N=1 Global Supersymmetry in D=4

- Susy algebra

$$\begin{aligned}
\left\{Q_\alpha, \bar{Q}^\beta\right\} &= -\tfrac{1}{2}(\gamma_\mu)_\alpha{}^\beta P^\mu \,, \\
\left[M_{[\mu\nu]}, Q_\alpha\right] &= -\tfrac{1}{2}(\gamma_{\mu\nu})_\alpha{}^\beta Q_\beta \,, \\
[P_\mu, Q_\alpha] &= 0\,.
\end{aligned}$$

equivalently

$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2}\left(\gamma_\mu C^{-1}\right)_{\alpha\beta} P^\mu \qquad C^{-1} = i\,\gamma^0$$

at quantum level

$$\left\{Q_\alpha, (Q^\dagger)^\beta\right\}_{\rm qu} = \tfrac{1}{2}\left(\gamma_\mu \gamma^0\right)_\alpha{}^\beta P^\mu$$

N=1 Global Supersymmetry in D=4

- Susy algebra

In Weyl basis

$$\{Q_{+\alpha}, Q_{-\beta}\} = -\frac{1}{2}\left(P_L\gamma_\mu C^{-1}\right)_{\alpha\beta} P^\mu$$
$$\{Q_{+\alpha}, Q_{+\beta}\} = 0$$
$$\{Q_{-\alpha}, Q_{-\beta}\} = 0$$

$$Q_+ = Q_L,\, Q_- = Q_R$$
$$P_+ = P_L, P_+ = P_R$$

In this form it is obvious the U(1) R symmetry

$$Q_\pm \to e^{\mp i\alpha} Q_\pm$$

$$[T_R, Q_\pm] = \mp i Q_\pm\,, \qquad [T_R, P_m] = 0\,, \qquad [T_R, J_{mn}] = 0$$

$$[T_R, Q_\alpha] = -\mathrm{i}(\gamma_*)_\alpha{}^\beta Q_\beta$$

N=1 Global Supersymmetry in D=4

- Susy algebra

We choose a Majorana representation for which all spinors are real.
In a quantum theory the real spinor charge Q becomes a hermitean operator.

$$\{Q_\alpha, Q_\beta\}_{qu} = \frac{1}{2}\left(\gamma_\mu \gamma^0\right)_{\alpha\beta} P^\mu$$

$$= \frac{1}{2}\delta_{\alpha\beta} H + \frac{1}{2}\left(\gamma_i \gamma^0\right)_{\alpha\beta} P^i$$

If we take the trace $H = \sum_{\alpha=1}^{4} Q_\alpha^2, \qquad E \geq 0$

N=1 Global Supersymmetry in D=4

- BPS states

Apart from the vacuum states, which preserve all supersymmetries, the Only states preserving some supersymmetry are states with null momentum

$$n = \dim \ker \{Q, Q\}$$

$$\text{If } n \neq 0 \text{ then } \dim \ker \{Q, Q\} \neq 0, \text{ so } \det \{Q, Q\} = 0$$

$$0 = \det \gamma \cdot P = \sqrt{\det(\gamma \cdot P)^2} = (P^2)^2$$

This shows that $n \neq 0$ implies $P^2 = 0$. $\quad P^{\mu} = (1, 0, 0, 1)$

$$\{Q, Q\} = \frac{1}{2}(1 - \gamma_{03})$$

Since $\gamma_{03}^2 = 1$ and $tr\, \gamma_{03} = 0$

We have a BPS state with n=2

N=1 Global Supersymmetry in D=4

- General properties about representations

Given a one-particle state $|p\rangle$ for a particle of 4-momentum p, the state $Q|p\rangle$ is either zero or another one-particle state with the same 4- momentum

One particle states preserving n-supersymmetries are in some representation of the Clifford algebra generated by (4-n) Qs

Massive particles. In the rest frame $\{Q_\alpha, Q_\beta\} = \frac{m}{2}\delta_{\alpha\beta} \qquad (\alpha, \beta = 1, 2, 3, 4)$

Thre is a unique 4 d irreducible representation. Therefore supermultiplets will be multiple of 4 states, In massless case n=2, supermultiplets multiple of two states

