Copyright

Dedication

To God who transformed my life in every sort of way.

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Summary

One of the fundamental problems of theoretical physics has become the search for a quantum theory of gravitation. This is because General Relativity when quantized renders a non-renormalizable theory. In this thesis, we offer a possible solution to this problem. We present a model of quantum gravity, whose principal characteristic is that it is finite in vacuum. This means that it does not even need to be renormalized since there are no divergences. We called this model Delta Gravity, which is a model of gravity with two symmetric tensors, based on a general procedure to modify, in this case, General Relativity. In Cosmology, it shows accelerated expansion without a cosmological constant. One of the main features of Delta Gravity is that, at the quantum level, it lives on shell for the original field and at one loop only, two facts that are crucial to proof the finiteness of the model. As one might expect, not everything can be good news as this model has some shortcomings. One of these is that the model has ghosts. The other is that it is finite only in vacuum, therefore it does not include matter, which is not the best of approximations, but we hope it to be one small step towards the long sought final dreamed theory of quantum gravity.

Introduction

The past century has been the scenario for two major revolutions in physics, the quantum theory and relativity, by relativity I mean the special and general theory. If we look at the history of physics tracing back to the days of Kepler, many of the most striking advances in theoretical physics have derived from the effort of finding a common theoretical framework for two theories which were apparently disconnected or, even more, in conflict with each other. This has always resulted in predicting new physical phenomena. For example, combining Keplerian orbits and Galilean physics led to Newtonian mechanics and its most celebrated three laws of motion. Maxwell's electromagnetism and Galilean relativity led to special relativity and the concept that time and space are relative. Special relativity and Newtonian gravity led to the General Theory of Relativity, giving rise to a geometrical interpretation of Gravity. Special Relativity and Quantum Mechanics led to Quantum Field Theory, which brought as a consequence the prediction of the existence of antimatter, soon confirmed by experimental observations.

As far as we know, there are four fundamental forces that govern the natural world: the electromagnetic force, the weak and strong nuclear forces, and Gravity. Gravity was the first to be explored and described by Newton's Universal law of gravitation, then Maxwell unified electric and magnetic effects in his four celebrated equations. Maxwell equations are classical, this is, not quantum, and since the beginning of the twenty century the quantum revolution made it through to stay, physicists began to look for a quantum version of electromagnetism. This was finally accomplished by Feynman, Schwinger, Tomonaga, and Dyson in the late forties, in a theory called QED (for Quantum Electrodynamics). Then, Enrico Fermi proposed a theory describing the weak interaction, but this theory had the problem of not being renormalizable.

In the late sixties, Glashow, Weinberg and Salam provided the correct description of the weak nuclear force and unified with the electromagnetic one in their electroweak theory. Also, in the late sixties and early seventies, the theory for describing the strong nuclear force was derived and was called QCD (for Quantum Chromodynamics). So far, these three quantum theories are Quantum Field Theories and conform what is known as 'The Standard Model'. But, what has happened with our understanding of Gravity? The Gravitational interaction is described by Einstein's General Theory of Relativity, also known as "geometrodynamics". This last name is good because the source of gravity, which is mass, which in turn as special relativity tell us is equivalent to energy produce the bending or curvature of spacetime so that the gravitational field manifest itself through the curvature of spacetime.

GR is a well established classical theory [1]. For example it has predicted the bending of light rays as they pass near a large mass (of the size of the Sun, for instance), the precession of the perihelion of Mercury, the gravitational redshift of light, and the expansion of the Universe. However, there are some more recent observations that suggest GR may be modified by a more encompassing theory to explain certain 'huge' effects as dark matter [2] and dark energy [3]. All in all, GR contains singularities [4], which are places in spacetime where the theory predicts infinite mass and therefore infinite curvature, and the theory breaks down. These singularities are expected to happen inside black holes and near the Big Bang, where the whole Universe was very small. In order to understand what happens in these scenarios, we need a quantum theory of gravity. But these aspects are not the only ones for having such a desire, for as we have already pointed out, the unification of GR with Quantum Theory may predict a huge amount of new physical phenomena and an understanding of them. To illustrate this point a little more, we can take as an example the discovery in the mid-seventies of black hole radiation by Hawking [5]. This result tempts us to suppose that it has exposed a small corner of a broad new area of fundamental physics in which gravity, quantum field theory, and thermodynamics are closely interwoven.

The understanding of a complete and correct theory of quantum gravity may help us to overcome other difficulties such as the ultraviolet divergencies that plague the renormalizable quantum field theories of the other three interactions. These divergencies may well be the result of the assumption that spacetime is, on all scales, a continuum. In this respect, it is important to notice that one of the current candidates for the long sought theory, the so-called Loop Quantum Gravity (LQG) [6], predicts the quantization of spacetime.

The principal problem with the attempt to quantize gravity is that the coupling constant, Newton's gravitational constant, has dimensions of energy to the power of minus two. What this actually means is that, when conventional field quantization methods based on the weak-field perturbation expansion are applied to General Relativity, it renders a non-renormalizable theory, so that the perturbative approach does not work. This may mean that either we have to search for a non-perturbative formulation of quantum gravity or, as some people suggest, it may mean that GR is, as the Fermi theory was for the weak interaction, an effective theory and that it is not of fundamental character, so that we still have to find the correct classical theory to replace GR for a renormalizable theory when quantized.

Another difficult issue which I think is good to point out is the apparent incompatibility of GR and Quantum Theory in the treatment of space-time and dynamical objects. In GR, space-time is dynamical, in quantum theory is a fixed background in which the dynamical objects, the quantum fields, act. On the other hand, in quantum theory, all dynamical objects are quantized but in GR space-time is dynamical, so we reach to a possible conclusion for the resulting theory: should space-time be quantized? Should the theory be background independent? One could say that at least for the former that these effects should manifest at the order of the Planck length, 10^{-33}cm, and that therefore the effects of quantum gravity are important only at this scale, but there is an argument that contradicts this point. Since gravity couples to everything because everything has energy and with the same strength, it also interacts with itself and so interactions of, say, photons with the gravitational field will be of the same importance as interactions of gravitons with a gravity background or field and so these effects will manifest at all scales [7].

Since the 1930's to these days, many paths have been followed to quantize the gravitational interaction. There have been the covariant approach [8], the canonical approach [9], sum over histories, supergravity [10], twistor theory [11], non-commutative geometry [12], loop quantum gravity [6], and string theory [13] [14], to name a few. Most methods are by now quite involved, and none has been accepted as the correct and final answer to the problem of quantum gravity. But all these efforts have been sparked by the problem of non-renormalizability of quantum GR. As Feynman put it once over a period (the 1950's) of his working life that he devoted to quantum gravity: "the consequences of quantum gravity might be a 'piece of cake' to work out, after all, gravity is really weak" Following the spectacular success of perturbative QED, he figured that there would be essentially no need to work out anything beyond first order [15]. To this comment we must add the fact that, if quantum GR is truncated to a certain number of loops in the perturbative regime, then it can be made renormalizable, because it would involve a finite number of counterterms [16].

In this thesis, we present a quantum gravity field theory model that we called Delta Gravity. We use the standard techniques of quantization used in conventional quantum field theory, this is, we base our analysis on weak quantum fluctuations of the field over a background metric space so that it is a covariant approach. The new and important ingredient is that, in contrast to quantum GR, this model only lives at one loop order in perturbation expansion. It is a natural and almost unique extension of GR that has two tensor fields. These is the graviton field $g_{\mu\nu}$ which transforms as a two covariant tensor under general coordinate transformations (GCT), plus $\tilde{g}_{\mu\nu}$, which transforms as a two covariant tensor under general coordinate transformations and under an additional symmetry. The classical aspects of the model were explored in [17], where it is shown that $\tilde{\delta}GR$ preserves the classical equations of the former metric $g_{\mu\nu}$. The equations of motion for both fields are second order, the newtonian limit is compatible with experiments, the equivalence principle is satisfied and, in Cosmology, accelerated expansion of the universe is obtained without introducing a cosmological constant. This last feature is also quite appealing, because not having a cosmological constant avoids the huge contradiction between its observational and theoretical predictions. In the present work, we show that all delta theories live at one loop.

Our model is not only renormalizable but, the divergent part of the effective action turns out to be twice of what was found in by 't Hooft and Veltman [8], which is proportional to a linear combination of the square of the Ricci tensor and scalar curvature. As the model lives at one loop, this is the exact effective action. Since the equation for the original field is preserved, this means the quantum corrections of the model are On-Shell in the $g_{\mu\nu}$ fields, so that also the Ricci tensor and thus the Ricci scalar vanish resulting in that the divergent part of the effective action vanishes. This implies that in our model the effective action at one loop is exact and finite in vacuum so that it does not need to be renormalized.

The problem that this model has is the apparently inevitable appearance of ghosts. Due to them, it may not be unitarity or stable. This in turn implies difficulties with the quantization of the model, but in [18] [19] [20] [21] [22], phantom fields are used to explain the accelerated expansion of the universe as an alternative to the cosmological constant and quintessence, a feature that our model [17] seems to introduce in a natural way. It would be possible that our ghosts could be related to phantom fields in $\tilde{\delta}$GR. This connection may be far reaching because the phantom idea has gained great popularity as an alternative to the cosmological constant. The present model could provide an arena to study the quantum properties of a phantom field, since the model has a finite quantum effective action. Moreover, the advantage of being a gauge-type model maintains open the possibility of fixing a gauge in which the model is unitary or impose a condition to restrict the physical Hilbert space in such a way that the model defined on this subspace is unitary. On the other hand, as [21] mentions, a choice could be made of having either ghosts or instabilities. There the author explains that, in order to save unitarity, we are forced to choose instabilities that would imply having a Hamiltonian not bounded from below.

Naturally, a theory of gravitation without matter is incomplete, but it serves as a motivation for future works where the research on this type of models can lead us to more realistic results. A possible solution is to use $\tilde{\delta}$ Supergravity models that contain matter fields [10] and could cure the phantom instability. Another possibility is to use the model we present here and add to it $\tilde{\delta}$ matter fields.

In **Chapter 1** we give the definition of the $\tilde{\delta}$ transformation, we present the general coordinate transformations and their corresponding extensions, we define the new gauge transformations, the generalizations of the covariant derivative, and the generalization of the affine geometry. In **Chapter 2**, we show the general form of the invariant action for general $\tilde{\delta}$ theories, we present and demonstrate the invariance of $\tilde{\delta}$ Gravity action, and give the general form of the classical equations of motion for general fields. In **Chapter 3**, we compute the effective action for a generic $\tilde{\delta}$ model and show that all of them live at one loop. **Chapter 4** shows how the gauge fixing and the corresponding Faddeev-Popov Lagrangian for the model are found using the BRST formalism. **Chapter 5** is the most important chapter of this work, here we apply what was seen in the previous chapters to the particular case of the Einstein-Hilbert theory. We show the classical equations of motion for the two fields and give solutions for the particular case of the

Schwarzschild metric. We apply the background field method (BFM), we present the relevant quadratic total Lagrangian. We also calculate the divergent part of the effective action at one loop using an algorithm developed in [23]. In **Chapter 6**, using the gauge fixing of the previous chapter, we explore the Hamiltonian formalism, redefine the fields and the creation and annihilation operators, and we see the existence of ghosts. In **Chapter 7**, we analyze the form of the finite quantum corrections to the effective action and we show the modification of the equations of motion due to the simplest type of corrections [24] [25] [26] [27]. Finally in **Chapter 8**, we present the conclusions of this thesis.

In **Appendix A**, we give a review of the Background Field Method following [29]. Finally, in **Appendix B**, we give a brief review of the algorithm developed in [23] for the computation of the divergent part of the Effective Action at one loop and we indicate the values of the parameters used in our case.

