu A^\mu - \frac{1}{\alpha}\partial^\nu\partial.A = -j^\nu, \Box D^\nu_\alpha - \partial^\nu\partial_\mu D^\mu_\alpha\left(1 + \frac{1}{\alpha}\right) = i\delta^\nu_\alpha\delta(x)$$

$$D^\nu_\alpha(x) = \int \frac{d^dk}{(2\pi)^d}D^\nu_\alpha(k)e^{-ik.x}$$

$$-k^2D^\nu_\alpha + k^\nu k_\mu D^\mu_\alpha\left(1 + \frac{1}{\alpha}\right) = i\delta^\nu_\alpha,\ S_\mu^{\ \nu} = k^2\delta^\nu_\mu - k^\nu k_\mu\left(1 + \frac{1}{\alpha}\right), A\delta^\nu_\mu + Bk^\nu k_\mu = S_\mu^{-1\nu}$$

$$(A\delta^\nu_\mu + Bk^\nu k_\mu)\left(k^2\delta^\mu_\beta - k^\mu k_\beta\left(1 + \frac{1}{\alpha}\right)\right) = \delta^\nu_\beta$$

$$Ak^2\delta^\nu_\beta - Ak^\nu k_\beta\left(1 + \frac{1}{\alpha}\right) + Bk^2k^\nu k_\beta - \qquad Bk^2k^\nu k_\beta\left(1 + \frac{1}{\alpha}\right)$$

$$Ak^2 = 1,\quad -A\left(1 + \frac{1}{\alpha}\right) + Bk^2 - Bk^2\left(1 + \frac{1}{\alpha}\right) = 0$$

$$Bk^2 = -\alpha\left(1 + \frac{1}{\alpha}\right)\frac{1}{k^2}$$

$$D^\mu_\nu = \frac{-i}{k^2 + i\varepsilon}\left(\delta^\mu_\nu - (\alpha + 1)\frac{k^\mu k_\nu}{k^2}\right)$$

Calcular, usando la integral funcional

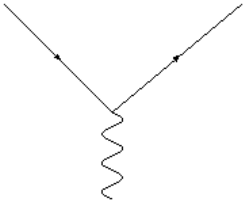
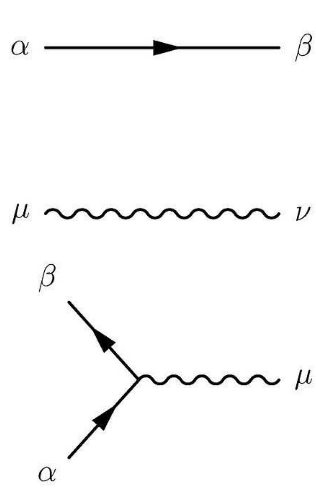


Figura 1.

$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}$, $D_\mu = \partial_\mu - ie\mathrm{QA}_\mu$, $\mathcal{L}_I = e\mathrm{QA}_\mu\bar{\psi}\gamma^\mu\psi$

Vértice: $ieQ\gamma^\mu$, El electrón tiene $Q = -1$



$$\rightarrow \quad \left(\frac{i}{\not{p} - m + i\varepsilon}\right)_{\beta\alpha}$$

$$\rightarrow \quad \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$

$$\rightarrow \quad -ie\gamma^{\mu}_{\beta\alpha}(2\pi)^4\delta^{(4)}(p_1 + p_2 + p_3).$$

Figura 2.

Además un loop cerrado de fermión agrega -1 y se debe calcular la traza sobre las matrices de Dirac.

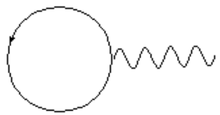
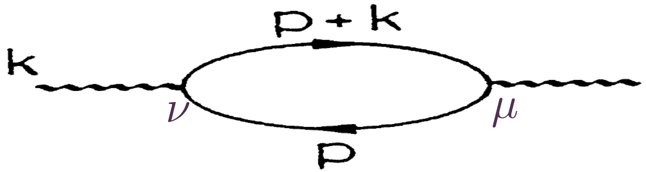


Figura 3.