N=1 Global Supersymmetry in D=4

- In any supermultiplet of one-particle states, the number of bosons equal to number of fermions

$$\xi_1 = \sqrt{\frac{2}{m}}\,(Q_1 + iQ_2)\,, \qquad \xi_2 = \sqrt{\frac{2}{m}}\,(Q_3 + iQ_4)$$

Creation and annihilation fermionic opearors

$$\{\xi_i, \xi_j\} = 0, \qquad \{\xi_i, \xi_j^\dagger\} = 1$$

Let $|\rangle$ be a *Clifford vacuum* bose state annihilated by the ξ_i

two bose states $(|\rangle, \xi_1^\dagger \xi_2^\dagger |\rangle)$ and two fermi states $(\xi_i^\dagger |\rangle)$

In the massless case we have only one set of fermion creation and annihilation Operator, so we have one boson and one fermion.

N=1 Global Supersymmetry in D=4

- Basic multiplets

The states of particles with momentum $\vec{p}$ and energy $E(\vec{p}) = \sqrt{\vec{p}^2 + m^2_{B,F}}$

$$|\vec{p}, B\rangle \text{ and } |\vec{p}, F\rangle$$

$$Q_\alpha|\vec{p}, B\rangle = |\vec{p}, F\rangle \text{ and } Q_\alpha|\vec{p}, F\rangle \propto |\vec{p}, B\rangle.$$

Since $[P^\mu\,,\,Q_\alpha] = 0$, $\quad m_B^2 = m_F^2$

chiral multiplet

complex spin 0 boson $Z(x) = (A(x) + \mathrm{i}B(x))/\sqrt{2}$

spin 1/2 fermion Majorana field $\chi(x)$ or Weyl spinor $P_L\chi$

N=1 Global Supersymmetry in D=4

- Basic multiplets

gauge multiplet

gauge field $A_\mu(x)$

spin 1/2 fermionic partner, the gaugino

Majorana spinor $\lambda(x)$ Weyl field $P_L\lambda$

Gauge gravity multiplet

$e^a_\mu(x)$ describing the graviton

$\Psi^i_\mu(x)$, $i = 1, \ldots, \mathcal{N}$, whose quanta are the gravitinos.

N=1 Global Supersymmetry in D=4

- Conserved super-currents

free gauge multiplet

Equations of motion $\partial^\mu F_{\mu\nu} = 0$ $\quad \not{\partial}\lambda = 0$

current $\mathcal{J}^\mu = \gamma^{\nu\rho} F_{\nu\rho} \gamma^\mu \lambda$ is conserved

$$\partial_\mu \mathcal{J}^\mu = \partial_\mu F_{\nu\rho} \gamma^{\nu\rho} \gamma^\mu \lambda + \gamma^{\nu\rho} F_{\nu\rho} \not{\partial}\lambda$$

If we use $\gamma^{\nu\rho}\gamma^\mu = \gamma^{\nu\rho\mu} + 2\gamma^{[\nu}\eta^{\rho]\mu}$

vanishes due to Maxwell equation and Bianchi identity

N=1 Global Supersymmetry in D=4

- Susy Yang-Mills Theory

Basic fields: gauge boson $A_\mu^A(x)$ the Majorana spinor gaugino $\lambda^A(x)$

$$S = \int \mathrm{d}^4x \, \left[-\tfrac{1}{4}F^{\mu\nu A}F^A_{\mu\nu} - \tfrac{1}{2}\bar{\lambda}^A\gamma^\mu D_\mu\lambda^A\right]$$

Equations of motion
plus Bianchi identity

$$D^\mu F^A_{\mu\nu} = -\tfrac{1}{2} g f_{BC}{}^A \bar{\lambda}^B \gamma_\nu \lambda^C \,,$$
$$D_\mu F^A_{\nu\rho} + D_\nu F^A_{\rho\mu} + D_\rho F^A_{\mu\nu} = 0 \,,$$
$$\gamma^\mu D_\mu \lambda^A = 0 \,.$$

The current $\mathcal{J}^\mu = \gamma^{\nu\rho} F^A_{\nu\rho} \gamma^\mu \lambda^A$ is conserved

N=1 Global Supersymmetry in D=4

$$\begin{aligned}\partial_\mu \mathcal{J}^\mu &= \partial_\mu F^A_{\nu\rho}\gamma^{\nu\rho}\gamma^\mu\lambda^A + \gamma^{\nu\rho}F^A_{\nu\rho}\gamma^\mu D_\mu\lambda^A \\ &= -D^\mu F^A_{\mu\nu}\gamma^\nu\lambda^A \quad = \frac{1}{2}f_{ABC}\gamma^\nu\lambda^A\bar{\lambda}^B\gamma_\nu\lambda^C\end{aligned}$$

