Quantum Gravity and Lorentz Invariance Violation in the Standard Model

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The most important problem of fundamental physics is the quantization of the gravitational field. A main difficulty is the lack of available experimental tests that discriminate among the theories proposed to quantize gravity. Recently, Lorentz invariance violation by quantum gravity (QG) has been the source of growing interest. However, the predictions depend on an *ad hoc* hypothesis and too many arbitrary parameters. Here we show that the standard model itself contains tiny Lorentz invariance violation terms coming from QG. All terms depend on one arbitrary parameter α that sets the scale of QG effects. This parameter can be estimated using data from the ultrahigh energy cosmic ray spectrum to be $|\alpha| < \sim 10^{-22} - 10^{-23}$.

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In recent years, several proposals have been advanced to select theories and predict new phenomena associated with the quantum gravitational field [1–4]. Most of the new phenomenology is associated with some sort of Lorentz invariance violations (LIV's) [5,6]. Recently [7], this approach has been subjected to severe criticism.

In this Letter, we assert that the main effect of quantum gravity (QG) is to deform the measure of integration of Feynman graphs at large four momenta by a tiny LIV. The classical Lagrangian is unchanged. Equivalently, we can say that QG deforms the metric of space-time, introducing a tiny LIV proportional to $(d-4)\alpha$; d being the dimension of space-time in dimensional regularization and α the only arbitrary parameter in the model. Such small LIV could be due to quantum fluctuations of the metric of space-time produced by QG: virtual black holes as suggested in [1], D branes as in [8], compactification of extra dimensions or spin-foam anisotropies [9]. A precise derivation of α will have to wait for additional progress in the available theories of QG [10].

It is possible to have modified dispersion relations (DSR) without a preferred frame [11]. Notice, however, that in our case the classical Lagrangian is invariant under usual linear Lorentz transformations but not under DSR. So our LIV is more akin to radiative breaking of the usual Lorentz symmetry than to DSR. Moreover the regulator R defined below and the deformed metric (5) are given in a particular inertial frame, where spatial rotational symmetry is preserved. That is why, in this Letter we are ascribing to the point of view of [6], which is widely used in the literature. The preferred frame is the one where the cosmic background radiation is isotropic.

Within the standard model (SM), such LIV implies several remarkable effects, which are wholly determined up to one arbitrary parameter (α). The main effects are:

The maximal attainable velocity for particles is not the speed of light, but depends on the specific couplings of the particles within the standard model. Noticeably, this LIV of the dispersion relations is the only one acceptable, according to the very stringent bounds coming from the ultrahigh energy cosmic ray (UHECR) spectrum [12,13]. Moreover, the specific interactions between particles in the SM, determine different maximum attainable velocities for each particle, a necessary requirement to explain the Greisen [14], Zatsepin, and Kuz'min [15] anomaly [6,13,16]. Since the Auger [17] experiment is expected to produce results in the near future, powerful tests of Lorentz invariance using the spectrum of UHECR will be available.

Also, birefringence occurs for charged leptons, but not for gauge bosons. In particular, photons and neutrinos have different maximum attainable velocities. This could be tested in the next generation of neutrino detectors such as NUBE [18,19].

Vertices in the SM will pick up a finite LIV.

Cutoff regulator.—To see what the implications are of the asymmetry in the measure for renormalizable theories, we will mimic the Lorentz asymmetry of the measure by the replacement

$$\int d^d k \to \int d^d k R\left(\frac{k^2 + \alpha k_0^2}{\Lambda^2}\right).$$

Here R is an arbitrary function and Λ is a cutoff with mass dimensions, that will go to infinity at the end of the calculation. We normalize $R(0) = 1$ to recover the original integral. $R(\infty) = 0$ to regulate the integral. α is a real parameter. Notice that we are assuming that rotational invariance in space is preserved. More general possibilities such as violation of rotational symmetry in space can be easily incorporated in our formalism.

This regulator has the property that for logarithmically divergent integrals, the divergent term is Lorentz invariant, whereas when the cutoff goes to infinity a finite LIV part proportional to α remains.

One loop.—Let D be the naive degree of divergence of a one particle irreducible (1PI) graph. The change in the measure induces modifications to the primitively log divergent integrals ($D = 0$). In this case, the correction amounts to a finite LIV. The finite part of 1PI Green functions will not be affected. Therefore, standard model predictions are intact, except for the maximum attainable velocity for particles [6] and interaction vertices, which receive a finite wholly determined contribution from QG. Let us analyze the primitively divergent 1PI graphs for bosons first.