Motivated by simplicity, we will use cosmological constant $\Lambda = 0$.

Chapter 1

$\tilde{\delta}$ Transformation

In this work, we will study a modification of models that consists in the application of a variation that we will define as $\tilde{\delta}$. As a variation it will have all properties of a usual variation such as:

$$\begin{aligned} \tilde{\delta}(AB) &= \tilde{\delta}(A)B + A\tilde{\delta}(B), \\ \tilde{\delta}\delta A &= \delta\tilde{\delta}A, \\ \tilde{\delta}(\Phi_{,\mu}) &= (\tilde{\delta}\Phi)_{,\mu}. \end{aligned} \tag{1.1}$$

The particular point with this variation is that when applied to a field (function, tensor, etc.) it will give a new element that we define as $\tilde{\delta}$ fields which is an entire new independent object from the original:

$$\tilde{\delta}(\Phi) = \tilde{\Phi}, \tag{1.2}$$

and to indicate this, is that we call this variation 'delta tilde' $\tilde{\delta}$.

We take throughout our work the convention that a tilde tensor is equal to the $\tilde{\delta}$ transformation of the original tensor associated to it when all its indexes are covariant. We raise and lower indexes using the metric g.

In this form, we will have:

$$\tilde{S}_{\mu\nu\alpha\ldots} \equiv \tilde{\delta}\left(S_{\mu\nu\alpha\ldots}\right), \tag{1.3}$$

and, for example:

$$\begin{aligned}\tilde{\delta}\left(S^{\mu}{}_{\nu\alpha\ldots}\right) &= \tilde{\delta}(g^{\mu\rho}S_{\rho\nu\alpha\ldots}),\\ &= \tilde{\delta}(g^{\mu\rho})S_{\rho\nu\alpha\ldots} + g^{\mu\rho}\tilde{\delta}\left(S_{\rho\nu\alpha\ldots}\right).\end{aligned} \tag{1.4}$$

It is known that $\delta(g^{\mu\nu}) = -\delta(g_{\alpha\beta})g^{\mu\alpha}g^{\nu\beta}$, so:

$$\tilde{\delta}\left(S^{\mu}{}_{\nu\alpha\ldots}\right) = -\tilde{g}^{\mu\rho}S_{\rho\nu\alpha\ldots} + \tilde{S}^{\mu}{}_{\nu\alpha\ldots}. \tag{1.5}$$

1.1 General Coordinate Transformation

With the previous notation in mind, we can work out the general transformations $\tilde{\delta}$ for any tensor with all its indexes covariant. (For mixed indices, please see (1.5).) We begin by considering general coordinate transformations or diffeomorphism in their infinitesimal form:

$$\begin{aligned}x'^{\mu} &= x^{\mu} - \xi_0^{\mu}(x),\\ \delta x^{\mu} &= -\xi_0^{\mu}(x).\end{aligned} \tag{1.6}$$

Where δ is the general coordinate transformation. Now, we define:

$$\xi_1^{\mu}(x) \equiv \tilde{\delta}\xi_0^{\mu}(x). \tag{1.7}$$

Now we see some examples:

I) A scalar $\Phi(x)$:

$$\begin{aligned}\Phi'(x') &= \Phi(x),\\ \delta\Phi(x) &= \xi_0^{\mu}\Phi_{,\mu}.\end{aligned} \tag{1.8}$$

Noting that $\tilde{\delta}$ commutes with δ, we can read the transformation rule for $\tilde{\Phi} = \tilde{\delta}\Phi$:

$$\delta\tilde{\Phi}(x) = \xi_1^{\mu}\Phi_{,\mu} + \xi_0^{\mu}\tilde{\Phi}_{,\mu}. \tag{1.9}$$

II) A vector $V_\mu(x)$:

$$\delta V_\mu(x) = \xi_0^\beta V_{\mu,\beta} + \xi^\alpha_{0,\mu} V_\alpha. \tag{1.10}$$

Therefore, using (1.5), our new transformation will be:

$$\delta\tilde{V}_\mu(x) = \xi_1^\beta V_{\mu,\beta} + \xi^\alpha_{1,\mu} V_\alpha + \xi_0^\beta \tilde{V}_{\mu,\beta} + \xi^\alpha_{0,\mu}\tilde{V}_\alpha. \tag{1.11}$$

III) Rank two covariant tensor $M_{\mu\nu}$:

$$\delta M_{\mu\nu}(x) \;=\; \xi_0^\rho M_{\mu\nu,\rho} + \xi^\beta_{0,\nu} M_{\mu\beta} + \xi^\beta_{0,\mu} M_{\nu\beta}, \tag{1.12}$$

and for $\tilde{M}_{\mu\nu}$,

$$\delta\tilde{M}_{\mu\nu}(x) = \xi_1^\rho M_{\mu\nu,\rho} + \xi^\beta_{1,\nu} M_{\mu\beta} + \xi^\beta_{1,\mu} M_{\nu\beta} + \xi_0^\rho \tilde{M}_{\mu\nu,\rho} + \xi^\beta_{0,\nu}\tilde{M}_{\mu\beta} + \xi^\beta_{0,\mu}\tilde{M}_{\nu\beta}. \tag{1.13}$$

We can define the new general coordinate transformations so that δ_0 is the transformation in ξ_0 and δ_1 in ξ_1. This new transformation is the basis of this type of model.

1.2 Symmetry, Algebra and Gauge

1.2.1 Gauge Transformations

In gravitation we have a model with two fields. The first is just the usual gravitational field $g_{\mu\nu}(x)$, and a second is $\tilde{g}_{\mu\nu}(x)$, which corresponds to the $\tilde{\delta}$ variation of the first. We will have two gauge transformations associated to a general coordinate transformation, given by (1.12) and (1.13):

$$\begin{aligned}
\delta g_{\mu\nu}(x) &= g_{\mu\rho}\xi^\rho_{0,\nu} + g_{\nu\rho}\xi^\rho_{0,\mu} + g_{\mu\nu,\rho}\xi^\rho_0, &(1.14)\\
\delta \tilde{g}_{\mu\nu}(x) &= g_{\mu\rho}\xi^\rho_{1,\nu} + g_{\nu\rho}\xi^\rho_{1,\mu} + g_{\mu\nu,\rho}\xi^\rho_1 + \tilde{g}_{\mu\rho}\xi^\rho_{0,\nu} + \tilde{g}_{\nu\rho}\xi^\rho_{0,\mu} + \tilde{g}_{\mu\nu,\rho}\xi^\rho_0, &(1.15)
\end{aligned}$$

where $\xi^\mu_0(x)$ and $\xi^\mu_1(x)$ are infinitesimal contravariant vectors of the gauge transformations. Studying the algebra of these transformations, we see:

$$[\delta_{\bar{\xi}_0}, \delta_{\xi_0}]g_{\mu\nu}(x) = g_{\mu\rho}\zeta^{\rho}_{0,\nu} + g_{\nu\rho}\zeta^{\rho}_{0,\mu} + g_{\mu\nu,\rho}\zeta^{\rho}_{0}, \tag{1.16}$$

with:

$$\zeta^{\lambda}_{0} = \bar{\xi}^{\lambda}_{0,\rho}\xi^{\rho}_{0} - \xi^{\lambda}_{0,\rho}\bar{\xi}^{\rho}_{0}, \tag{1.17}$$

and:

$$[\delta_{\bar{\xi}}, \delta_{\xi}]\tilde{g}_{\mu\nu}(x) = g_{\mu\rho}\zeta^{\rho}_{1,\nu} + g_{\nu\rho}\zeta^{\rho}_{1,\mu} + g_{\mu\nu,\rho}\zeta^{\rho}_{1} + \tilde{g}_{\mu\rho}\zeta^{\rho}_{0,\nu} + \tilde{g}_{\nu\rho}\zeta^{\rho}_{0,\mu} + \tilde{g}_{\mu\nu,\rho}\zeta^{\rho}_{0} = \delta_{\zeta}\tilde{g}_{\mu\nu}(x), \tag{1.18}$$

where ζ_0 is as before and:

$$\zeta^{\lambda}_{1} = \bar{\xi}^{\lambda}_{0,\rho}\xi^{\rho}_{1} + \bar{\xi}^{\lambda}_{1,\rho}\xi^{\rho}_{0} - \xi^{\lambda}_{0,\rho}\bar{\xi}^{\rho}_{1} - \xi^{\lambda}_{1,\rho}\bar{\xi}^{\rho}_{0}. \tag{1.19}$$

It can be seen from the above equations that both transformations form a closed algebra.

1.2.2 Covariant Differentiation

First, it is good to note that, in this thesis, we always use torsion equal to zero: $T^{\rho}{}_{\mu\nu} = 0$ so that,

$$\Gamma_{\mu\nu}{}^{\alpha} = \frac{1}{2}g^{\alpha\beta}(\partial_{\nu}g_{\beta\mu} + \partial_{\mu}g_{\nu\beta} - \partial_{\beta}g_{\mu\nu}). \tag{1.20}$$

As it is usual, we define the covariant derivative as:

$$D_{\nu}A_{\alpha} = A_{\alpha;\nu} = A_{\alpha,\nu} - \Gamma_{\alpha\nu}{}^{\lambda}A_{\lambda}, \tag{1.21}$$

where A_{α} is a covariant vector. Now we generalize the definition of the covariant derivative when it acts on 'tilde' tensors e.g:

$$\nabla_{\nu}\tilde{A}_{\alpha} = \tilde{\delta}(D_{\nu}A_{\alpha}) = \tilde{A}_{\alpha,\nu} - \Gamma_{\alpha\nu}{}^{\lambda}\tilde{A}_{\lambda} - \tilde{\delta}(\Gamma_{\alpha\nu}{}^{\lambda})A_{\lambda}, \tag{1.22}$$

where $\tilde{A}_\alpha = \tilde{\delta} A_\alpha$, and we reserve the D notation for the usual covariant derivative and ∇ for the generalized one so that:

$$\nabla_\nu \tilde{A}_\alpha = D_\nu \tilde{A}_\alpha - \tilde{\delta}(\Gamma_{\alpha\nu}{}^{\lambda}) A_\lambda. \tag{1.23}$$

Where:

$$\tilde{\delta}(\Gamma_{\alpha\nu}{}^{\lambda}) = \frac{1}{2} g^{\lambda\rho} \left(D_\nu \tilde{g}_{\rho\alpha} + D_\alpha \tilde{g}_{\nu\rho} - D_\rho \tilde{g}_{\alpha\nu} \right), \tag{1.24}$$

further, the infinitesimal transformation of the modified connection is:

$$\delta(\tilde{\delta}\Gamma_{\mu\nu}{}^{\varepsilon}) = \nabla_\mu \nabla_\nu \xi_1^\varepsilon + R^\varepsilon{}_{\nu\gamma\mu} \xi_1^\gamma + \tilde{\delta}(R^\varepsilon{}_{\nu\gamma\mu}) \xi_0^\gamma, \tag{1.25}$$

with:

$$\tilde{\delta}(R^\varepsilon{}_{\nu\gamma\mu}) = D_\gamma \left[\tilde{\delta}(\Gamma_{\mu\nu}{}^{\varepsilon}) \right] - D_\mu \left[\tilde{\delta}(\Gamma_{\gamma\nu}{}^{\varepsilon}) \right]. \tag{1.26}$$

As $D_\nu A_\alpha$ is a two covariant tensor, $\nabla_\nu \tilde{A}_\alpha$ is a tilde tensor of rank two and transforms according to equation (1.13). This definition of covariant derivative will be used in **Chapter 5**. We notice that an analogous type of generalization of covariant derivative was used in [30].