Now we need a Fierz rearragement

$$\frac{1}{2}f_{ABC}\gamma^\nu\lambda^A\bar{\lambda}^B\gamma_\nu\lambda^C = \frac{1}{2^m}f^{ABC}\sum_A v_D\Gamma^D\lambda^A\bar{\lambda}^B\Gamma_D\lambda^C$$

$$v_A = (-)^{r_A}(D-2r_A)$$

r_A Is the tensor rank of the Clifford basis element $\Gamma_A \longrightarrow \gamma_{\nu_1\nu_2\dots\nu_{r_A}}$

For anticommuting Majorana spinors , each bilinear $\bar{\Psi}_1\Gamma_A\Psi_2$ has a definite Symmetry under the interchange of $\Psi_1 \leftrightarrow \Psi_2$

Super Yang Mills

Since the Lie algebra indices of $\bar{\lambda}^B \Gamma_A \lambda^C$ are anti-symmetrized

the choices are $r_A = 1, 2$ bilinears $v_A = -2, 0$ for $r_A = 1, 2$

$$f_{ABC} \gamma^\nu \lambda^A \bar{\lambda}^B \gamma_\nu \lambda^C = -\frac{1}{2} f_{ABC} \gamma^\nu \lambda^A \bar{\lambda}^B \gamma_\nu \lambda^C$$

Therefore the supercurrent is conserved. It also conserved in other situations

i) Majorana spinors in $D = 3$,
ii) Majorana (or Weyl) spinors in $D = 4$,
iii) Weyl spinors in $D = 6$, *and*
iv) Majorana-Weyl spinors in $D = 10$.

N=1 Global Supersymmetry in D=4

- Susy field theories of the chiral multiplet

transformation rules

$$
\begin{aligned}
\delta Z &= \frac{1}{\sqrt{2}}\bar{\epsilon}P_L\chi\,, \\
\delta P_L\chi &= \frac{1}{\sqrt{2}}P_L(\partial\!\!\!/ Z + F)\epsilon \\
\delta F &= \frac{1}{\sqrt{2}}\bar{\epsilon}\,\partial\!\!\!/ P_L\chi\,.
\end{aligned}
$$

N=1 Global Supersymmetry in D=4

- Transformations rules of the antichiral multiplet

$$\begin{aligned}\delta\bar{Z} &= \frac{1}{\sqrt{2}}\bar{\epsilon}P_R\chi\,,\\ \delta P_R\chi &= \frac{1}{\sqrt{2}}P_R(\partial\!\!\!/\bar{Z}+\bar{F})\epsilon\\ \delta\bar{F} &= \frac{1}{\sqrt{2}}\bar{\epsilon}\,\partial\!\!\!/P_R\chi\,.\end{aligned}$$

the variations $\delta\bar{Z}$, $\delta P_R\chi$, $\delta\bar{F}$ *are precisely the adjoints of* δZ, $\delta P_L\chi$, δF

N=1 Global Supersymmetry in D=4

- Action

$$S_{\rm kin} = \int {\rm d}^4x \left[-\partial^\mu \bar{Z}\partial_\mu Z - \bar{\chi}\not{\partial} P_L \chi + \bar{F}F\right]$$

$$S_F = \int {\rm d}^4x \left[F W'(Z) - \tfrac{1}{2}\bar{\chi} P_L W''(Z)\chi\right]$$

W(Z) superpotential, arbitrary holomorphic function of Z

the action S_F is not Hermitian $\qquad S_{\bar{F}} = (S_F)^\dagger$

Complete action $\qquad S = S_{\rm kin} + S_F + S_{\bar{F}}$

$F \quad \bar{F}$ Are not a dynamical field, their equations of motion are algebraic

