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Quantum gravity induced Lorentz invariance violation in the standard model: Hadrons

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In a previous letter, we have observed that the infinities of the standard model (SM) are a source of Quantum Gravity effects at lower energies. This analysis implies the existence of Lorentz invariance violation (LIV) within the SM. In this paper we obtain the LIV for mesons and nucleons using chiral lagrangian and Heavy Baryon Chiral Perturbation Theory. We use them to study several effects, including the GZK anomaly.

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I. INTRODUCTION

In recent times Quantum Gravity Phenomenology has seen a revival[1–4]; Most of it involve some sort of Lorentz invariance violations(LIV's)[5–7]. Recently [8], some of the previous results has been subjected to severe criticism.

In [9], we realized that the main effect of QG is to affect the measure of integration of Feynman graphs at large four momenta by a tiny LIV. The classical lagrangian is unchanged. Similarly, it can be stated that QG deforms the metric of space-time, introducing a tiny LIV proportional to (d-4)α, d being the dimension of space time in Dimensional Regularization and α being the only arbitrary parameter in the model. This deformation of the integration measure (or space-time metric) could be due to quantum fluctuations of the metric of space-time produced by QG: virtual black holes as in [1], D-branes as in [10], compactification of extra-dimensions or spin-foam anisotropies [11]. A theoretical derivation of α will have to wait for additional progress in the theories of QG[1]

Modified dispersion relations could exist without a preferred frame (DSR) [14]. But, in our proposal the classical lagrangian is invariant under usual linear Lorentz transformations but not under DSR. So our LIV is closer to radiative breaking of usual Lorentz symmetry than to DSR. The regulator R defined below and the deformed metric of Sec. III are given in a particular inertial frame, where spatial rotational symmetry is preserved. The preferred frame is the one where the Cosmic Background Radiation is isotropic.

In the standard model, such LIV implies several effects truly remarkable, which depend on one arbitrary parameter (α).The main effects are:

The maximal attainable velocity (MAV) for particles is not the speed of light, but depends on the specific couplings of the particles within the standard model. According to the very stringent bounds coming from the Ultra High Energy Cosmic Rays (UHECR) spectrum [12,13], this LIV of the dispersion relations is the only compatible with the data.

Also, the couplings between particles in the SM, determine different MAV for each particle, which is needed to explain the Greisen [15], Zatsepin and Kuz'min [16](GZK) anomaly [6,13,17]. In a few years, the Pierre Auger Observatory [18] will provide abundant new data in the highest energy range of the spectrum, which can be used as powerful tests of Lorentz invariance.

Birrefringence occurs for charged leptons, but not for gauge bosons: Photons and neutrinos have different MAV, which could be tested in the next generation of neutrino detectors such as NuBE [19,20]; Left and right helicity electron have different MAV.

Vertices in the SM will pick up a finite LIV.

The previous analysis can be applied only when perturbation theory is justified. In the standard model this is true for leptons, weak gauge bosons and the photon; and QCD due to asymptotic freedom. But in the case of hadrons we cannot rely on usual perturbation theory to compute the LIV. Most of the relevant physics involve hadrons, so we have to extend the methods of [9] to this case.

In the present work, we compute the LIV for hadrons using chiral lagrangian and Heavy Baryon Chiral Perturbation Theory. These are the standard effective theories used to describe the QCD physics at low energies, where the description in terms of gluons and quarks fails.

As a result, several effects become available to test the predictions of [9]. In particular, we give precise criteria to determine the sign and size of α.

This paper is organized as follows: In Secs. II and III we review the main arguments of [9]. In Sec. II, we present the LIV cutoff regulator and study its effects on One Particle Irreducible Green functions (1PI); Sec. III defines LIV dimensional regularization and explain how to proceed in the case of fermions; Explicit one-loop computations are contained in Sec. IV, using the cutoff regulator and LIV

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[1]The theory must explain why the LIV parameter is so small. Progress in this direction is in [3,4,13]. There α appears as $(l_P/L)^2$, where l_P is Planck's lenght and L is defined by the semiclassical gravitational state in Loop Quantum Gravity. If $L \sim 10^{11} l_P$, an α of the right order is obtained

dimensional regularization; the LIV for mesons and baryons are found in Sec. V; Reactions thresholds are contained in Sec. VI; Bounds on α are derived in Sec. VII; Section VIII contains our conclusions.