1.2.3 Affine Geometry

We know that the Riemann curvature and the torsion tensors give us useful pieces of information about the geometry of the manifold that we are studying. In particular, they provide a link between the properties of the space in question with the commutation of the covariant derivatives defined on it. What we find for our modified theory of gravity is an almost predictable generalization of the standard results known for the usual theory.

First we cite the elementary result true for a A_α:

$$[D_\mu, D_\nu] A_\alpha = -R^\rho{}_{\alpha\mu\nu} A_\rho - T^\rho{}_{\nu\mu} (D_\rho A_\alpha), \tag{1.27}$$

where $R^{\rho}{}_{\alpha\mu\nu}$ is the Riemann Tensor given by [23]:

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_{\mu}\Gamma_{\nu\beta}{}^{\alpha} - \partial_{\nu}\Gamma_{\mu\beta}{}^{\alpha} + \Gamma_{\mu\gamma}{}^{\alpha}\Gamma_{\nu\beta}{}^{\gamma} - \Gamma_{\nu\gamma}{}^{\alpha}\Gamma_{\mu\beta}{}^{\gamma}, \tag{1.28}$$

with the Ricci Tensor $R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$, the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$ and:

$$T^{\rho}{}_{\nu\mu} = \Gamma_{\nu\mu}{}^{\rho} - \Gamma_{\mu\nu}{}^{\rho} \tag{1.29}$$

is the torsion tensor. Now, we have a new ingredient that comes with $\tilde{A}_{\alpha}$:

$$[\nabla_{\mu}, \nabla_{\nu}]\tilde{A}_{\alpha} = -R^{\rho}{}_{\alpha\mu\nu}\tilde{A}_{\rho} - \tilde{\delta}(R^{\rho}{}_{\alpha\mu\nu})A_{\rho} - T^{\rho}{}_{\nu\mu}(\nabla_{\rho}\tilde{A}_{\alpha}) - \tilde{\delta}(T^{\rho}{}_{\nu\mu})(D_{\rho}A_{\alpha}). \tag{1.30}$$

Where:

$$\tilde{\delta}(T^{\rho}{}_{\nu\mu}) = \tilde{\delta}(\Gamma_{\nu\mu}{}^{\rho}) - \tilde{\delta}(\Gamma_{\mu\nu}{}^{\rho}) \tag{1.31}$$

In this thesis, we always use $\tilde{\delta}(T^{\rho}{}_{\mu\nu}) = 0$.

Now that we have established the notation and the definitions, we can start to look for the structure of the modified models. In the next chapter, we will define the new invariant action and find the classical equations of motion.

Chapter 2

Modified Model

As the general coordinate transformations were extended, we can look for an invariant action. We start by considering a model based on a given action $S_0[\phi_I]$ where ϕ_I are generic fields, then we add to it a piece that is equal to a $\tilde{\delta}$ variation with respect to the fields, and we let $\tilde{\delta}\phi_J = \tilde{\phi}_J$, so that we have:

$$S[\phi, \tilde{\phi}] = S_0[\phi] + \kappa_2 \int d^4x \frac{\delta S_0}{\delta \phi_I(x)}[\phi]\tilde{\phi}_I(x), \tag{2.1}$$

with κ_2 an arbitrary constant and the indexes I can represent any kind of indexes. For more details of the definition of $\tilde{\delta}$, please see *Appendix A* of [17]. This new defined action shows the standard structure used to define any modified element or function for $\tilde{\delta}$ type models, for example the gauge fixing and Faddeev Popov. Next, we verify that this form of action is indeed the correct one for $\tilde{\delta}$ Gravity and so is invariant to the new general coordinate transformation.

2.1 The Modified Model's Invariance

In this thesis, we will investigate the $\tilde{\delta}$ Gravity action, obtained by the procedure sketched above:

$$S[g, \tilde{g}] = \int d^dx \sqrt{-g}\left(-\frac{1}{2\kappa}R\right) + \kappa_2 \int \left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\sqrt{-g}\tilde{g}_{\mu\nu}d^dx. \tag{2.2}$$

Now we must verify that (2.2) is invariant under the following transformations:

$$
\begin{aligned}
\delta g_{\mu\nu}(x) &= g_{\mu\rho}\xi^{\rho}_{0,\nu} + g_{\nu\rho}\xi^{\rho}_{0,\mu} + g_{\mu\nu,\rho}\xi^{\rho}_{0} = \xi_{0\mu;\nu} + \xi_{0\nu;\mu}, \\
\delta \tilde{g}_{\mu\nu}(x) &= \xi_{1\mu;\nu} + \xi_{1\nu;\mu} + \tilde{g}_{\mu\rho}\xi^{\rho}_{0,\nu} + \tilde{g}_{\nu\rho}\xi^{\rho}_{0,\mu} + \tilde{g}_{\mu\nu,\rho}\xi^{\rho}_{0}.
\end{aligned}
$$

We can see that (2.2) is obviously invariant under transformations generated by ξ^{ρ}_{0}, since these are general coordinate transformations and we declared $\tilde{g}_{\mu\nu}$ to be a two covariant tensor. Under transformations generated by $\xi^{\rho}_{1}(\delta_1)$, $g_{\mu\nu}$ does not change, so we have:

$$
\begin{aligned}
\delta_1 S(g,\tilde{g}) &= \kappa_2 \int \left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\sqrt{-g}(\delta_1\tilde{g}_{\mu\nu})d^d x, \\
&= \kappa_2 \int \left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\sqrt{-g}(\xi_{1\mu;\nu} + \xi_{1\nu;\mu})d^d x, \\
&= -2\kappa_2 \int \left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)_{;\nu}\sqrt{-g}\,\xi_{1\mu}d^d x = 0.
\end{aligned}
\tag{2.3}
$$

2.2 Classical Equation

Now that we know that our action is invariant, we can start to study the model. To begin with, we will analyze the classical equations of motion. When varying (2.1) with respect to $\tilde{\phi}_I$, we obtain the classical equation for ϕ_I:

$$
\frac{\delta S_0}{\delta \phi_I(x)}[\phi] = 0, \tag{2.4}
$$

and when varying with respect to ϕ_I, we obtain the equation for $\tilde{\phi}_I$:

$$
\frac{\delta S_0}{\delta \phi_I(y)}[\phi] + \kappa_2 \int d^4x \frac{\delta^2 S_0}{\delta\phi_I(y)\delta\phi_J(x)}[\phi]\tilde{\phi}_J(x) = 0. \tag{2.5}
$$

Simplifying this equation using (2.4), we obtain:

$$
\int d^4x \frac{\delta^2 S_0}{\delta\phi_I(y)\delta\phi_J(x)}[\phi]\tilde{\phi}_J(x) = 0, \tag{2.6}
$$

where we notice that $\frac{\delta^2 S_0}{\delta\phi_J\delta\phi_I}[\phi]$ is a differential operator acting on $\tilde{\phi}_J$. $\tilde{\phi}_J$ belongs to the kernel of this differential operator. It turns out that the kernel is not zero, a fact that can be clearly seen in this thesis, for the case of gravitation, in equation (5.2) and below.

Having studied the classical model, we can begin to look for the quantum aspects of it. In the next chapter we will compute the quantum corrections using a path integral approach.

Chapter 3

Quantum Modified Model

In this chapter, we derive the exact effective action for a generic $\tilde{\delta}$ model and apply the result to the Einstein-Hilbert action in **Chapter 5**. We saw that the classical action for a $\tilde{\delta}$ model is (2.1). This in turn implies that we now have two fields to be integrated in the generating functional of Green functions:

$$Z(j,\tilde{j}) = e^{iW(j,\tilde{j})} = \int \mathcal{D}\phi\mathcal{D}\tilde{\phi}e^{i\left(S_0+\int d^Nx\frac{\delta S_0}{\delta\phi_I}\tilde{\phi}_I+\int d^Nx(j_I(x)\phi_I(x)+\tilde{j}_I(x)\tilde{\phi}_I(x))\right)}. \tag{3.1}$$

We can readily appreciate that, because of the linearity of the exponent on $\tilde{\phi}_J$, what we have is the integral representation of a Dirac delta function, so that our modified model once integrated over $\tilde{\phi}_J$, gives a model with a constraint making the original model live on shell.

$$Z(j,\tilde{j}) = \int \mathcal{D}\phi e^{i\left(S_0+\int d^Nx j_I(x)\phi_I(x)\right)}\delta\left(\frac{\delta S_0}{\delta\phi_I(x)}+\tilde{j}_I(x)\right). \tag{3.2}$$

A first glance at equation (3.2) could lead us to believe that this model is purely classical. But we can see by doing a short and simple analysis that this is not so. For this, we follow [31]. (See also [32].)

Let φ_I solve the classical equation of motion:

$$\frac{\delta S_0}{\delta\phi_I(x)}|_{\varphi_I}+\tilde{j}_I(x) = 0. \tag{3.3}$$

We have:

$$\delta\left(\frac{\delta S_0}{\delta\phi_I(x)}+\tilde{j}_I(x)\right)=\overset{-1}{\det}\left(\frac{\delta^2 S_0}{\delta\phi_I(x)\delta\phi_J(y)}|_{\varphi_I}\right)\delta(\phi_I-\varphi_I). \tag{3.4}$$

Therefore:

$$\begin{aligned}
Z(j,\tilde{j}) &= \int \mathcal{D}\phi e^{i(S_0+\int d^N x j_I(x)\phi_I(x))}\delta\left(\frac{\delta S_0}{\delta\phi_I(x)}+\tilde{j}_I(x)\right), \\
&= e^{i(S_0(\varphi)+\int d^N x j_I(x)\varphi_I(x))}\overset{-1}{\det}\left(\frac{\delta^2 S_0}{\delta\phi_I(x)\delta\phi_J(y)}|_{\varphi_I}\right).
\end{aligned} \tag{3.5}$$

Notice that φ is a functional of $\tilde{j}$. The generating functional of connected Green functions is:

$$W(j,\tilde{j})=S_0(\varphi)+\int d^N x j_I(x)\varphi_I(x)+i\mathrm{Tr}\left(\log\left(\frac{\delta^2 S_0}{\delta\phi_I(x)\delta\phi_J(y)}|_{\varphi_I}\right)\right). \tag{3.6}$$

Define:

$$\begin{aligned}
\Phi_I(x) &= \frac{\delta W}{\delta j_I(x)}, \\
&= \varphi_I(x) \\
\tilde{\Phi}_I(x) &= \frac{\delta W}{\delta \tilde{j}_I(x)}.
\end{aligned}$$

The effective action is defined by:

$$\Gamma(\Phi,\tilde{\Phi})=W(j,\tilde{j})-\int d^N x\left\{j_I(x)\Phi_I(x)+\tilde{j}_I(x)\tilde{\Phi}_I(x)\right\}.$$

We get, using equations (3.3) and (3.6):

$$\Gamma(\Phi,\tilde{\Phi})=S_0(\Phi)+\int d^N x\frac{\delta S_0}{\delta\Phi_I(x)}\tilde{\Phi}_I(x)+i\mathrm{Tr}\left(\log\left(\frac{\delta^2 S_0}{\delta\Phi_I(x)\delta\Phi_J(y)}\right)\right). \tag{3.7}$$

This is the exact effective action for $\tilde{\delta}$ theories. In this proof, it is assumed that all the relevant steps for fixing the gauge have been made in (3.1), so S_0 includes the Gauge Fixing and Faddeev-Popov Lagrangian, which will be the matter of the next chapter.

Comparing equation (16.42) of [31] with equation (3.7), we see that the one-loop contribution to the effective action of δ theories is exact and the $\tilde{\delta}$ modified model lives only to one loop because higher corrections simply do not exist. Finally it is twice the one loop contribution of the original theory from which the $\tilde{\delta}$ model was derived. This results from having doubled the number of degrees of freedom. We also see that this term does not depend on the $\tilde{\phi}_I$ fields.