$F = -\overline{W}'(\bar{Z}) \qquad \bar{F} = -W'(Z)$ we can eliminate them

N=1 Global Supersymmetry in D=4

- Wess-Zumino model

$$W = \tfrac{1}{2} m Z^2 + \tfrac{1}{3} g Z^3$$

Eliminating the auxiliary field F

$$\begin{aligned} S_{WZ} &= \int \mathrm{d}^4x \left[\frac{1}{2}(-(\partial A)^2 - m^2 A^2 - (\partial B)^2 - m^2 B^2 - \bar{\chi}(\partial\!\!\!/ - m)\chi) \right. \\ &\quad \left. + \frac{g}{\sqrt{2}} \bar{\chi}(A + \mathrm{i}\gamma_* B)\chi + \frac{mg}{\sqrt{2}}(A^3 + AB^2) + \frac{g^2}{4}(A^2 + B^2)^2 \right] . \end{aligned}$$

N=1 Global Supersymmetry in D=4

- The action is invariant under susy transformations

The conserved supercurrent is given by

$$\mathcal{J}^\mu = \frac{1}{\sqrt{2}}[P_L(\partial\!\!\!/ \bar{Z} - F) + P_R(\partial\!\!\!/ Z - \bar{F})]\gamma^\mu \chi$$

N=1 Global Supersymmetry in D=4

- Susy algebra

Note that the anticommutator $\{Q,\bar{Q}\}$ is realized as the commutator of two variations with parameters $\epsilon_1,\,\epsilon_2$

$$\begin{aligned}
[\delta_1,\delta_2]\Phi(x) &= \left[\bar{\epsilon}_1 Q,\left[\bar{Q}\epsilon_2,\Phi(x)\right]\right]-(\epsilon_1\leftrightarrow\epsilon_2)\\
&= \bar{\epsilon}_1^{\alpha}[\{Q_\alpha,\bar{Q}^\beta\},\Phi(x)]\epsilon_{2\beta}\\
&= -\tfrac{1}{2}\bar{\epsilon}_1\gamma^\mu\epsilon_2\,\partial_\mu\Phi(x)\,.
\end{aligned}$$

$\bar{\epsilon}Q=\bar{Q}\epsilon$ for Majorana spinors

If we compute the left hand side, this dones not the anticommutator of the fermionic charges because any bosonic charge that commutes with field will not contribute

N=1 Global Supersymmetry in D=4

- Susy algebra

$$
\begin{aligned}
[\delta_1, \delta_2] Z &= \frac{1}{\sqrt{2}} \delta_1 (\bar{\epsilon}_2 P_L \chi) - [1 \leftrightarrow 2] \\
&= \frac{1}{2} \bar{\epsilon}_2 P_L (\partial\!\!\!/ Z + F) \epsilon_1 - [1 \leftrightarrow 2] \\
&= -\frac{1}{2} \bar{\epsilon}_1 \gamma^\mu \epsilon_2 \partial_\mu Z \,.
\end{aligned}
$$

The symmetry properties of Majorana spinor bilinears has been used

N=1 Global Supersymmetry in D=4

- Susy algebra

$$[\delta_1, \delta_2] P_L \chi = -\frac{1}{2} \bar{\epsilon}_1 \gamma^\mu \epsilon_2 P_L \partial_\mu \chi$$

Fierz rearrangement is required

$$[\delta_1, \delta_2] F = -\frac{1}{2} \bar{\epsilon}_1 \gamma^\mu \epsilon_2 \partial_\mu F$$

We have recovered the susy algebra via the transformations of fields

N=1 Global Supersymmetry in D=4

Consider the theory after elimination of F and $\bar{F}$

$$F = -\overline{W}'(\bar{Z})$$

$$S = \int \mathrm{d}^4 x \left[-\partial^\mu \bar{Z} \partial_\mu Z - \bar{\chi} \partial\!\!\!/ P_L \chi - \overline{W}' W' + \tfrac{1}{2} \bar{\chi} (P_L W'' + P_R \overline{W}'') \chi \right]$$

Now the symmetry algebra only closes on-shell

$$[\delta_1, \delta_2] P_L \chi = -\tfrac{1}{4} \bar{\epsilon}_1 \gamma^\mu \epsilon_2 P_L \left[\partial_\mu \chi - \gamma_\mu (\partial\!\!\!/ - \overline{W}'') \chi \right]$$

the extra factor apart from translation is a symmetric combination of the equation of the fermion field