Ejercicio: Use la integral funcional para encontrar

$$\Pi_{\mu\nu} = -(-i\,e)^2 \int d_n p \; \frac{\mathrm{Tr}(\gamma_\mu\, i(\not{p} + \not{k} + m)\gamma_\nu i(\not{p} + m))}{[(p+k)^2 - m^2 + i\varepsilon][p^2 - m^2 + i\varepsilon]}$$



$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} 1_n$

$\mathrm{Tr}(\gamma_\mu \gamma_\nu) = 4\eta_{\mu\nu}$. Notar el 4 en cualquier número de dimensiones. Esto es $\mathrm{Tr}\, 1_n = 4$

$\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta) = 4\eta^{\beta\mu}\eta^{\nu\alpha} - 4\eta^{\beta\nu}\eta^{\mu\alpha} + 4\eta^{\beta\alpha}\eta^{\nu\mu}$

En efecto:

$$\begin{aligned}
\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta) = -\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\beta \gamma^\alpha) + 2\eta^{\alpha\beta}\mathrm{Tr}(\gamma^\mu \gamma^\nu) &= \\
\mathrm{Tr}(\gamma^\mu \gamma^\beta \gamma^\nu \gamma^\alpha) - 2\eta^{\nu\beta}\mathrm{Tr}(\gamma^\mu \gamma^\alpha) + 2\eta^{\alpha\beta}\mathrm{Tr}(\gamma^\mu \gamma^\nu) &= \\
-\mathrm{Tr}(\gamma^\beta \gamma^\mu \gamma^\nu \gamma^\alpha) + 2\eta^{\mu\beta}\mathrm{Tr}(\gamma^\nu \gamma^\alpha) - 2\eta^{\nu\beta}\mathrm{Tr}(\gamma^\mu \gamma^\alpha) + 2\eta^{\alpha\beta}\mathrm{Tr}(\gamma^\mu \gamma^\nu) &
\end{aligned}$$

La traza es cíclica:

$$2\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta) = 2\eta^{\mu\beta}\mathrm{Tr}(\gamma^\nu \gamma^\alpha) - 2\eta^{\nu\beta}\mathrm{Tr}(\gamma^\mu \gamma^\alpha) + 2\eta^{\alpha\beta}\mathrm{Tr}(\gamma^\mu \gamma^\nu)$$

$$\mathrm{Tr}(\gamma_\mu (\not p + \not k + m)\gamma_\nu(\not p + m)) = \mathrm{Tr}(\gamma_\mu (\not p + \not k)\gamma_\nu \not p) + 4m^2\eta_{\mu\nu} =$$

$$(p+k)_\alpha p_\beta 4(\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) + 4m^2\eta_{\mu\nu} = (4m^2 - 4(p^2 + p.k))\eta_{\mu\nu} + 4(p+k)_\mu p_\nu + 4(p+k)_\nu p_\mu = 4N_{\mu\nu}$$

$$\frac{1}{ab} = \int_0^1 \frac{dx}{(ax + b(1-x))^2}$$
$$I_n^\gamma = -4e^2 \int_0^1 dx \int d_n p \frac{N_{\mu\nu}}{D^2}$$
$$D = (p^2 - m^2 + i\varepsilon)x + (1-x)((p+k)^2 - m^2 + i\varepsilon)$$
$$N_{\mu\nu} = (m^2 - p^2 - p.k)\eta_{\mu\nu} + (p+k)_\mu p_\nu + (\mu \leftrightarrow \nu)$$

Completamos el cuadrado en el denominador:

$$D = p^2 - m^2 + i\varepsilon + (1-x)(2p.k + k^2) =$$
$$(p + k(1-x))^2 - k^2(1-x)^2 + k^2(1-x) - m^2 + i\varepsilon$$

Redefinir:

$$p \longrightarrow p - k(1-x)$$
$$D = p^2 + k^2 x(1-x) - m^2 + i\varepsilon$$
$$N_{\mu\nu} = \eta_{\mu\nu}(m^2 - p^2 - k^2(1-x)^2 + k^2(1-x)) + 2p_\mu p_\nu - 2k_\mu k_\nu x(1-x) \quad =$$
$$\eta_{\mu\nu}(m^2 - p^2 + k^2(1-x)x) + 2p_\mu p_\nu - 2k_\mu k_\nu x(1-x) \quad =$$

$$\eta_{\mu\nu}(m^2 - p^2 + k^2(1-x)x) + \frac{2}{n}p^2\eta_{\mu\nu} - 2k_\mu k_\nu x(1-x) \quad =$$

$$\eta_{\mu\nu}\left(m^2 - p^2\left(1 - \frac{2}{n}\right) + k^2(1-x)x\right) - 2k_\mu k_\nu x(1-x) = \quad N_{\mu\nu}$$