We see from equation (3.7) that the equations of motion for the original field $\Phi_I(x)$ do not receive quantum corrections:

$$\begin{aligned} \frac{\delta}{\delta\tilde{\Phi}_I(z)}\Gamma(\Phi,\tilde{\Phi}) &= 0, \\ \frac{\delta S_0}{\delta\Phi_I(z)} &= 0. \end{aligned} \tag{3.8}$$

On the other side, when varying with respect to ϕ_I, one obtains that the equations of motion for the new field $\tilde{\phi}_I$ do receive quantum corrections:

$$\begin{aligned} \frac{\delta}{\delta\Phi_I(x)}\Gamma(\Phi,\tilde{\Phi}) &= 0, \\ \int d^N x \frac{\delta^2 S_0}{\delta\Phi_I(z)\delta\Phi_J(x)}\tilde{\Phi}_J(x) + i\frac{\delta}{\delta\Phi_I(z)}\mathrm{Tr}\left(\log\left(\frac{\delta^2 S_0}{\delta\Phi_I(x)\delta\Phi_J(y)}\right)\right) &= 0. \end{aligned} \tag{3.9}$$

In conclusion, the quantum corrections behave as a source that only affects the equations of the new field, while those of the original field remain unchanged. This is clearly seen when we compare (3.8) and (3.9) with (2.4) and (2.6).

In general, $\mathrm{Tr}\left(\log\left(\frac{\delta^2 S_0}{\delta\Phi_I(x)\delta\Phi_J(y)}\right)\right)$ could be divergent and needs to be renormalized (see [30]). From equation (3.7), we see that the $\tilde{\delta}$ model will be renormalizable if the original theory is renormalizable. But, due to equation (3.8), originally non-renormalizable theories could be finite or renormalizable in their $\tilde{\delta}$ version. This term can be calculated in many ways, for example by Zeta function regularization (see, for instance, [33]), perturbation theory (Feynman diagrams), etc. For gravitation, the calculation of this term is quite difficult for any of the above methods, so we will use an alternative method developed in [23].

In the present work, the $\tilde{\delta}$ Gravity model contains two dynamical fields, $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, both of which are important to describe the gravitational field in this approach. (see [17]). So, we must consider the effective action of the model for the two fields. We saw that $g_{\mu\nu}$ always satisfies the classical equations. This is the meaning of equation (3.2).

However, the equation of motion for $\tilde{g}_{\mu\nu}$ do receive quantum corrections. Moreover, One Particle Irreducible Graphs containing $g_{\mu\nu}$ external legs are non trivial and subjected to Quantum effects.

Chapter 4

Gauge Fixing and Faddeev-Popov Lagrangian via BRST Formalism

We start by using the background field method (See **Appendix A**) in which each field is separated into a classical background $g, \tilde{g}$ and a quantum part $h, \tilde{h}$:

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu} \qquad \tilde{g}_{\mu\nu} \to \tilde{g}_{\mu\nu} + \tilde{h}_{\mu\nu} \tag{4.1}$$

Now, we give the BRST transformations $\bar{\delta}$ of our model:

$$\begin{aligned} \xi_0^\mu(x) &= \lambda c_0^\mu(x), \\ \xi_1^\mu(x) &= \lambda c_1^\mu(x), \end{aligned} \tag{4.2}$$

where λ is a Grassmann constant and c_0^μ, c_1^μ are the two ghosts of our model. Starting from the gauge transformations for our quantum fields $h_{\mu\nu}$ and $\tilde{h}_{\mu\nu}$ [34], we obtain to zeroth order in h and $\tilde{h}$:

$$\bar{\delta} h_{\mu\nu} = c_{0\mu;\nu} + c_{0\nu;\mu}, \tag{4.3}$$

$$\bar{\delta} \tilde{h}_{\mu\nu} = c_{1\mu;\nu} + c_{1\nu;\mu} + \tilde{g}_{\mu\nu;\lambda} c_0^\lambda + \tilde{g}_{\mu\lambda} c^\lambda_{0;\nu} + \tilde{g}_{\nu\lambda} c^\lambda_{0;\mu}, \tag{4.4}$$

and we also have:

$$\begin{aligned} \bar{\delta} c_0^\mu &= c_0^\rho c^\mu_{0,\rho}, \\ \bar{\delta} c_1^\mu &= c_0^\rho c^\mu_{1,\rho} + c_1^\rho c^\mu_{0,\rho}, \\ \bar{\delta} \bar{c}_0^\mu &= i b_0^\mu(x), \\ \bar{\delta} \bar{c}_1^\mu &= i b_1^\mu(x), \end{aligned} \tag{4.5}$$

for the corresponding anti-ghosts $\bar{c}$ and where the b's are the auxiliary Nakanishi-Lautrup fields which satisfy:

$$\bar{\delta} b^{\mu}_{0,1} = 0. \tag{4.6}$$

It has been verified that these transformations are nilpotent. Now, we choose for our gauge fixing term:

$$\mathrm{GF} = -\sqrt{-g}\frac{C^2}{2} - \tilde{\delta}\left(\kappa_2\sqrt{-g}\frac{C^2}{2}\right). \tag{4.7}$$

We see that this is a good choice for our gauge fixing since it is invariant under both transformations δ_0 and δ_1 (see **2.1**), where [8] [23]:

$$\begin{aligned} C^2 &= g^{\alpha\beta}C_\alpha C_\beta, \\ C_\mu &= D_\nu h^\nu_\mu - \frac{1}{2}D_\mu h^\nu_\nu. \end{aligned} \tag{4.8}$$

In this way, we have:

$$\begin{aligned} \mathrm{GF} &= -\sqrt{-g}\left[\left(1+\frac{\kappa_2}{2}g^{\alpha\beta}\tilde{g}_{\alpha\beta}\right)\frac{C^2}{2} + \kappa_2\tilde{\delta}\left(\frac{g^{\mu\rho}C_\mu C_\rho}{2}\right)\right], \\ &= -\sqrt{-g}\left[\left(1+\frac{\kappa_2}{2}g^{\alpha\beta}\tilde{g}_{\alpha\beta}\right)\frac{C_\mu C^\mu}{2} + \kappa_2\left(\tilde{C}_\mu C^\mu - \frac{\tilde{g}_{\mu\beta}C^\mu C^\beta}{2}\right)\right], \end{aligned} \tag{4.9}$$

where:

$$\tilde{C}_\mu = \tilde{\delta}C_\mu = \tilde{\delta}\left[D_\nu h^\nu_\mu - \frac{1}{2}D_\mu h^\nu_\nu\right] = g^{\nu\rho}\left[\nabla_\nu\tilde{h}_{\rho\mu} - \frac{1}{2}\nabla_\mu\tilde{h}_{\rho\nu}\right] - \tilde{g}^{\nu\rho}\left[D_\nu h_{\rho\mu} - \frac{1}{2}D_\mu h_{\rho\nu}\right] \tag{4.10}$$

This can be written in the form:

$$\mathrm{GF} = -\sqrt{-g}H_\mu C^\mu, \tag{4.11}$$

with:

$$H_\mu = \left[\left(1+\frac{\kappa_2}{2}\tilde{g}^\alpha_\alpha\right)\frac{C_\mu}{2} + \kappa_2\left(\tilde{C}_\mu - \tilde{g}_{\mu\beta}\frac{C^\beta}{2}\right)\right]. \tag{4.12}$$

Having established the form of the gauge-fixing term, we can now by a standard procedure (the BRST method) find the associated Faddeev-Popov Lagrangian. Following [28], now we do:

$$\mathcal{L}_{\mathrm{GF+FP}} = -i\bar{\delta}(P), \tag{4.13}$$

where P in our case is:

$$P = \bar{c}_0^\mu H_\mu + \bar{c}_1^\mu C_\mu + \beta_1 \bar{c}_1^\mu b_{0\mu} + \beta_2 \bar{c}_0^\mu b_{1\mu}, \tag{4.14}$$

where the β's are arbitrary constants to be fixed shortly, so we have:

$$\mathcal{L}_{\mathrm{GF+FP}} = -i(ib_0^\mu H_\mu + ib_1^\mu C_\mu + i(\beta_1+\beta_2) b_1^\mu b_{0\mu} - \bar{c}_0^\mu(\bar{\delta} H_\mu) - \bar{c}_1^\mu(\bar{\delta} C_\mu)), \tag{4.15}$$

and so:

$$\begin{aligned}
\mathcal{L}_{\mathrm{GF}} &= b_0^\mu H_\mu + b_1^\mu C_\mu + (\beta_1+\beta_2) b_1^\mu b_{0\mu}, &(4.16)\\
\mathcal{L}_{\mathrm{FP}} &= i(\bar{c}_0^\mu(\bar{\delta} H_\mu) + \bar{c}_1^\mu(\bar{\delta} C_\mu)). &(4.17)
\end{aligned}$$

Now, for the gauge fixing part, we can use the equations of motion for the auxiliary fields to make them disappear,

$$\begin{aligned}
\frac{\partial \mathcal{L}_{\mathrm{GF}}}{\partial b_1^\mu} &= C_\mu + (\beta_1+\beta_2) b_{0\mu} = 0 \longrightarrow b_{0\mu} = -\frac{C_\mu}{(\beta_1+\beta_2)},\\
\frac{\partial \mathcal{L}_{\mathrm{GF}}}{\partial b_0^\mu} &= H_\mu + (\beta_1+\beta_2) b_{1\mu} = 0 \longrightarrow b_{1\mu} = -\frac{H_\mu}{(\beta_1+\beta_2)},
\end{aligned} \tag{4.18}$$

substituting in $\mathcal{L}_{\mathrm{GF}}$ we get:

$$\mathcal{L}_{\mathrm{GF}} = -\frac{C^\mu H_\mu}{(\beta_1+\beta_2)} - \frac{C^\mu H_\mu}{(\beta_1+\beta_2)} + \frac{(\beta_1+\beta_2) C^\mu H_\mu}{(\beta_1+\beta_2)^2} = -\frac{C^\mu H_\mu}{(\beta_1+\beta_2)}, \tag{4.19}$$

so we see we recover our initial gauge fixing if we set $(\beta_1+\beta_2) = 1$. Now, for the Faddeev-Popov Lagrangian, we have:

$$\mathcal{L}_{\mathrm{FP}} = i\left(\bar{c}_0^\mu(\bar{\delta} H_\mu) + \bar{c}_1^\mu(\bar{\delta} C_\mu)\right). \tag{4.20}$$

It is well known that [8] [23]:

$$\bar{\delta}C_\mu = D_\nu D^\nu c_{0\mu} + R_{\mu\nu}c_0^\nu, \tag{4.21}$$

and using:

$$\begin{aligned}
\bar{\delta}h_{\nu\rho} &= D_\nu c_{0\rho} + D_\rho c_{0\nu}, \\
\bar{\delta}\tilde{h}_{\nu\rho} &= \nabla_\nu c_{1\rho} + \nabla_\rho c_{1\nu},
\end{aligned} \tag{4.22}$$

we get:

$$\begin{aligned}
\bar{\delta}H_\mu &= \left[\left(1 + \frac{\kappa_2}{2}\tilde{g}^\alpha_\alpha\right)\frac{\bar{\delta}C_\mu}{2} + \kappa_2\left(\bar{\delta}\tilde{C}_\mu - \tilde{g}_{\mu\beta}\frac{\bar{\delta}C^\beta}{2}\right)\right], \\
\bar{\delta}\tilde{C}_\mu &= \nabla_\nu\nabla^\nu c_{1\mu} + R_{\mu\nu}c_1^\nu - g^{\rho\nu}\tilde{\delta}(R^\alpha{}_{\rho\nu\mu})c_{0\alpha} - \tilde{g}^{\nu\rho}[D_\nu D_\rho c_{0\mu} + c_{0\sigma}R^\sigma_{\rho\mu\nu}].
\end{aligned} \tag{4.23}$$

So, evaluating in (4.20), we will obtain (5.14).