N=1 Global Supersymmetry in D=4

The $\mathrm{U}(1)_R$ symmetry is a phase transformation

$$
\begin{aligned}
\delta_R Z &= \mathrm{i}\rho r Z\,, \\
\delta_R P_L \chi &= \mathrm{i}\rho(r-1) P_L \chi\,, \\
\delta_R F &= \mathrm{i}\rho(r-2) F\,.
\end{aligned}
$$

the different weights are implied by the relation

$$
[\delta_R(\rho), \delta(\epsilon)] = \rho\bar{\epsilon}^{\alpha}\,[T_R, Q_\alpha] = -\mathrm{i}\rho\bar{\epsilon}^{\alpha}(\gamma_*)_\alpha{}^\beta Q_\beta
$$

N=1 Global Supersymmetry in D=4

One can show that $r_W = 2$

$\mathrm{U}(1)_R$ invariant provided we assign $r = 2/k$ as the R-charge to theelemntary field with a superpotential $W(Z) = Z^k$

In the WZ model $W(Z) = mZ^2/2 + gZ^3/3$

If $g = 0$, $r = 1$ If $m = 0$ $r = 2/3$.

Super Yang Mills

- Susy transformations

The variation of

$$S = \int \mathrm{d}^4x \; [-\tfrac{1}{4}F^{\mu\nu A}F^A_{\mu\nu} - \tfrac{1}{2}\bar{\lambda}^A\gamma^\mu D_\mu\lambda^A]$$

$$\delta S = \int \mathrm{d}^4x \, [\delta A^A_\nu D^\mu F^A_{\mu\nu} - \delta\bar{\lambda}^A\gamma^\mu D_\mu\lambda^A + \tfrac{1}{2}f_{ABC}\delta A^A_\mu\bar{\lambda}^B\gamma^\mu\lambda^C]$$

Consider the transformations

$$\delta A^A_\mu = -\tfrac{1}{2}\bar{\epsilon}\gamma_\mu\lambda^A\,, \qquad \delta\lambda^A = \tfrac{1}{4}\gamma^{\rho\sigma}F^A_{\rho\sigma}\epsilon$$

$[\epsilon] = -1/2,\ [A_\mu] = 1,\ [\lambda] = 3/2$ in units of mass

Super Yang Mills

- The last term of the variation vanishes by the Fierz rearrangement

$$\begin{aligned}\delta S &= -\tfrac{1}{2}\int \mathrm{d}^4x\,\left[\bar{\epsilon}\gamma^\nu\lambda^A D^\mu F^A_{\mu\nu} + \tfrac{1}{2}\bar{\epsilon}\gamma^{\rho\sigma}\gamma^\mu\lambda^A D_\mu F^A_{\rho\sigma} + \tfrac{1}{2}\partial_\mu\bar{\epsilon}\gamma^{\rho\sigma}\gamma^\mu F^A_{\rho\sigma}\lambda^A\right] \\ &= -\tfrac{1}{2}\int \mathrm{d}^4x\,[\bar{\epsilon}\gamma^\nu\lambda^A D^\mu F^A_{\mu\nu} - \bar{\epsilon}\gamma^\nu\lambda^A D^\mu F^A_{\mu\nu} + \tfrac{1}{2}\partial_\mu\bar{\epsilon}\gamma^{\rho\sigma}\gamma^\mu F^A_{\rho\sigma}\lambda^A]\,, \qquad (\end{aligned}$$

The supercurrent coincides with the one obtained before

$$\mathcal{J}^\mu = \gamma^{\nu\rho}F^A_{\nu\rho}\gamma^\mu\lambda^A \qquad \text{identify } \tfrac{1}{4}\mathcal{J}^\mu \text{ as the Noether current}$$

Super Yang Mills

SUSY transformation rules

$$\delta\Phi(x) = \{\bar{\epsilon}^{\alpha} Q_{\alpha}, \Phi(x)\}_{\mathrm{PB}} = -\mathrm{i}[\bar{\epsilon}^{\alpha} Q_{\alpha}, \Phi(x)]_{\mathrm{qu}}$$

$$Q = \int d^3x\, \gamma^{\nu\rho}\gamma^0\lambda^A F^A_{\nu\rho} \quad \bar{Q} = -\int d^3x\, \lambda^A\gamma^0\gamma^{\nu\rho}F^A_{\nu\rho}$$

$$\{Q_{\alpha}, Q_{\beta}\} \;\; = \;\; -\frac{1}{2}\left(\gamma_{\mu}C^{-1}\right)_{\alpha\beta} P^{\mu}$$