II. REGULATOR OF THE INTEGRATION MEASURE AND 1PI GRAPHS

For renormalizable theories, we consider the effect of the Lorentz asymmetry of the measure using the replacement

$$\int d^d k \to \int d^d k R\left(\frac{k^2 + \alpha k_0^2}{\Lambda^2}\right)$$

R is an arbitrary function, Λ is a cutoff with mass dimensions, that will go to infinity at the end of the calculation. To recover the original integral, $R(0) = 1$. $R(\infty) = 0$ to regulate the integral. α is a real parameter. For logarithmically divergent integrals, the divergent term is Lorentz invariant whereas a finite LIV part proportional to α remains when the cutoff goes to infinity.

1PI graphs are modified as follows:.

Self-energy Bosons: $\chi(p) = \chi(0) + A^{\mu\nu} p_\mu p_\nu +$ convergent,

$$A^{\mu\nu} = c_2 \eta^{\mu\nu} + a^{\mu\nu}$$

c_2 is the wave function renormalization counterterm; $a^{\mu\nu}$ is a finite LIV.

Self-energy Fermions:$\Sigma(p) = \Sigma(0) + s^{\mu\nu}\gamma_\nu p_\mu$,

$$s^{\mu\nu} = s\eta^{\mu\nu} + a^{\mu\nu}/2$$

s is a wave function renormalization counterterm; $a^{\mu\nu}$ is a finite LIV.

The on-shell condition is:

$$p^2 - m^2 - a^{\mu\nu} p_\mu p_\nu = 0$$

If spatial rotational invariance is preserved, the nonzero components of the matrix a are:

$$a^{00} = a_0; a^{ii} = -a_1$$

So the maximum attainable velocity (MAV) for this particle will be:

$$c_m = \sqrt{\frac{1 - a_1}{1 - a_0}} \sim 1 - (a_1 - a_0)/2 \tag{1}$$

The dependence on R amounts to a multiplicative factor. So ratios of LIV's are uniquely determined.

Vertex correction This graph has $D = 0$, so the regulator R will induce a tiny LIV.

Gauge Bosons [2]

$$\Pi^{\nu\beta}(p) = c^{\mu\nu\alpha\beta} p_\alpha p_\mu \Pi(p) \tag{2}$$

$c^{\mu\nu\alpha\beta}$ is antisymmetric in $\mu\nu$ and $\alpha\beta$ and symmetric by $(\alpha, \beta) <-> (\mu, \nu)$; $c^{\mu\nu\alpha\beta}$ is given by a logarithmically divergent integral.As in the previous cases we obtain:

$$c^{\mu\nu\alpha\beta} = c_2(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\beta}\eta^{\nu\alpha}) + a^{\mu\nu\alpha\beta} \tag{3}$$

c_2 is a Lorentz invariant counterterm and $a^{\mu\nu\alpha\beta}$ is a LIV.

III. LIV DIMENSIONAL REGULARIZATION

We consider dimensional regularization in a d-dimensional space with an arbitrary constant, positive metric $g_{\mu\nu}$. The Minkowski metric is obtained by Wick rotation. Here $g = det(g_{\mu\nu})$, $k^2 = g^{\mu\nu} k_\mu k_\nu$ and $\epsilon = 2 - \frac{d}{2}$.

LIV dimensional regularization consists in:

1) Calculating the d-dimensional integrals using a general metric $g_{\mu\nu}$.

2) Gamma matrix algebra is generalized to a general metric $g_{\mu\nu}$,$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

3) At the end of the calculation, replace $g^{\mu\nu} = \eta^{\mu\nu} + (4\pi)^2 \alpha \eta^{\mu 0}\eta^{\nu 0} Res_{\epsilon=0}$ and then take the limit $\epsilon -> 0$.

$Res_{\epsilon=0}$ is the residue of the pole at $\epsilon = 0$.