Integramos sobre p: $\int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2-\Delta+i\varepsilon)^n} = i(-1)^n \frac{1}{(4\pi)^{d/2}} \frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)} \left(\frac{1}{\Delta - i\varepsilon}\right)^{n-\frac{d}{2}}$

$$\int d_n p \frac{1}{(p^2-M^2+i\varepsilon)^2} = i\frac{\pi^{\frac{n}{2}}}{(M^2)^{2-\frac{n}{2}}}\Gamma\left(2-\frac{n}{2}\right)$$

$$\int d_n p \frac{p^2}{(p^2-M^2+i\varepsilon)^2} = -i\frac{\pi^{\frac{n}{2}}}{(M^2)^{2-\frac{n}{2}}}\Gamma\left(1-\frac{n}{2}\right)M^2\frac{n}{2}$$

Se obtiene, con $M^2 = -k^2 x(1-x) + m^2$,

$$-4e^2 i\pi^{\frac{n}{2}} \int_0^1 \frac{dx}{(M^2)^{2-\frac{n}{2}}}$$
$$\left[\Gamma\left(2-\frac{n}{2}\right)(\eta_{\mu\nu}(m^2+k^2(1-x)x) - 2k_\mu k_\nu x(1-x))\right.$$
$$+\eta_{\mu\nu}\left(1-\frac{2}{n}\right)\Gamma\left(1-\frac{n}{2}\right)M^2\frac{n}{2}] =$$
$$-4e^2 i\pi^{\frac{n}{2}}\Gamma\left(2-\frac{n}{2}\right)\int_0^1 \frac{dx}{(M^2)^{2-\frac{n}{2}}}$$
$$[(\eta_{\mu\nu}(m^2+k^2(1-x)x) - 2k_\mu k_\nu x(1-x) - \eta_{\mu\nu}(-k^2x(1-x)+m^2)] \ =$$
$$8e^2 i\pi^{\frac{n}{2}}\Gamma\left(2-\frac{n}{2}\right)\int_0^1 \frac{dx}{(M^2)^{2-\frac{n}{2}}} x(1-x)(k^2\eta_{\mu\nu} - k_\mu k_\nu)$$

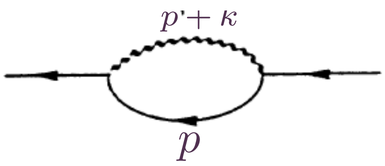


Figura 4.

$$I_n^e = (-ie)^2 i(-i)\int d_n p \frac{\gamma^\mu(\not{p}+m)\gamma_\mu}{(p^2-m^2+i\varepsilon)((p+k)^2+i\varepsilon)}$$

$$\gamma^\mu\gamma_\mu = n1$$

$$\gamma^\mu\gamma_\nu\gamma_\mu = -n\gamma_\nu + 2\eta_{\mu\nu}\gamma_\mu = (2-n)\gamma_\nu$$

$$I_n^e = -e^2\int d_n p \frac{(\not{p}(2-n)+nm)}{(p^2-m^2+i\varepsilon)((p+k)^2+i\varepsilon)} =$$

$$-e^2\int_0^1 dx \int d_n p \frac{(\not{p}(2-n)+nm)}{[(p^2-m^2+i\varepsilon)x+(1-x)((p+k)^2+i\varepsilon)]^2}$$

$$N = i\not{p}(2-n)+nm$$

$$D = p^2 - m^2 x + i\varepsilon + 2p.k(1-x) + k^2(1-x) =$$

$$(p+k(1-x))^2 - m^2 x + i\varepsilon + k^2(1-x)x$$

$$p \to p - k(1-x)$$

$$N = -(2-n)\not{k}(1-x)+nm$$

$$I_n^e = -e^2\int_0^1 dx \frac{-(2-n)\not{k}(1-x)+nm}{\left(M'^2\right)^{2-\frac{n}{2}}} i\pi^{\frac{n}{2}}\Gamma\left(2-\frac{n}{2}\right), \quad M'^2 = m^2 x - k^2(1-x)x$$

- Ramond, Capítulo 8.4
- Peskin, Capítulo 6

Lamb Shift

- Ramond, página 269 y problemas
- Peskin página 253.