In the next chapter, we will study $\tilde{\delta}$ Gravity. We will see that the divergent part of the quantum corrections to the effective action give a null contribution to the equations of motion for pure gravity and without a cosmological constant, which means that under these conditions we have a finite model of gravity.

Chapter 5

$\tilde{\delta}$ Gravity

Until now, we have studied $\tilde{\delta}$ models in general. We found the invariant action given by (2.1), with the classical equations of motion (2.4) and (2.6). Then, we demonstrated that $\tilde{\delta}$ models live only to one loop and the effective action is given by (3.7). In this chapter, we apply these results to gravity. In the first part, we will present the classical equations of motion for both fields and show the solutions in two cases. Then we will apply the Background Field Method (BFM) to obtain the quadratic Lagrangians and finally we calculate the divergent part of the effective action for $\tilde{\delta}$ Gravity.

5.1 Classical Equations of Motion and Solutions

Now we are ready to study the modifications to gravity. In this case, we have that $\phi_I \to g_{\mu\nu}$ and $\tilde{\phi}_I \to \tilde{g}_{\mu\nu}$. So, using (2.1), we obtain:

$$\begin{aligned} L_0[g_{\mu\nu}] &= \sqrt{-g}\left(-\frac{1}{2\kappa}R\right), \\ L[g_{\mu\nu},\tilde{g}_{\mu\nu}] &= \sqrt{-g}\left[-\frac{1}{2\kappa}R + \kappa_2 G^{\mu\nu}\tilde{g}_{\mu\nu}\right]. \end{aligned} \tag{5.1}$$

If we vary this action, we obtain the equations of motion:

$$\begin{aligned} G^{\mu\nu} &= 0, \\ F^{(\mu\nu)(\alpha\beta)\rho\lambda} D_\rho D_\lambda \tilde{g}_{\alpha\beta} &= 0, \end{aligned} \tag{5.2}$$

with:

$$
\begin{aligned}
F^{(\mu\nu)(\alpha\beta)\rho\lambda} &= P^{((\rho\mu)(\alpha\beta))}g^{\nu\lambda} + P^{((\rho\nu)(\alpha\beta))}g^{\mu\lambda} - P^{((\mu\nu)(\alpha\beta))}g^{\rho\lambda} - P^{((\rho\lambda)(\alpha\beta))}g^{\mu\nu}, \\
P^{((\alpha\beta)(\mu\nu))} &= \frac{1}{4}\left(g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\beta}g^{\mu\nu}\right). \qquad (5.3)
\end{aligned}
$$

Where $(\mu\nu)$ tells us that the μ and ν are in a totally symmetric combination. An important thing to notice is that both equations are of second order in derivatives, which is needed to preserve causality.

One particular solution to equations (5.2) is the following:

For the vacuum, we have for example the case of Schwarzschild:

$$
g_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{\alpha}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{\alpha}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta) \end{pmatrix}, \qquad (5.4)
$$

which has a solution for $\tilde{g}_{\alpha\beta}$ of the form:

$$
\tilde{g}_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{2\alpha+\beta}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1+\frac{\beta}{r}}{\left(1-\frac{\alpha}{r}\right)^2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta) \end{pmatrix}. \qquad (5.5)
$$

Here it as been imposed that $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ approach Minkowski space when $r \to \infty$, and α and β are determined by boundary conditions.

5.2 Quadratic Lagrangians

We proceed to calculate the quadratic Lagrangians for $\tilde{\delta}$ Gravity and Faddeev-Popov. These expressions are needed to obtain the one-loop corrections of the model. For this, we use the Background Field Method (See **Appendix A**). That is $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ and $\tilde{g}_{\mu\nu} \to \tilde{g}_{\mu\nu} + \tilde{h}_{\mu\nu}$, this is:

$$
\begin{aligned}
L_0[g_{\mu\nu} + h_{\mu\nu}] &= \frac{\sqrt{-g}}{2\kappa}\left(-\bar{R} - \frac{1}{2}C^2\right), \\
L[g_{\mu\nu} + h_{\mu\nu}] &= \frac{\sqrt{-g}}{2\kappa}\left(-\bar{R} - C^\mu H_\mu + \kappa_2 \bar{G}^{\mu\nu}\tilde{g}_{\mu\nu}\right), \qquad (5.6)
\end{aligned}
$$

with $\bar{R} = R[g+h]$ and $\bar{G}^{\mu\nu} = G^{\mu\nu}[g+h]$. We have included the original gauge fixing $C_\mu = h^\nu_{\mu;\nu} - \frac{1}{2}h^\nu_{\nu;\mu}$ and the new part $H_\mu = \frac{1}{2}\left(1 + \frac{\kappa_2}{2}\tilde{g}^\alpha_\alpha\right)C_\mu + \kappa_2\left(\tilde{C}_\mu - \frac{1}{2}\tilde{g}_{\mu\rho}C^\rho\right)$. When we calculate the quadratic part in the quantum gravitational fields, $h_{\mu\nu}$ and $\tilde{h}_{\mu\nu}$, we obtain:

$$L_{quad} = \frac{1}{2}\sqrt{-g}\vec{h}^T_{(\alpha\beta)}P^{((\alpha\beta)(\mu\nu))}\left(\left[K^{(\gamma\varepsilon)}_{(\mu\nu)}\right]^{(\lambda\eta)}\nabla_\lambda\nabla_\eta + \left[W^{(\gamma\varepsilon)}_{(\mu\nu)}\right]\right)\vec{h}_{(\gamma\varepsilon)}, \tag{5.7}$$

and:

$$\vec{h}_{(\alpha\beta)} = \begin{pmatrix} h_{\alpha\beta} \\ \tilde{h}_{\alpha\beta} \end{pmatrix} \tag{5.8}$$

$$\left[K^{(\gamma\varepsilon)}_{(\mu\nu)}\right]^{(\lambda\eta)} = \frac{1}{2\kappa}g^{\lambda\eta}\begin{pmatrix} \left(1+\frac{\kappa_2}{2}\tilde{g}^\sigma_\sigma\right)\delta^{\gamma\varepsilon}_{\mu\nu} + \kappa_2 P^{-1}_{((\mu\nu)(\sigma\rho))}\tilde{\delta}(P^{((\sigma\rho)(\gamma\varepsilon))}) & \kappa_2\delta^{\gamma\varepsilon}_{\mu\nu} \\ \kappa_2\delta^{\gamma\varepsilon}_{\mu\nu} & 0 \end{pmatrix} - \frac{\kappa_2}{2\kappa}\tilde{g}^{\lambda\eta}\delta^{\gamma\varepsilon}_{\mu\nu}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{5.9}$$

$$\left[W^{(\gamma\varepsilon)}_{(\mu\nu)}\right] = \frac{1}{\kappa}\begin{pmatrix} \left(1+\frac{\kappa_2}{2}\tilde{g}^\sigma_\sigma\right)X^{(\gamma\varepsilon)}_{(\mu\nu)} + \kappa_2\tilde{\delta}(X^{(\gamma\varepsilon)}_{(\mu\nu)}) + \kappa_2 P^{-1}_{((\mu\nu)(\sigma\rho))}\tilde{\delta}(P^{((\sigma\rho)(\alpha\beta))})X^{(\gamma\varepsilon)}_{(\alpha\beta)} & \kappa_2 X^{(\gamma\varepsilon)}_{(\mu\nu)} \\ \kappa_2 X^{(\gamma\varepsilon)}_{(\mu\nu)} & 0 \end{pmatrix} \tag{5.10}$$

Where:

$$X^{(\gamma\varepsilon)}_{(\mu\nu)} = \frac{1}{2}\left(R_\mu{}^\gamma{}_\nu{}^\varepsilon + R_\mu{}^\varepsilon{}_\nu{}^\gamma + \frac{1}{2}\left(\delta^\gamma_\mu R^\varepsilon_\nu + \delta^\varepsilon_\mu R^\gamma_\nu + \delta^\gamma_\nu R^\varepsilon_\mu + \delta^\varepsilon_\nu R^\gamma_\mu\right) - \delta^{\gamma\varepsilon}R_{\mu\nu} - \delta_{\mu\nu}R^{\gamma\varepsilon} - \frac{1}{2}R\left(\delta^\gamma_\mu\delta^\varepsilon_\nu + \delta^\varepsilon_\mu\delta^\gamma_\nu - \delta_{\mu\nu}\delta^{\gamma\varepsilon}\right)\right) \tag{5.11}$$

where $P^{((\alpha\beta)(\mu\nu))}$ is defined in (5.3) and $\delta^{\gamma\varepsilon}_{\mu\nu}$ is the symmetrized Kronecker delta. Moreover, the covariant derivative works on $\vec{h}_{(\gamma\varepsilon)}$ vector like:

$$\nabla_\lambda\vec{h}_{(\gamma\varepsilon)} = \partial_\lambda\vec{h}_{(\gamma\varepsilon)} - \left[\Gamma_{\lambda\gamma}{}^\beta\right]\vec{h}_{(\beta\varepsilon)} - \left[\Gamma_{\lambda\varepsilon}{}^\beta\right]\vec{h}_{(\gamma\beta)}, \tag{5.12}$$

with:

$$\left[\Gamma_{\lambda\gamma}{}^\beta\right] = \begin{pmatrix} \Gamma_{\lambda\gamma}{}^\beta & 0 \\ \tilde{\delta}(\Gamma_{\lambda\gamma}{}^\beta) & \Gamma_{\lambda\gamma}{}^\beta \end{pmatrix}, \tag{5.13}$$

And using the BRST method, we obtain the Faddeev-Popov Lagrangian:

$$L_{FP} = \vec{\bar{c}}^T_\mu\sqrt{-g}\left(\left[K^{\mu\lambda}_{FP}\right]^{(\rho\nu)}\nabla_\rho\nabla_\nu + \left[W^{\mu\lambda}_{FP}\right]\right)\vec{c}_\lambda, \tag{5.14}$$

Where:

$$\vec{c}_\lambda = \begin{pmatrix} c_{0\lambda} \\ c_{1\lambda} \end{pmatrix} \tag{5.15}$$

$$\left[K^{\mu\lambda}_{FP}\right]^{(\rho\nu)} = ig^{\nu\rho}\begin{pmatrix} \frac{1}{2}\left(1+\frac{\kappa_2}{2}\tilde{g}^\sigma_\sigma\right)g^{\mu\lambda} - \frac{\kappa_2}{2}\tilde{g}^{\mu\lambda} & \kappa_2 g^{\mu\lambda} \\ g^{\mu\lambda} & 0 \end{pmatrix} - i\kappa_2\tilde{g}^{\nu\rho}g^{\mu\lambda}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{5.16}$$

$$\left[W^{\mu\lambda}_{FP}\right] = i\begin{pmatrix} \frac{1}{2}\left(1+\frac{\kappa_2}{2}\tilde{g}^\sigma_\sigma\right)R^{\mu\lambda} - \kappa_2\tilde{g}_{\alpha\beta}R^{\mu\alpha\lambda\beta} - \frac{\kappa_2}{2}\tilde{g}^{\mu\alpha}R^\lambda_\alpha - g^{\alpha\beta}g^{\mu\gamma}\tilde{\delta}\left(R^\lambda{}_{\alpha\beta\gamma}\right) & \kappa_2 R^{\mu\lambda} \\ R^{\mu\lambda} & 0 \end{pmatrix} \tag{5.17}$$

with:

$$\nabla_\lambda \vec{c}_\mu = \partial_\lambda \vec{c}_\mu - \left[\Gamma_{\lambda\mu}^{\ \ \beta}\right] \vec{c}_\beta \tag{5.18}$$