In 10d there is a topological term in the right hand side. Tensor cahrges not Carried by any particle could, there is no direct contradiction with the Coleman-Mandula theorem

N=1 Global Supersymmetry in D=4

- More SYM action

$$S = \int \mathrm{d}^4 x \left[-\tfrac{1}{4} F^{\mu\nu A} F^A_{\mu\nu} - \tfrac{1}{2} \bar{\lambda}^A \gamma^\mu D_\mu \lambda^A + \tfrac{1}{2} D^A D^A \right]$$

real pseudoscalar field in the adjoint representation D^A

Susy transformations

$$\begin{aligned}
\delta A^A_\mu &= -\tfrac{1}{2} \bar{\epsilon} \gamma_\mu \lambda^A \,, \\
\delta \lambda^A &= \left[\tfrac{1}{4} \gamma^{\mu\nu} F^A_{\mu\nu} + \tfrac{1}{2} \mathrm{i} \gamma_* D^A \right] \epsilon \,, \\
\delta D^A &= \tfrac{1}{2} \mathrm{i} \, \bar{\epsilon} \gamma_* \gamma^\mu D_\mu \lambda^A \,, \qquad D_\mu \lambda^A \equiv \partial_\mu \lambda^A + \lambda^C A_\mu{}^B f_{BC}{}^A
\end{aligned}$$

$\delta S = 0.$ *Only terms involving* D^A *need to be examined*

N=1 Global Supersymmetry in D=4

- Internal symmetries

$$\begin{aligned} \delta(\theta) A_\mu^A &= \partial_\mu \theta^A + \theta^C A_\mu{}^B f_{BC}{}^A \,, \\ \delta(\theta) \lambda^A &= \theta^C \lambda^B f_{BC}{}^A \,, \\ \delta(\theta) D^A &= \theta^C D^B f_{BC}{}^A \,. \end{aligned}$$

Commutator of susy transformations

$$\begin{aligned} \left[\delta_1, \delta_2\right] A_\mu^A &= -\tfrac{1}{2}\bar{\epsilon}_1 \gamma^\nu \epsilon_2 F_{\nu\mu}^A \,, \\ \left[\delta_1, \delta_2\right] \lambda^A &= -\tfrac{1}{2}\bar{\epsilon}_1 \gamma^\nu \epsilon_2 D_\nu \lambda^A \,, \\ \left[\delta_1, \delta_2\right] D^A &= -\tfrac{1}{2}\bar{\epsilon}_1 \gamma^\nu \epsilon_2 D_\nu D^A \,. \end{aligned}$$

$$[\delta_1, \delta_2] = \delta_{susy} + \delta_{gauge}$$

the gauge field dependent transformation is $\theta^A = \frac{1}{2}\bar{\epsilon}_1 \gamma^\nu \epsilon_2 A_\nu^A$

N=1 Global Supersymmetry in D=4

- Representations

$$\{Q_\alpha, Q_\beta\}_{qu} = \frac{1}{2}\left(\gamma_\mu \gamma^0\right)_{\alpha\beta} P^\mu$$

P^2 is a Casimir of Super poincare but W^2 is not

$$Z^m = W^m - \frac{1}{32}\bar{Q}\gamma^m\gamma_5 Q$$

New Casimir $$\mathcal{C} = (Z \cdot P)^2 - Z^2 P^2$$

$$[Z^\mu, Z^\nu] = \varepsilon^{\mu\nu\rho\sigma} Z_\rho P_\sigma$$

In the rest frame $$\mathcal{C} = m^2 Z_0^2 + m^2 Z^2 = m^2 |\mathbf{Z}|^2$$

N=1 Global Supersymmetry in D=4

- Representations

$$\left[Z^i, Z^j\right] = m\varepsilon^{ijk} Z^k$$

Values of the Casimir $\mathcal{C} = m^4 Y(Y+1)\,, \qquad (Y = 0, \frac{1}{2}, 1, \ldots)$

Y superspin

$$\{Q_\alpha, Q_\beta\}_{qu} \quad = \quad m\delta^{\alpha\beta}$$

$Q_{-\alpha}|Y\rangle = 0 \qquad (\text{for all } \alpha)$ Clifford vacuum

supermultiplet $|Y\rangle, \qquad Q_{+\alpha}|Y\rangle, \qquad (\bar{Q}_+ Q_+)|Y\rangle$

Number of staes is a mutiple of four