Consideremos la integral funcional:

$$\int \mathcal{D}\phi^a e^{iS(\phi)} G(\phi)$$

La transformación infinitesimal de ϕ:$\delta\phi^a$ deja invariante la acción y la medida de integración. Realizando el cambio de variables $\phi^a \to \phi^a + \delta\phi^a$ obtenemos:

$$\int \mathcal{D}\phi^a e^{iS(\phi)} G(\phi) = \int \mathcal{D}\phi^a e^{iS(\phi)} G(\phi + \delta\phi)$$

$$\int \mathcal{D}\phi^a e^{iS(\phi)} \delta G(\phi) = 0 \qquad \delta G(\phi) = \frac{\delta G}{\delta \phi^a} \delta\phi^a$$

La ecuación en rojo es la identidad de Ward correspondiente a la simetría $\delta\phi^a$.

Encontremos la identidad de Ward asociada a la simetría BRST de la electrodinámica, para $G(A_\mu, C, \bar{C}) = \bar{C}(x)A_\nu(y)$

$$\int \mathcal{D}A_\mu \mathcal{D}\bar{C}\mathcal{D}\mathrm{C}\mathrm{e}^{i\,(S+S_{\mathrm{GF}})}\left(-\frac{\partial_\mu A_\mu(x)}{2\alpha}A_\nu(y) - \bar{C}(x)\partial_\nu^y C(y)\right)$$

Es decir

$$-\frac{1}{2\alpha}\partial_\mu^x D_{\mu\nu}(x-y) + \partial_\nu F(x-y) = 0$$

D es el propagador vestido del fotón. En espacio de momentum se tiene:

$$D_{\mu\nu}(k) = D^0_{\mu\nu}(k) + D^0_{\mu\alpha}(k)\Pi_{\alpha\beta}(k)D^0_{\beta\nu}(k) + \ldots.$$

Dado que $\partial_\nu F(x-y)$ no tiene correcciones radiativas, la identidad se satura con el propagador del fotón libre. Por lo tanto:

$$k_\mu D^0_{\mu\alpha}(k)\Pi_{\alpha\beta}(k)D^0_{\beta\nu}(k) = 0 \quad k_\mu D^0_{\mu\alpha}(k) = A(k^2)k_\alpha$$

Esto es: $k_\alpha \Pi_{\alpha\beta}(k) = 0.$

$$\int \mathcal{D}\phi^a e^{iS(\phi)} \delta G(\phi) = 0, G = e^{i\int dx\, j_a(x)\phi_a(x)}$$

Esto es:

$$\int \mathcal{D}\phi^a e^{iS(\phi)+i\int dx\, j_a(x)\phi_a(x)} \int dx\, j_a(x)\delta\phi_a(x) = 0 \quad j_a(x) = \frac{\delta\Gamma(\varphi)}{\delta\varphi_a(x)}$$

Obtenemos la identidad de Slavnov Taylor:

$$\int dx < \delta\phi_a(x) >_j \frac{\delta\Gamma(\varphi)}{\delta\varphi_a(x)} = 0$$

Esto dice que la acción efectiva es invariante bajo las transformaciones infinitesimales dadas por $\delta\varphi_a(x) = <\delta\phi_a(x)>_j$. En general estas transformaciones difieren de las que dejan invariante la acción S. Ambas coinciden para transformaciones lineales.

Problema #1

Considere un campo escalar complejo, acoplado minimalmente al campo electromagnético A_μ. El Lagrangiano es:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu\phi)^*(D^\mu\phi) - m^2\phi^*\phi\,,\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\;,\; D_\mu = \partial_\mu + i\,e\,A_\mu$$

1- Encuentre en espacio de momentos

i) el propagador del campo escalar;

ii) Los vértices que describen la interacción del fotón con el campo ϕ.

2-Calcule la contribución del campo escalar a la polarización del vacío del fotón, usando regularización dimensional. Note que hay dos diagramas. Para escribir la respuesta en la forma transversal:

$$\Pi^{\mu\nu}(q^2) = (g^{\mu\nu}q^2 - q^\mu q^\nu)\Pi(q^2)$$

es conveniente sumar los dos diagramas al comienzo, poniéndolos sobre un denominador común antes de introducir parámetros de Feynman. Además use la simetría del integrando bajo el cambio de variables $x - > 1 - x$. Escriba explícitamente la parte divergente y la parte finita. Explique claramente de donde viene la dependencia en μ.