5.3 Divergent Part of the Effective Action

In **Chapter 3**, we demonstrated that the quantum corrections to the effective action do not depend on the tilde fields, in this case $\tilde{g}_{\mu\nu}$. On the other side, Renormalization Theory tells us that its divergent corrections can only be local terms. So, by power counting and invariance of the Background Field Effective Action under general coordinate transformations, we know that the divergent part to L loops is [8] [35]:

$$\Delta S^L_{div} \propto \int d^4x \sqrt{-g} R^{L+1}, \tag{5.19}$$

where R^{L+1} is any scalar contraction of $(L+1)$ Riemann tensors. As our model lives only to one loop,

$$L^{div}_Q = \sqrt{-g}(a_1 R^2 + a_2 R_{\alpha\beta} R^{\alpha\beta}). \tag{5.20}$$

We do not use $R_{\alpha\beta\gamma\lambda}R^{\alpha\beta\gamma\lambda}$ because we have the topological identity in four dimensions:

$$\sqrt{-g}\left(R_{\alpha\beta\gamma\lambda}R^{\alpha\beta\gamma\lambda} - 4R_{\alpha\beta}R^{\alpha\beta} + R\right) = \text{Total derivative.} \tag{5.21}$$

To calculate the divergent part of the Effective Action in our model (i.e. a_1 and a_2 in (5.20)), we made a FORM program [36] to implement the algorithm developed in [23], obtaining in our case (**See Appendix B**):

$$\begin{aligned}
L^{div}_{Q,grav} &= \sqrt{-g}\frac{\hbar c}{\varepsilon}\left(\frac{7}{12}R^2 + \frac{7}{6}R_{\alpha\beta}R^{\alpha\beta}\right), \\
L^{div}_{Q,ghost} &= -2 \times \sqrt{-g}\frac{\hbar c}{\varepsilon}\left(\frac{17}{60}R^2 + \frac{7}{30}R_{\alpha\beta}R^{\alpha\beta}\right), \\
L^{div}_{Q} &= \sqrt{-g}\frac{\hbar c}{\varepsilon}\left(\frac{1}{60}R^2 + \frac{7}{10}R_{\alpha\beta}R^{\alpha\beta}\right),
\end{aligned} \tag{5.22}$$

with $\varepsilon = 8\pi^2(N-4)$. When we compare with the usual result in gravitation [8] [23], we can see that we obtain twice the divergent term of General Relativity. Divergences also double in Yang-Mills [30].

Moreover, since Einstein's equations of motion are exactly valid at the quantum level,

$$\left(\frac{\delta\Gamma(g,\tilde{g})}{\delta\tilde{g}_{\mu\nu}}\right) = R^{\mu\nu} = 0, \tag{5.23}$$

where $\Gamma(g,\tilde{g})$ is the Effective Action in the Background Field Method. It follows that the contribution of (5.22) to the equation of motion vanishes:

$$\hbar c\left[\frac{\sqrt{-g}}{\varepsilon}\left(\frac{1}{2}g^{\mu\nu}\left(\frac{1}{60}R^2+\frac{7}{10}R_{\alpha\beta}R^{\alpha\beta}\right)+\frac{1}{30}R\frac{\delta R}{\delta g_{\mu\nu}}+\frac{7}{10}R_{\alpha\beta}\frac{\delta R^{\alpha\beta}}{\delta g_{\mu\nu}}+\frac{7}{10}R^{\alpha\beta}\frac{\delta R_{\alpha\beta}}{\delta g_{\mu\nu}}\right)\right]_{R_{\alpha\beta}=0} = 0 \tag{5.24}$$

Therefore, $\tilde{\delta}$ Gravity is a finite model of gravitation if we do not have matter and a cosmological constant. The finiteness of our model implies that Newton's Constant does not run at all, neither with time nor energy scale, which would be supported by the very stringent experimental bounds set on its change [37] [38]. We must notice that this model is finite only in four dimensions because we need (5.21). Moreover, in more dimensions there could appear more terms in (5.20) that contains $R^{\mu_1\mu_2\cdots\mu_N}$ with N the dimension of space, that give a non-zero contribution to the equations of motion.

In spite of these apparent successes, there seems to be a problem with this model, namely is the possible existence of ghosts. This issue will be dealt with in the next chapter.

Chapter 6

Ghosts

In this chapter, we discuss the fact that our model has ghosts, as well as the lost of unitarity due to them. In order to proceed with this endeavor, we first write the quadratic Lagrangian (5.7) for a non-interacting model (this is, with the backgrounds both equal to the Minkowski metric tensor) and calculate from it the canonical conjugate momenta to the quantum fields. It is important to notice that, for the Lagrangian (5.7), a gauge has been chosen. Thus, it is possible to show, that under these conditions and in this gauge, the quantum fields obey the wave equation and an expansion in plane waves is possible where the Fourier coefficients are promoted to creation and annihilation operators much in the same way as can be done for the electromagnetic potential. We use the canonical commutation relations for fields and momenta to work out the corresponding canonical commutation relations for the creation and annihilation operators. We also show first the Hamiltonian in terms of fields and momenta and then in terms of annihilation and creation operators.

To study the existence of ghosts in the model we will study small perturbations to flat space. This is done by taking expression (5.7) and putting the backgrounds equal to the Minkowski metric $g_{\mu\nu} = \eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, thus obtaining:

$$S[h, \tilde{h}] = -\frac{1}{2\kappa} \int d^4x P^{((\alpha\beta)(\mu\nu))} \left(\frac{(1-\kappa_2)}{2} \partial_\rho h_{\alpha\beta} \partial^\rho h_{\mu\nu} + \kappa_2 \partial_\rho \tilde{h}_{\alpha\beta} \partial^\rho h_{\mu\nu} \right), \tag{6.1}$$

where now:

$$P^{((\alpha\beta)(\mu\nu))} = \frac{1}{4} \left(\eta^{\alpha\mu}\eta^{\beta\nu} + \eta^{\alpha\nu}\eta^{\beta\mu} - \eta^{\alpha\beta}\eta^{\mu\nu} \right), \tag{6.2}$$

and the equations of motion for the fields are:

$$\partial^2 h_{\mu\nu} = 0,$$
$$\partial^2 \tilde{h}_{\mu\nu} = 0. \tag{6.3}$$

With $\partial^2 = \eta^{\rho\lambda}\partial_\rho\partial_\lambda$. This corresponds to the wave equation with energy $E_{\mathbf{p}} = |\mathbf{p}|$. Here we notice that in order to obtain these equations, we have made use of a particular gauge fixing term (4.11) in the Lagrangian (5.7).

It is well known that for a diffeomorfism-invariant Lagrangian, the canonical Hamiltonian is zero. This is so in delta-gravity as well as in General Relativity: the total Hamiltonian is a linear combination of the first-class constraints (See [9]). After gauge fixing, the Hamiltonian is:

$$H = \int d^3x \left(\frac{2\kappa}{\kappa_2} P^{-1}_{((\alpha\beta)(\mu\nu))}\left(\tilde{\Pi}^{\alpha\beta}\Pi^{\mu\nu} - \frac{(1-\kappa_2)}{2\kappa_2}\tilde{\Pi}^{\alpha\beta}\tilde{\Pi}^{\mu\nu}\right)\right) \tag{6.4}$$
$$+ \int d^3x \left(\frac{\kappa_2}{2\kappa} P^{((\alpha\beta)(\mu\nu))}\left(\partial_i\tilde{h}_{\alpha\beta}\partial_i h_{\mu\nu} + \frac{(1-\kappa_2)}{2\kappa_2}\partial_i h_{\alpha\beta}\partial_i h_{\mu\nu}\right)\right),$$

with:

$$P^{-1}_{((\alpha\beta)(\mu\nu))} = \eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu} - \eta_{\alpha\beta}\eta_{\mu\nu} = 4P_{((\alpha\beta)(\mu\nu))}, \tag{6.5}$$

and where the conjugate momenta are:

$$\Pi^{\mu\nu} = \frac{\delta\mathcal{L}}{\delta\dot{h}_{\mu\nu}} = \frac{1}{2\kappa}P^{((\alpha\beta)(\mu\nu))}\left((1-\kappa_2)\dot{h}_{\alpha\beta} + \kappa_2\dot{\tilde{h}}_{\alpha\beta}\right), \tag{6.6}$$
$$\tilde{\Pi}^{\mu\nu} = \frac{\delta\mathcal{L}}{\delta\dot{\tilde{h}}_{\mu\nu}} = \frac{\kappa_2}{2\kappa}P^{((\alpha\beta)(\mu\nu))}\dot{h}_{\alpha\beta}. \tag{6.7}$$

We can write our fields h y $\tilde{h}$ the following way:

$$h_{\mu\nu}(\mathbf{x},t) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_{\mathbf{p}}}}\left[\chi^{(AB)}_{(\mu\nu)}(\mathbf{p})a_{(AB)}(\mathbf{p})e^{ip\cdot x} + \chi^{(AB)}_{(\mu\nu)}(\mathbf{p})a^{+}_{(AB)}(\mathbf{p})e^{-ip\cdot x}\right]|_{p_0=E_{\mathbf{p}}}$$
$$\tilde{h}_{\mu\nu}(\mathbf{x},t) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_{\mathbf{p}}}}\left[\chi^{(AB)}_{(\mu\nu)}(\mathbf{p})\tilde{a}_{(AB)}(\mathbf{p})e^{ip\cdot x} + \chi^{(AB)}_{(\mu\nu)}(\mathbf{p})\tilde{a}^{+}_{(AB)}(\mathbf{p})e^{-ip\cdot x}\right]|_{p_0=E_{\mathbf{p}}} \tag{6.8}$$

where $\chi^{(AB)}_{(\mu\nu)}(\mathbf{p})$ is a polarization tensor and $a_{(AB)}(\mathbf{p})$ and $\tilde{a}_{(AB)}(\mathbf{p})$ are promoted to annihilation operators when we quantize it. $a^{+}_{(AB)}(\mathbf{p})$ and $\tilde{a}^{+}_{(AB)}(\mathbf{p})$ correspond to the creation operators. A and B are indices of polarization that work like Lorentz indices, this is, they go from 0 to 3 and are moved up and down with η^{AB}. As these indices are presented symmetrically, we will have ten polarization tensors, enough to make a complete basis. For quantization of the model, we must impose the canonical commutation relations, the only non vanishing commutators are:

$$[h_{\mu\nu}(t,\mathbf{x}), \Pi^{\alpha\beta}(t,\mathbf{y})] = [\tilde{h}_{\mu\nu}(t,\mathbf{x}), \tilde{\Pi}^{\alpha\beta}(t,\mathbf{y})] = i\delta^{\alpha\beta}_{\mu\nu}\delta^3(\mathbf{x}-\mathbf{y}). \tag{6.9}$$

When expressed using (6.8) the non-vanishing commutators are:

$$[a^{AB}(\mathbf{p}), \tilde{a}^{+}_{CD}(\mathbf{p}')] = [\tilde{a}^{AB}(\mathbf{p}), a^{+}_{CD}(\mathbf{p}')] = \frac{4\kappa}{\kappa_2}\delta^{AB}_{CD}\delta^3(\mathbf{p}-\mathbf{p}'), \tag{6.10}$$

$$[\tilde{a}^{AB}(\mathbf{p}), \tilde{a}^{+}_{CD}(\mathbf{p}')] = -\frac{4\kappa(1-\kappa_2)}{\kappa_2^2}\delta^{AB}_{CD}\delta^3(\mathbf{p}-\mathbf{p}'). \tag{6.11}$$

There is a slight subtlety in calculating the above commutators. Basically, the expression that appears at one stage of the calculus is:

$$\sum_{ABCD}\chi^{(AB)}_{(\mu\nu)}P^{(\alpha\beta)}_{(\gamma\epsilon)}\chi^{(\gamma\epsilon)}_{CD} = \sum_{ABCD}\chi^{(AB)}_{(\mu\nu)}\frac{1}{2}\delta^{(\alpha\beta)}_{(\gamma\epsilon)}\chi^{(\gamma\epsilon)}_{CD} - \frac{1}{4}\eta^{\alpha\beta}\chi^{(AB)}_{(\mu\nu)}Tr(\chi), \tag{6.12}$$

and since we have the completeness relation:

$$\sum_{ABCD}\chi^{(AB)}_{(\mu\nu)}\chi^{(\alpha\beta)}_{(CD)}\delta^{(CD)}_{(AB)} = \delta^{(\alpha\beta)}_{(\mu\nu)}, \tag{6.13}$$

we must impose $Tr(\chi) = 0$, which in turn means that $Tr(h) = Tr(\tilde{h}) = 0$. This can always be done, because the gauge fixing being used does not fix the gauge freedom entirely, and this further condition can be imposed (see [39]).