To define the counterterms, we use the minimal substraction scheme(MSS); that is we substract the poles in ϵ from the 1PI graphs.

These definitions preserve gauge invariance, because the integration measure is invariant under shifts.

As a concrete example, let us consider QED. The electron self-energy to one loop is given by:

$$-i\Sigma_2(q) = (-ie)^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{g}} \gamma^\mu \frac{(i\not{k} + m)}{k^2 - m^2 + i0} \times \gamma_\mu \frac{-i}{(k-q)^2 - \mu^2 + i0} \tag{4}$$

To obtain the LIV, we have to evaluate (we have introduced a parameter Δ and put it to zero afterwards):

$$-iL\Sigma_2(q) = 2i(-ie)^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{g}} \frac{\gamma^\mu \not{k} \gamma_\mu k.q}{(k^2 - \Delta + i0)^3} \tag{5}$$

$$= -2i(-ie)^2 (d-2) \int \frac{d^d k}{(2\pi)^d} \frac{1}{\sqrt{g}} \frac{\not{k} k.q}{(k^2 - \Delta + i0)^3} \tag{6}$$

$$= (-ie)^2 \frac{(d-2)}{2} \frac{1}{(4\pi)^{d/2}} \not{q} \frac{\Gamma(2 - d/2)}{\Delta^{2-d/2}} \tag{7}$$

but

$$\not{q} \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} = q_\mu \left(\delta_a^\mu - \frac{(4\pi)^2 \alpha Res_{\epsilon=0}}{2} \delta_0^\mu \delta_a^0\right) \times \gamma^a \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} = \tag{8}$$

$$\not{q} - \frac{(4\pi)^2 \alpha}{2} q_0 \gamma^0 \tag{9}$$

[2] A Chern-Simons term is absent due to the symmetry $k_\mu -> -k_\mu$, which is preserved by the regulator.

The last $\not{q}$ uses the Minkowski metric. It follows that,

$$-iL\Sigma_2(q) = \frac{e^2\alpha}{2} q_0\gamma^0 \tag{10}$$

LIV Dimensional Regularization makes stronger our claim that these tiny LIV's come from Quantum Gravity. In fact the sole change of the metric of space time is a correction of order ϵ to the Minkowsky metric which can be due to quantum fluctuations of the gravitational field.

IV. EXPLICIT ONE-LOOP COMPUTATIONS

A. Example: $g\phi^3$ in six space-time dimensions

To illustrate the method using the cutoff regulator, we consider $g\phi^3$ in six space-time dimensions.

The one-loop contribution to the self energy of the particle is:

$$i\chi(q) = \frac{(-gi)^2}{2}\int \frac{d^6k}{(2\pi)^6}\frac{1}{k^2 - m^2 + i0} \times \frac{1}{(k-q)^2 - m^2 + i0} \tag{11}$$

The term containing the LIV is:

$$L(i\chi) = -2q^\mu q^\nu g^2 B_{\mu\nu} \tag{12}$$

$$B_{\mu\nu} = \int \frac{d^6k}{(2\pi)^6}\frac{k_\mu k_\nu}{(k^2 - m^2 + i0)^4} \tag{13}$$

To evaluate $B_{\mu\nu}$, introduce the regulator of the integration measure,

$$R = \frac{-\Lambda^2}{k^2 - \Lambda^2 + ak_0^2 + i0} \tag{14}$$

define $k = \Lambda p$ and take the limit $\Lambda - >\infty$. In this way we verify that the LIV is mass independent. Since $a < <1$, we keep only the first order in a. We end up with:

$$LB_{\mu\nu} \sim a\int \frac{d^6p}{(2\pi)^6}\frac{p_0^2 p_\mu p_\nu}{(p^2 - 1 + i0)^2(p^2 + i0)^4} \tag{15}$$

Therefore:

$$L(i\chi) = -\frac{g^2 a q_0^2 i}{24(4\pi)^3} \tag{16}$$

B. LIV in the standard model

We follow [21,22] and use LIV Dimensional Regularization.