Self-energy.—$\chi(p) = \chi(0) + A^{\mu\nu} p_\mu p_\nu +$ convergent, $A^{\mu\nu} = \frac{1}{2}\partial_\mu \partial_\nu \chi(0)$. We have

$$A^{\mu\nu} = c_2 \eta^{\mu\nu} + a^{\mu\nu}.$$

c_2 is the log divergent wave function renormalization counterterm; $a^{\mu\nu}$ is a finite LIV. The on shell condition is

$$p^2 - m^2 - a^{\mu\nu} p_\mu p_\nu = 0.$$

If spatial rotational invariance is preserved, the nonzero components of the matrix a are

$$a^{00} = a_0; \qquad a^{ii} = -a_1.$$

So the maximum attainable velocity for this particle will be

$$v_m = \sqrt{\frac{1 - a_1}{1 - a_0}} \sim 1 - (a_1 - a_0)/2. \tag{1}$$

For fermions, we have the self-energy graph

$$\Sigma(p) = \Sigma(0) + s^{\mu\nu}\gamma_\nu p_\mu,$$

$s^{\mu\nu}\gamma_\nu = \partial_\mu \Sigma(0)$. Moreover,

$$s^{\mu\nu} = s\eta^{\mu\nu} + a^{\mu\nu}/2.$$

s is a log divergent wave function renormalization counterterm; $a^{\mu\nu}$ is a finite LIV. The maximum attainable velocity of this particle will be given again by Eq. (1).

By doing explicit computations for all particles in the SM, we get definite predictions for the LIV, assuming a particular regulator R. However, the dependence on R amounts to a multiplicative factor. So ratios of LIV's are uniquely determined.

Vertex correction.—This graph has $D = 0$, so the regulator R will induce a tiny LIV.

Gauge bosons.—Consider the most general quadratic Lagrangian which is gauge invariant, but could permit LIV's [20],

$$L = c^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}.$$

$c^{\mu\nu\alpha\beta}$ is antisymmetric in $\mu\nu$ and $\alpha\beta$ and symmetric by $(\alpha, \beta)\langle-\rangle(\mu, \nu)$. It implies that the most general expression for the self-energy of the gauge boson will be

$$\Pi^{\nu\beta}(p) = c^{\mu\nu\alpha\beta} p_\alpha p_\mu \Pi(p). \tag{2}$$

We see that

$$p_\nu \Pi^{\nu\beta}(p) = 0.$$

$c^{\mu\nu\alpha\beta}$ is given by a logarithmically divergent integral. We get

$$c^{\mu\nu\alpha\beta} = c_2(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\beta}\eta^{\nu\alpha}) + a^{\mu\nu\alpha\beta}. \tag{3}$$

c_2 is a Lorentz invariant counterterm and $a^{\mu\nu\alpha\beta}$ is a LIV.

It is clear that the same argument applies to massive gauge bosons that got their mass by spontaneous gauge symmetry breaking as well as to the graviton in linearized gravity. Explicit computations are simplified by using dimensional regularization as explained below.

LIV dimensional regularization.—We generalize dimensional regularization to a d dimensional space with an arbitrary constant metric $g_{\mu\nu}$. We work with a positive definite metric first and then Wick rotate. We will illustrate the procedure with an example. Here $g = \det(g_{\mu\nu})$ and $\Delta > 0$.

$$\frac{1}{\sqrt{g}} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{(k^2 + \Delta)^n} = \frac{1}{\sqrt{g}\Gamma(n)} \int_0^\infty dt t^{n-1} \int \frac{d^d k}{(2\pi)^d} k_\mu k_\nu e^{-t(g^{\alpha\beta}k_\alpha k_\beta + \Delta)} = \frac{1}{(4\pi)^{d/2}} \frac{g_{\mu\nu}}{2} \frac{\Gamma(n-1-d/2)}{\Gamma(n)} \frac{1}{\Delta^{n-1-d/2}}. \tag{4}$$

In the same manner, after Wick rotation, we obtain Appendix A4 of [21].

These definitions preserve gauge invariance because the integration measure is invariant under shifts. To get a LIV measure, we assume that

$$g^{\mu\nu} = \eta^{\mu\nu} + (4\pi)^2 \alpha \eta^{\mu 0}\eta^{\nu 0} \mathrm{Res}_{\epsilon=0} \tag{5}$$

where $\epsilon = 2 - \frac{d}{2}$ and $\mathrm{Res}_{\epsilon=0}$ is the residue of the pole at $\epsilon = 0$. A formerly divergent integral will have a pole at $\epsilon = 0$, so when we take the physical limit, $\epsilon \to 0$, the answer will contain a LIV term.

That is, LIV dimensional regularization consists in (1) calculating the d-dimensional integrals using a general metric $g_{\mu\nu}$, (2) gamma matrix algebra is generalized to a general metric $g_{\mu\nu}$, and (3) at the end of the calculation, replace $g^{\mu\nu} = \eta^{\mu\nu} + (4\pi)^2\alpha\eta^{\mu 0}\eta^{\nu 0}\mathrm{Res}_{\epsilon=0}$ and then take the limit $\epsilon \to 0$.