The Hamiltonian expressed in terms of creation and annihilation operators is:

$$H = \int\frac{d^3p}{4\kappa}E_{\mathbf{p}}\left((1-\kappa_2)a^{+}_{AB}a^{AB} + \kappa_2 a^{+}_{AB}\tilde{a}^{AB} + \kappa_2\tilde{a}^{+}_{AB}a^{AB}\right), \tag{6.14}$$

where we have subtracted an infinite constant. Looking at this Hamiltonian, we notice that it has cross-products of operators, which obscures its physical interpretation. Something analogous happens when we observe the commutators (6.10) and (6.11), and so it is difficult to define their action over states. Because of this, we redefine our annihilation (and therefore also the creation) operators, for which we return to our action (6.1), defining:

$$\begin{aligned} h_{\mu\nu} &= A\bar{h}^1_{\mu\nu} + B\bar{h}^2_{\mu\nu}, \\ \tilde{h}_{\mu\nu} &= C\bar{h}^1_{\mu\nu} + D\bar{h}^2_{\mu\nu}, \end{aligned} \tag{6.15}$$

where A, B, C and D are real constants, so that the new fields, $\bar{h}^1$ and $\bar{h}^2$, are real fields. When replacing this in (6.1), we obtain:

$$\begin{aligned} S[\bar{h}^1, \bar{h}^2] &= \frac{1}{2\kappa}\int d^4x P^{((\alpha\beta)(\mu\nu))}\left(\frac{A}{2}(A - \kappa_2 A + 2\kappa_2 C)\bar{h}^1_{\alpha\beta}\partial^2\bar{h}^1_{\mu\nu} + \frac{B}{2}(B - \kappa_2 B + 2\kappa_2 D)\bar{h}^2_{\alpha\beta}\partial^2\bar{h}^2_{\mu\nu}\right) \\ &\quad + P^{((\alpha\beta)(\mu\nu))}(AB - \kappa_2 AB + \kappa_2 AD + \kappa_2 BC)\bar{h}^1_{\alpha\beta}\partial^2\bar{h}^2_{\mu\nu}. \end{aligned} \tag{6.16}$$

With the objective of decoupling the new fields, we make the last term in (6.16) null. It can be demonstrated that imposing the above criteria, it is inevitable that one (and only one) of two fields will be a ghost. We make the choice of $\bar{h}^2$ as the corresponding ghost. Taking the above considerations plus the condition that (6.16) to have the usual form of an action with real fields, we impose that the coefficients of the first and second terms in it are $\frac{1}{2}$ and $-\frac{1}{2}$, respectively. This means:

$$\begin{aligned} A &= B, \\ C &= \frac{1 - (1 - \kappa_2)B^2}{2\kappa_2 B}, \\ D &= -\frac{1 + (1 - \kappa_2)B^2}{2\kappa_2 B}, \end{aligned} \tag{6.17}$$

where B is left as an arbitrary real constant. Here we make the point that, if we had chosen $\bar{h}^1$ as the ghost, then the real constants change such that $C \leftrightarrow D$.

Thus, the action we are finally left with is:

$$S[\bar{h}^1, \bar{h}^2] = \frac{1}{2\kappa}\int d^4x P^{((\alpha\beta)(\mu\nu))}\left(\frac{1}{2}\bar{h}^1_{\alpha\beta}\partial^2\bar{h}^1_{\mu\nu} - \frac{1}{2}\bar{h}^2_{\alpha\beta}\partial^2\bar{h}^2_{\mu\nu}\right). \tag{6.18}$$

Following this same line of reasoning, we can find the annihilation operators for $\bar{h}^1$ and $\bar{h}^2$:

$$
\begin{aligned}
b^1_{AB}(\vec{p}) &= \frac{1+B^2(1-\kappa_2)}{2B}a_{AB}(\vec{p}) + \kappa_2 B\tilde{a}_{AB}(\vec{p}), && (6.19)\\
b^2_{AB}(\vec{p}) &= \frac{1-B^2(1-\kappa_2)}{2B}a_{AB}(\vec{p}) - \kappa_2 B\tilde{a}_{AB}(\vec{p}), && (6.20)
\end{aligned}
$$

where we have used (6.15). It can be verified that the only non vanishing commutators are now:

$$
[b^{1(AB)}(\vec{p}), b^{1+}_{CD}(\vec{p'})] = 4\kappa\delta^{AB}_{CD}\delta^3(\vec{p}-\vec{p'}), \tag{6.21}
$$

$$
[b^{2(AB)}(\vec{p}), b^{2+}_{CD}(\vec{p'})] = -4\kappa\delta^{AB}_{CD}\delta^3(\vec{p}-\vec{p'}). \tag{6.22}
$$

These commutators indicate that b^1 and b^2 have a vanishing inner product and that b^2 is the annihilation operator for the ghost. On the other hand, the Hamiltonian expressed in terms of these operators is:

$$
H = \int \frac{d^3p}{4\kappa} E_{\mathbf{p}}(b^{1+}_{AB}b^{1AB} - b^{2+}_{AB}b^{2AB}). \tag{6.23}
$$

Due to the existence of the ghost, it is possible that this model will not be unitary. To analyze this in greater depth, it is necessary to do a more profound study of the S-Matrix, but to do this for gravitation is a colossal task that would take us beyond the original scope of this work. On the other side, the existence of ghost or phantom fields has been proposed by some authors to explain the accelerated expansion of the universe [18] [19] [20] [21] [22], a feature that our model presents [17]. The problem with these models is that, when they are quantized, either there is a loss of unitarity or there is negative energy, which means loss of stability. Looking at (6.18), we find that the propagators of $\bar{h}^1$ and $\bar{h}^2$ are, respectively:

$$
-2\kappa P^{-1}_{((\alpha\beta)(\mu\nu))}\frac{i}{p^2 - i\varepsilon}, \tag{6.24}
$$

$$
2\kappa P^{-1}_{((\alpha\beta)(\mu\nu))}\frac{i}{p^2 \pm i\varepsilon}, \tag{6.25}
$$

where the sign $\pm$ in the phantom propagator, $\bar{h}^2$, will decide whether unitarity and negative energy solutions or nonunitary and positive energy solutions will be present in the model [21].

The advantage that our model has against other models that use scalar fields for the phantoms is that, being a gauge model, the possibility remains, open of fixing a gauge in which the model is unitary, keeping the model's good attributes, as in the BRST canonical quantization [40].

It is important to indicate, that the existence of ghosts is a general feature of all delta theories and not only subscribed to Delta Gravity, as can clearly be seen in [30] (see there Appendix B, where the hamiltonian of the model is not bounded by below).

The fact that our model has ghosts permits us to avoid a no go theorem [41][42] on the non possibility of having models with more than one consistent interacting gravitons (spin two fields). Thus, in our case, we have a model with two interacting gravitons, but with a hamiltonian not bounded by below (instability) as exhibited by (6.23).

On the other hand, as a possible solution to the case of instability, we may consider $\tilde{\delta}$ Supergravity, which may solve the unboundedness from below of the Hamiltonian. The last argument comes from the fact that in supersymmetry one defines the Hamiltonian as the square of an Hermitian charge, making it positive definite [43] [44].

Having explained the problem that our model has, now we discuss the new physics that our model might predict. For this, we will analyze the type of some finite quantum corrections and how the simplest of these affect the equations of motion of the model.

Chapter 7

Finite Quantum Corrections

The finite quantum corrections to our modified model of gravity can be separated into two groups. The first are the non-local terms, which are characterized by the presence of a logarithm, in the form [27]:

$$\begin{aligned} \sqrt{-g} R_{\mu\nu} \ln \left(\frac{\nabla^2}{\mu^2} \right) R^{\mu\nu} \\ \sqrt{-g} R \ln \left(\frac{\nabla^2}{\mu^2} \right) R \end{aligned} \tag{7.1}$$

where $\nabla^2 = g^{\alpha\beta} \nabla_\alpha \nabla_\beta$, ∇_β being the covariant derivative. There are no terms like the above ones but quadratic in the Riemann tensor because these terms always occur like:

$$\frac{1}{\epsilon} + \ln \left(\frac{\nabla^2}{\mu^2} \right), \tag{7.2}$$

and it is known that the terms that appear with the pole are purely Ricci tensors and Ricci scalars [8] [23] (see eq. (5.22) too), which in turn is due to (5.21). Now, when looking at the quantum corrections and Eq. (5.23), we need to care about the variations of (7.1) with respect to $g_{\mu\nu}$. Taking this into consideration, for the non-local terms we have:

$$
\begin{aligned}
\delta\left(\sqrt{-g}\right) R_{\mu\nu} \ln\left(\frac{\nabla^2}{\mu^2}\right) R^{\mu\nu} &= 0, \\
\sqrt{-g} R_{\mu\nu} \delta\left(\ln\left(\frac{\nabla^2}{\mu^2}\right) R^{\mu\nu}\right) &= 0, \\
\sqrt{-g}\delta(R_{\mu\nu}) \ln\left(\frac{\nabla^2}{\mu^2}\right) R^{\mu\nu} &= ,0 \\
\delta\left(\sqrt{-g}\right) R \ln\left(\frac{\nabla^2}{\mu^2}\right) R &= 0, \\
\sqrt{-g} R \delta\left(\ln\left(\frac{\nabla^2}{\mu^2}\right) R\right) &= 0, \\
\sqrt{-g}\delta(R) \ln\left(\frac{\nabla^2}{\mu^2}\right) R &= 0,
\end{aligned} \tag{7.3}
$$

because our model lives on shell, i.e. $R_{\mu\nu} \equiv 0$ and $R \equiv 0$. So, we see that the only relevant quantum corrections will come from the second group, that is, from the local terms that correspond to a series expansion in powers of the curvature tensor. The linear term is basically R, which corresponds to the original action, and the quadratic terms when taking into account their contribution is null due to (5.21). The next terms to consider are cubic in the Riemann tensor. In principle, any power of the curvature tensor will appear, but we now want to discuss only the cubic ones because they are the simpler to be dealt with [24]. The most general form of these corrections is:

$$
L_Q^{fin} = \sqrt{-g}\left(c_1\ R_{\mu\nu\lambda\sigma} R^{\alpha\beta\lambda\sigma} R^{\mu\nu}{}_{\alpha\beta}\ +\ c_2\ R^{\mu\nu}{}_{\lambda\sigma} R_{\mu\alpha}{}^{\lambda\beta} R^{\alpha\sigma}{}_{\nu\beta}\ +\ c_3\ R_{\mu\nu} R^{\mu\alpha\beta\gamma} R^{\nu}{}_{\alpha\beta\gamma}\ +\ c_4\ R R_{\mu\nu\lambda\kappa} R^{\mu\nu\lambda\kappa}\right). \tag{7.4}
$$