Photons

In the SM the photon self-energy can be written:

$$i\Pi^{\mu\nu} = i(q^2 g^{\mu\nu} - q^\mu q^\nu)\left(\frac{-23e^2}{48\pi^2\epsilon} + \text{finite}\right) \tag{17}$$

so that the LIV photon self-energy in the SM is:

$$L\Pi^{\mu\nu}(q) = -\frac{23}{3}e^2\alpha q_\alpha q_\beta(\eta^{\alpha\beta}\delta_0^\mu\delta_0^\nu + \eta^{\mu\nu}\delta_0^\alpha\delta_0^\beta - \eta^{\nu\beta}\delta_0^\mu\delta_0^\alpha - \eta^{\mu\alpha}\delta_0^\nu\delta_0^\beta) \tag{18}$$

It follows that the maximal attainable velocity is

$$c_\gamma = 1 - \frac{23}{6}e^2\alpha \tag{19}$$

We have included the coupling to quarks and charged leptons as well as 3 generations and color.

C. Fermions

Similarly, in the SM, the fermion self-energy is given by:

$$(4\pi^2)\Sigma(q) = -\frac{1}{\epsilon}\not{q}\sum_{\text{graphs}}(|c_V + c_A|^2 P_L + (|c_V - c_A|^2 P_R) + \text{finite} \tag{20}$$

where the fermion-gauge boson vertex is:

$$i\gamma^k(c_V - c_A\gamma^5) \tag{21}$$

and $P_L(P_R)$ are the L(R) helicity projectors.

Therefore

$$L\Sigma(q) = \frac{\alpha}{2}q_0\gamma^0\sum_{graphs}(|c_V + c_A|^2 P_L + (|c_V - c_A|^2 P_R) \tag{22}$$

Using this last result, we obtained the MAV for neutrinos and charged bosons [9]. Since we will use them later, we write the answer below.

MAV Neutrinos:

$$c_\nu = 1 - (3 + \tan^2\theta_w)\frac{g^2\alpha}{8} \tag{23}$$

MAV Electron. Birrefringence:

Define: $e_L = \frac{1-\gamma^5}{2}e$, $e_R = \frac{1+\gamma^5}{2}e$, where e is the electron field. We get

$$c_L = 1 - \left(\frac{g^2}{\theta_w \cos^2}(sin^2\theta_w - 1/2)^2 + e^2 + g^2/2\right)\frac{\alpha}{2}; \tag{24}$$

$$c_R = 1 - \left(e^2 + \frac{g^2\sin^4\theta_w}{\cos^2\theta_w}\right)\frac{\alpha}{2} \tag{25}$$

The difference in maximal speed for the left and right helicities is $\sim(5\times10^{-24})$.

It can be readely checked, employing R_ξ-gauges, that the LIV is gauge invariant. The gauge parameter affects the Lorentz invariant part only.

V. MESONS AND BARYONS

In order to apply our results to the computation of the UHECR spectrum and other phenomena, we must calculate the maximal attainable velocity (MAV) of hadrons. As we mentioned before, the problem is hadronization. One way to get an estimation of the effect is using effective lagrangians.

We use the results of [23,24] for the wave function renormalization of pions and nucleons in the chiral lagrangian and Heavy Baryon Chiral Perturbation Theory. They get:

$$Z_\pi^{-1} = 1 - \frac{4m_\pi^2}{3(4\pi)^2F^2}\frac{1}{\epsilon} + \text{finite} \tag{26}$$

$$Z_N^{-1} = 1 - \frac{9g_A^2m_\pi^2}{4(4\pi)^2F^2}\frac{1}{\epsilon} + \text{finite} \tag{27}$$

Here, m_π is the renormalized pion mass, F is the renormalized decay constant of pions and g_A is the axial vector coupling constant, in the chiral limit.

Using the LIV metric, we can read off the MAV for pions and nucleons:

$$c_\pi = 1 + \frac{2m_\pi^2\alpha}{3F^2} \qquad c_N = 1 + \frac{9m_\pi^2g_A^2\alpha}{8F^2} \tag{28}$$

VI. REACTION THRESHOLDS

Knowing the LIV for nucleons, pions, photons and electrons, we proceed to study the reactions involved in the GZK cutoff. We follow the discussion in [6,13]. The main difference with these works is that our MAV are correlated, because all of them are dependent on one parameter α.