Sol:

$$\Delta_F(p) = \frac{i}{p^2 - m^2 + i\epsilon}$$

$$\Pi1(k)_{\mu\nu} = (-ie)^2 \int \frac{d^dp}{(2\pi)^d} \quad (2p+k)_\mu \quad (2p+k)_\nu \quad \frac{i}{(p+k)^2 - m^2 + i\epsilon} \frac{i}{p^2 - m^2 + i\epsilon}$$

$$\Pi2(k)_{\mu\nu} = 2ie^2 \int \frac{d^dp}{(2\pi)^d} \frac{i}{p^2 - m^2 + i\epsilon}\, g_{\mu\nu}$$

$$\Pi(k)_{\mu\nu} = e^2 \int \frac{d^dp}{(2\pi)^d} \cdot \frac{(2p+k)_\mu (2p+k)_\nu - 2\, g_{\mu\nu}((p+k)^2 - m^2)}{(p^2 - m^2 + i\epsilon)((p+k)^2 - m^2 + i\epsilon)} =$$
$$e^2 \int_0^1 dx \int \frac{d^dp}{(2\pi)^d} \frac{(2p+k)_\mu (2p+k)_\nu - 2g_{\mu\nu}((p+k)^2 - m^2)}{[(p^2 - m^2 + i\epsilon)x + (1-x)((p+k)^2 - m^2 + i\epsilon)]^2}$$

$$D = p^2 - m^2 + 2(1-x)pk + (1-x)k^2 = [p + (1-x)k]^2 + k^2 x(1-x) - m^2 + i\epsilon$$

$p -> p - (1-x)k$:

$$N_{\mu\nu} = (2p + k(-1+2x))_\mu (2p + k(-1+2x))_\nu - 2\,g_{\mu\nu}((p+kx)^2 - m^2) =$$
$$4p_\mu p_\nu + k_\mu k_\nu (1-2x)^2 - 2\,g_{\mu\nu}(p^2 + k^2\,x^2 - m^2)$$

$$\Pi(k)_{\mu\nu} = e^2 \int_0^1 dx \int \frac{d^d p}{(2\pi)^d} \frac{4p_\mu p_\nu + k_\mu k_\nu (1-2x)^2 - 2\,g_{\mu\nu}(p^2 + k^2\,x^2 - m^2)}{(p^2 + k^2 x(1-x) - m^2 + i\,\epsilon)^2} =$$

$$e^2 \frac{i}{(4\pi)^{\frac{d}{2}}} \Gamma(1 - \frac{d}{2}) \int_0^1 dx\, M^{\frac{d}{2}-2} \Big(-2 g_{\mu\nu}(m^2 - k^2 x(1-x)) + k_\mu k_\nu (1-2x)^2 (1 - \frac{d}{2}) - 2\,g_{\mu\nu} \Big[$$

$$-m^2 + k^2(\frac{d}{2} x(1-x) + x^2(1 - \frac{d}{2})) \Big]$$

Coeff(m^2)=0

$$C(k^2 g_{\mu\nu}) = 2x(1-x) - 2\,(\frac{d}{2}x(1-x) + x^2(1-\frac{d}{2})) = (1-\frac{d}{2})[4x(1-x)-1]$$

$$\Pi(k)_{\mu\nu} = e^2 \frac{i}{(4\pi)^{\frac{d}{2}}} \Gamma(2-\frac{d}{2})(k_\mu k_\nu - g_{\mu\nu}k^2)\int_0^1 \,dx\,M^{\frac{d}{2}-2}(1-2x)^2$$

$\epsilon = 2 - \frac{d}{2}$

$$2[A] + 2 - d = 0, [A] = 1-\epsilon, 1 = [e] + [A], [e] = \epsilon$$

$$\Pi(k)_{\mu\nu} = e^2 \frac{i}{(4\pi)^2} \Gamma(\epsilon)(k_\mu k_\nu - g_{\mu\nu}k^2)\int_0^1 \,dx(\frac{M}{4\pi\mu^2})^{-\epsilon}(1-2x)^2$$

$$\mathrm{PF}\Pi(k)_{\mu\nu} = -e^2 \frac{i}{(4\pi)^2}(k_\mu k_\nu - g_{\mu\nu}k^2)\int_0^1 \,dx\,ln(\frac{M}{4\pi\mu^2})(1-2x)^2$$

$$\mathrm{PP}\Pi(k)_{\mu\nu} = e^2 \frac{i}{(4\pi)^2}\frac{1}{\epsilon}(k_\mu k_\nu - g_{\mu\nu}k^2)\int_0^1 \,dx(1-2x)^2$$