To define the counterterms, we used the minimal substraction scheme; that is, we substract the poles in ϵ from the 1PI graphs.

As a concrete example, let us evaluate a typical one loop integral that appears in the calculation of self-energy graphs:

$$A^{\mu\nu} = \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu}{[k^2 - m^2 + i0]^3} = \tag{6}$$

$$\frac{i}{(4\pi)^{d/2}}\frac{g^{\mu\nu}}{2}\frac{\Gamma(2-\frac{d}{2})}{2}\frac{1}{(m^2)^{2-d/2}} \tag{7}$$

$$=\frac{i}{(4\pi)^{d/2}}\frac{\eta^{\mu\nu}+(4\pi)^2\alpha\delta_0^\mu\delta_0^\nu\mathrm{Res}_{\epsilon=0}}{2}\frac{\Gamma(2-\frac{d}{2})}{2}\frac{1}{(m^2)^{2-d/2}} \tag{8}$$

$$=\frac{i}{4(4\pi)^2}\Big(\frac{\eta^{\mu\nu}}{\epsilon}+(4\pi)^2\alpha\delta_0^\mu\delta_0^\nu\Big)+\text{a finite LI term.} \tag{9}$$

LIV dimensional regularization reinforces our claim that these tiny LIV's originate in quantum gravity. In fact the sole change of the metric of space-time is a correction of order ϵ to the Minkowsky metric, and this is the source of the effects studied above. Quantum gravity is the strongest candidate to produce such effects because the gravitational field is precisely the metric of space-time and tiny LIV modifications to the flat Minkowsky metric may be produced by quantum fluctuations.

Using data from the UHECR spectrum [13] (see also [6]) we get the order of magnitude of the LIV: $(a1-a0)/2\sim 10^{-22}-10^{-23}$. From the results listed below, we get $|\alpha|<\sim 10^{-22}-10^{-23}$.

Explicit one loop computations.—We follow [21,22] and use LIV dimensional regularization.

Photons.—The LIV photon self-energy in the SM is

$$L\Pi^{\mu\nu}(q)=-\frac{23}{3}e^2\alpha q_\alpha q_\beta(\eta^{\alpha\beta}\delta_0^\mu\delta_0^\nu+\eta^{\mu\nu}\delta_0^\alpha\delta_0^\beta-\eta^{\nu\beta}\delta_0^\mu\delta_0^\alpha-\eta^{\mu\alpha}\delta_0^\nu\delta_0^\beta). \tag{10}$$

It follows that the maximal attainable velocity is

$$v_\gamma=1-\frac{23}{6}e^2\alpha. \tag{11}$$

We have included coupling to quarks and charged leptons as well as three generations and color.

Neutrinos.—The maximal attainable velocity is

$$v_\nu=1-(3+\tan^2\theta_w)\frac{g^2\alpha}{8}. \tag{12}$$

In this scenario, we predict that neutrinos [18] emitted simultaneously with photons in gamma ray bursts will not arrive simultaneously to Earth . The time delay during a flight from a source situated at a distance D will be of the order of $(10^{-22}-10^{-23})D/c\sim 10^{-5}-10^{-6}$ s, assuming $D=10^{10}$ light years. No dependence of the time delay on the energy of high energy photons or neutrinos should be observed (contrast with [1]). Photons will arrive earlier (later) if $\alpha<0(\alpha>0)$. These predictions could be tested in the next generation of neutrino detectors [19].

Using R_ξ gauges we have checked that the LIV is gauge invariant. The gauge parameter affects the Lorentz invariant part only.

Electron self-energy in the Weinberg-Salam model. Birefringence.—Define $e_L=\frac{1-\gamma^5}{2}e$, $e_R=\frac{1+\gamma^5}{2}e$, where e is the electron field. We get

$$v_L=1-\left(\frac{g^2}{\cos^2\theta_w}(\sin^2\theta_w-1/2)^2+e^2+g^2/2\right)\frac{\alpha}{2}; \tag{13}$$

$$v_R=1-\left(e^2+\frac{g^2\sin^4\theta_w}{\cos^2\theta_w}\right)\frac{\alpha}{2}. \tag{14}$$

The difference in maximal speed for the left and right helicities is $\sim(10^{-23}-10^{-24})$.

Higher order loops.—The standard model in the LIV background metric studied here is a renormalizable and unitary theory.

If the coupling constants are small as in the electroweak theory, the dominant LIV is the one loop contribution. This is true also for QCD due to asymptotic freedom, but extrapolation to lower energies is not simple due to hadronization.

We have computed the main effects of the LIV metric in the standard model, but other extensions of it could be considered as well.

Our results are generic: All particles will have a modified maximum attainable velocity and birefringence occurs for charged leptons, but not for gauge bosons, due to the chiral nature of the electroweak couplings.

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