A. Photo-Pion Production $\gamma + p \to p + \pi$

We start our discussion with the photo-pion production $\gamma + p \to p + \pi$. Using the corrections provided in the dispersion relation (28) for pions and nucleons, we note that, for the photo-pion production to be allowed, the following condition must be satisfied

$$2\delta c E_\pi^2 + 4E_\pi\omega \geq \frac{m_\pi^2(2m_p + m_\pi)}{m_p + m_\pi}. \tag{29}$$

where E_π is the produced pion energy and $\delta c = c_p - c_\pi$.

B. Pair Creation $\gamma + p \to p + e^+ + e^-$

Pair creation, $\gamma + p \to p + e^+ + e^-$, is abundant in the part of the spectrum previous to the GZK limit. When the dispersion relations for fermions are considered for both protons and electrons, we find

$$\delta c \frac{m_e}{m_p}E^2 + E\omega \geq m_e(m_p + m_e), \tag{30}$$

where E is the incident proton energy and $\delta c = c_p - c_e$.

VII. BOUNDS ON α

In order to analyze the threshold conditions (29) and (30), in the context of the GZK anomaly, we must establish some criteria. In the first place, as it is done in [13,25], the conventionally obtained theoretical spectrum provides a very good description of the phenomena up to an energy $\sim 4 \times 10^{19}$ eV. The main reaction taking place in this well described region is the pair creation $\gamma + p \to p + e^+ + e^-$ and, therefore, no modifications are present for this reaction up to $\sim 4 \times 10^{19}$ eV. As a consequence, and since threshold conditions offer a measure of how modified kinematics is, we will require that the threshold condition (30) for pair creation not be substantially altered by the new corrective terms.

Now, we want to explain the GZK anomaly. Since for energies greater than $\sim 8 \times 10^{19}$ eV the conventional theoretical spectrum does not fit the experimental data well, we shall impose that QG corrections be able to offer a violation of the GZK-cutoff. The dominant reaction in the violated $E > 8 \times 10^{19}$ region is the photo-pion production and, so, we must require that the new corrective terms present in the kinematical calculations be able to shift the threshold significantly to forbid the reaction.

We begin our analysis with the threshold condition for pair production. In this case we have:

$$\delta c \frac{m_e}{m_p}E^2 + E\omega \geq m_e(m_p + m_e), \tag{31}$$

with $\delta c = c_p - c_e$. From the above condition, the minimum soft-photon energy $\omega_{\min}$ for the pair production to occur, is

$$\omega_{\min} = \frac{m_e}{E}(m_p + m_e) - \delta c \frac{m_e}{m_p}E. \tag{32}$$

It follows therefore that the condition for a significant increase or decrease in the threshold energy for pair production becomes $|\delta c| \geq m_p(m_p + m_e)/E^2$. In this way, if we do not want kinematics to be modified up to a reference energy $E_{\text{ref}} = 3 \times 10^{19}$, we must impose the following bound

$$|c_p - c_e| < \frac{(m_p + m_e)m_p}{E_{\text{ref}}^2} = 9.8 \times 10^{-22}. \tag{33}$$

Similar treatments can be found for the analysis of other astrophysical signals like the Mkn 501 γ-rays [26], when the absence of anomalies is accepted.

Let us now consider the threshold condition for the photo-pion production. We have

$$2\delta c E_\pi^2 + 4E_\pi \omega \geq \frac{m_\pi^2(2m_p + m_\pi)}{m_p + m_\pi}. \tag{34}$$

In order that the above condition be violated for all energies E_π of the emerging pion, and therefore no reaction take place, the following inequality must hold

$$c_\pi - c_p > \frac{2\omega^2(m_p + m_\pi)}{m_\pi^2(2m_p + m_\pi)} = 3.3 \times 10^{-24}[\omega/\omega_0]^2. \tag{35}$$

where $\omega_0 = KT = 2.35 \times 10^{-4}$ eV is the thermal CMBR energy.

Combining the two reactions and the standard values, $m_\pi = 139$ MeV, $g_A = 1.26$, $F = 92.4$ MeV, we get an upper and lower bound on α

$$2.2 \times 10^{-21} > -\alpha > 1.3 \times 10^{-24} \tag{36}$$

We see that $\alpha < 0$, in order to suppress the photo-pion production, thus removing the GZK-cutoff. This implies that photons are the fastest particles and they arrive before neutrinos coming from the same source of GRB. Moreover, photons become unstable. They decay in a electron-positron pair above an energy E_0[6]. See below.

Since $c_{\text{photon}} > c_{\text{proton}}$, the strong bound of [27] is avoided: Proton is stable under Cerenkov radiation in vacuum.

If no GZK anomaly is confirmed in future experimental observations, then a stronger bound for the difference $c_\pi - c_p$ will follow. Using the same assumptions to set the restriction (33) when the primordial proton reference energy is $E_{\text{ref}} = 2 \times 10^{20}$ eV, we find

$$|c_\pi - c_p| < 2.3 \times 10^{-23}. \tag{37}$$

In terms of α, this last bound may be read as

$$|\alpha| < 9.1 \times 10^{-24}, \tag{38}$$

which is a stronger bound over α than (33), obtained from pair creation.

Photon unstability

It has been pointed out in [6,27] that if $c_{\text{photon}} > c_{\text{electron}}$ then the process $\gamma \rightarrow e^+ + e^-$ is allowed above an energy E_0:

$$E_0 = m_e\sqrt{\frac{2}{\delta c}} \tag{39}$$

where $\delta c = c_\gamma - c_e$.

In our case, we have:

$$\delta c_L = -\alpha\left(\frac{23}{6}e^2 - \left(\frac{g^2}{cos^2\theta_w}(sin^2\theta_w - 1/2)^2 + e^2 + g^2/2\right)\Big/2\right) \tag{40}$$

$$\delta c_R = -\alpha\left(\frac{23}{6}e^2 - \left(e^2 + \frac{g^2 sin^4\theta_w}{cos^2\theta_w}\right)\Big/2\right) \tag{41}$$

Therefore,

$$EL_0 = 2.3 \times 10^8 \text{ GeV} \qquad ER_0 = 1.9 \times 10^8 \text{ GeV} \tag{42}$$

So, we should not detect photons with energies above 2.3×10^8 GeV

Neutral pion Stability

Following [6] we study the main decay process of neutral pion $\pi_0 \rightarrow \gamma + \gamma$. This becomes suppressed if $c_\gamma > c_\pi$ and above an energy

$$E_\pi = \frac{m_\pi}{\sqrt{2(c_\gamma - c_\pi)}} \tag{43}$$

Using the bound $c_\gamma - c_\pi < 10^{-22}$ obtained in [27], we get

$$|\alpha| < 5.4 \times 10^{-23} \tag{44}$$

In our numerical estimates we have chosen $\alpha = -5 \times 10^{-23}$.

We get $E_\pi = 10^{19}$ eV. Therefore we expect that neutral pions above this energy are stable, so they could be a primary component of UHECR. Photons will be unstable above this energy by the same mechanism. Notice however that photons are unstable at a lower energy due to electron-positron pair creation (42).

VIII. CONCLUSIONS

In this paper we have computed the LIV induced by Quantum Gravity on Baryons and Mesons, using the Chiral Lagrangian approach. This permitted to fix that $\alpha < 0$, in order to explain the GZK anomaly. Studying several available processes, we found bounds on α:

From pair creation and absence of photo-pion creation: $2.2 \times 10^{-21} > -\alpha > 1.3 \times 10^{-24}$.

From pion stability and the most stringent experimental bound found in [28]: $|\alpha| < 5.4 \times 10^{-23}$.

Then, several predictions are obtained: Photons are unstable above an energy 2.3×10^8 GeV.

Neutral pions are stable above an energy $E_\pi = 10^{19}$ eV; so they could be a primary component of UHECR, thus evading the GZK cutoff.

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