This type of corrections will affect the equations of motion for $\tilde{g}_{\mu\nu}$. So, using (3.9), we obtain:

$$
F^{(\mu\nu)(\alpha\beta)\rho\lambda} D_\rho D_\lambda \tilde{g}_{\alpha\beta} = -\frac{1}{\kappa_2}\left(M^{(\mu\nu)} + c_1 N^{(\mu\nu)} + c_2 B^{(\mu\nu)} + 3\left\{D_\rho\ ,\ D\sigma\right\} E^{[\sigma\mu][\nu\rho]}\right), \tag{7.5}
$$

with:

$$
\begin{aligned}
M^{(\mu\nu)} &= \frac{1}{2}\left(D_\alpha D^\nu A^{(\alpha\mu)} + D_\alpha D^\mu A^{(\alpha\nu)} - D_\alpha D^\alpha A^{(\mu\nu)} - g^{\mu\nu} D_\alpha D_\beta A^{(\alpha\beta)}\right), && (7.6)\\
A^{(\mu\nu)} &= c_3 R^{\mu\alpha\beta\gamma} R^{\nu}{}_{\alpha\beta\gamma} + c_4 g^{\mu\nu} R^{\alpha\beta\gamma\epsilon} R_{\alpha\beta\gamma\epsilon}, && (7.7)\\
N^{(\mu\nu)} &= \frac{1}{2} g^{\mu\nu} R_{\rho\epsilon\lambda\sigma} R^{\lambda\sigma\alpha\beta} R_{\alpha\beta}{}^{\rho\epsilon} + 3 R_{\rho\epsilon\lambda\sigma} R_{\alpha}{}^{\nu\epsilon\rho} R^{\alpha\mu\lambda\sigma}, && (7.8)\\
B^{(\mu\nu)} &= \frac{1}{2} g^{\mu\nu} R_{\rho\epsilon\lambda\sigma} R^{\rho\alpha\lambda\beta} R_{\alpha}{}^{\sigma\epsilon}{}_{\beta} + 3 R_{\rho\epsilon\lambda\sigma} R^{\nu\sigma\rho}{}_{\beta} R^{\mu\epsilon\beta\lambda}, && (7.9)\\
E^{[\sigma\mu][\nu\rho]} &= c_1 R^{\sigma\mu}{}_{\alpha\beta} R^{\alpha\beta\nu\rho} + \frac{1}{2} c_2\left(R^{\nu}{}_{\alpha}{}^{\sigma}{}_{\beta} R^{\rho\beta\alpha\mu} - R^{\rho}{}_{\alpha}{}^{\sigma}{}_{\beta} R^{\nu\beta\alpha\mu}\right), && (7.10)
\end{aligned}
$$

where $[\mu\nu]$ means that μ and ν are in a antisymmetric combination, and $F^{(\mu\nu)(\alpha\beta)\rho\lambda}$ was defined in (5.3). Obviously, if we do not have quantum corrections, i.e: $c_1 = c_2 = c_3 = c_4 = 0$, (7.5) is transformed in (5.2). It is possible to demonstrate that one solution to (5.2) is $\tilde{g}_{\mu\nu} = g_{\mu\nu}$, a fact that is necessary so that the predictions of the original theory of Einstein-Hilbert are still fulfilled in vacuum. This means, the solution of (7.5) must come to be small perturbations to $g_{\mu\nu}$.

$\tilde{\delta}$ Gravity will provide finite answers for the constants c_i. Due to the general structure of the finite quantum corrections, they will be relevant only at very short distances and strong curvatures. So the natural scenario to test the predictions of the model is the inflationary epoch of the Universe. The computation of the c_i and the phenomenological implications of Quantum $\tilde{\delta}$ Gravity will be discussed elsewhere.

Chapter 8

Conclusions

We have shown, following [30], that the $\tilde{\delta}$ transformation, applied to any theory, produces physical models that live only at one loop. This is achieved introducing new fields that generate a new constraint through a functional Dirac's delta inside the path integral (3.2). We have seen that the original symmetries are generalized when we apply the $\tilde{\delta}$ transformation. Moreover, the modified model is invariant under the generalized symmetries.

Now, going to $\tilde{\delta}$ Gravity, we calculated the divergent part of the action to one loop and we obtained twice the well-known result of [8]. We see that this factor of two appears also in [30]. The divergent part at one loop is zero in the absence of matter and on shell, so $\tilde{\delta}$ Gravity is a finite quantum model in four-dimensional space-time. This in turn implies that Newton's Gravitational Constant does not run with scale, which agrees with the very stringent experimental bounds that restrict its variation [37] [38].

We have shown that perturbing around the Minkowski vacuum and using a particular Lorentz-invariant gauge, we can redefine the gravitational fields in such a way that the free part of the action is decoupled. In this redefinition, it is seen that one of the new fields is a ghost. In spite of that, this may bring unitary or unstable problems (negative energies), these ghosts (phantoms) can explain at a classical level the accelerated expansion of the universe [17]. Scalar phantoms have been introduced in order to explain Dark Energy in [18] and discussed in many papers, for instance, [19] [20] [21] [22]. This connection may be far reaching, because the phantom idea has gained great popularity as an alternative to the cosmological constant.The present model could provide an arena to study the quantum properties of a phantom field, since the model has a finite quantum effective action. In this respect, the advantage of the present model is that, being a gauge model, it could give us the possibility to solve the problem of lack of unitarity using standard techniques of gauge theories as the BRST method. This is something that needs to be studied further but goes beyond the original scope of this work.

We want to point out that Supergravity with matter is finite at the one-loop level [10]. According to the general argument developed in this thesis, $\tilde{\delta}$ Supergravity will be a one-loop model that has a strong possibility to be a finite quantum model of gravity plus matter, and it may also solve the instability of negative energies since in supersymmetry one has a Hermitian charge whose square is equal to the Hamiltonian operator meaning that the Hamiltonian is positive definite [43] [44].

Finally, we have shown that the contribution of quadratic local and non-local logarithmic terms is zero due to the on-shell condition of the modified model. We have also shown how the cubic corrections in the Riemann tensor affect the equation of motion (7.5). Given the general form of the quantum corrections in quantum $\tilde{\delta}$ Gravity, they might be important during the inflationary epoch of the Universe.

Appendix A: Background Field Method

The Background Field Method (BFM) is a mechanism used to calculate the effective action at any order of perturbation theory without losing explicit gauge invariance. This simplifies the calculations and the comprehension of the model. The importance of the effective action is due to the fact that it contains all the quantum information of the theory and that from it all One-Particle-Irreducible (1PI) Feynman diagrams can be computed. Stringing them together, we can compute all connected Feynman diagrams in a more efficient manner [29] and from them the S-matrix can be calculated.

Next we calculate the effective action Γ for a general model using the BFM. One begins by defining the generating functional of disconnected diagrams $Z[J]$:

$$Z[J] = \int \mathcal{D}\varphi e^{i(S[\varphi]+J\cdot\varphi)}, \tag{8.1}$$

where S is the action of the system and where we will be using the notation $J \cdot \varphi \equiv \int J\varphi d^4x$. In the background field method, we identify $\varphi \rightarrow \varphi + \phi$ inside the action, where ϕ is an arbitrary background. So now we have:

$$\hat{Z}[J,\phi] = \int \mathcal{D}\varphi e^{i(S[\varphi+\phi]+J\cdot\varphi)}. \tag{8.2}$$

Now the generating functional of connected diagrams $W[J]$ is:

$$W[J] = -i \ln Z[J], \tag{8.3}$$

so we define:

$$\hat{W}[J,\phi] = -i \ln \hat{Z}[J,\phi], \tag{8.4}$$

and

$$\bar{\varphi} = \frac{\delta W}{\delta J}, \tag{8.5}$$

so here

$$\hat{\varphi} = \frac{\delta \hat{W}}{\delta J}, \tag{8.6}$$

with all these definitions it is possible to give the formula for the usual Effective Action:

$$\Gamma[\bar{\varphi}] = W[J] - J \cdot \bar{\varphi}, \tag{8.7}$$

and the background field effective action:

$$\hat{\Gamma}[\hat{\varphi}, \phi] = \hat{W}[J, \phi] - J \cdot \hat{\varphi}, \tag{8.8}$$

now we do the shift $\varphi \to \varphi - \phi$ so that:

$$\hat{Z}[J, \phi] = Z[J] e^{-iJ\cdot\phi}, \tag{8.9}$$

from which it follows (after taking logarithms):

$$\hat{W}[J, \phi] = W[J] - J \cdot \phi, \tag{8.10}$$

taking now the functional derivative with respect to J:

$$\hat{\varphi} = \bar{\varphi} - \phi, \tag{8.11}$$

but now we can appreciate that:

$$\begin{aligned}
\hat{\Gamma}[\hat{\varphi}, \phi] &= W[J] - J \cdot \phi - J \cdot \hat{\varphi}, \\
&= W[J] - J \cdot \phi - J \cdot (\bar{\varphi} - \phi), \\
&= W[J] - J \cdot \bar{\varphi}, \\
\hat{\Gamma}[\hat{\varphi}, \phi] &= \Gamma[\hat{\varphi} + \phi]. \qquad (8.12)
\end{aligned}$$

In particular if we take $\hat{\varphi} = 0$, we have:

$$\hat{\Gamma}[0, \phi] = \Gamma[\phi]. \tag{8.13}$$

This means that the Effective Action of the theory Γ can be computed from the background field Effective Action $\hat{\Gamma}$ by taking the quantum field to zero and with the presence of the background ϕ. Since the derivatives of the Effective Action with respect to the fields generate the 1PI diagrams, the last equation means that if we treat ϕ perturbatively what we will have will be diagrams with external legs corresponding to the background field ϕ and with internal lines corresponding to the quantum field φ.

And so, to study the quantum effects it only suffices to do an expansion in the quantum fields in the action S or in the lagrangian L using the identification of the Background Field Method. This means:

$$\begin{aligned} \phi_I &\to \phi_I + \varphi_I, \\ \tilde{\phi}_I &\to \tilde{\phi}_I + \tilde{\varphi}_I, \end{aligned} \tag{8.14}$$

We use (8.14) in $\tilde{\delta}$ Gravity, where $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ and $\tilde{g}_{\mu\nu} \to \tilde{g}_{\mu\nu} + \tilde{h}_{\mu\nu}$.

Appendix B: Divergent Part of the Effective Action at One Loop.

As was mentioned in **Chapter 3** there are various ways to calculate the divergent part of the effective action at one loop, but they are quite complicated. So, we have resolved to follow an algorithm developed in [23].

The effective action Γ to one loop can be written as:

$$\Gamma[\phi] = S[\phi] + \frac{i}{2}\hbar \mathrm{Tr}\ln D + O(\hbar^2), \tag{8.15}$$

where:

$$D_i{}^j = \frac{\delta^2 S}{\delta\phi_i \delta\phi_j}[\phi], \tag{8.16}$$

is a differential operator depending on the background field ϕ_i. Its most general form is:

$$\begin{aligned} D_i{}^j &= K^{\mu_1\mu_2\ldots\mu_L}{}_i{}^j \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_L} + S^{\mu_1\mu_2\ldots\mu_{L-1}}{}_i{}^j \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-1}} \\ &+ W^{\mu_1\mu_2\ldots\mu_{L-2}}{}_i{}^j \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-2}} + N^{\mu_1\mu_2\ldots\mu_{L-3}}{}_i{}^j \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-3}} \\ &+ M^{\mu_1\mu_2\ldots\mu_{L-4}}{}_i{}^j \nabla_{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_{L-4}} + \ldots \end{aligned} \tag{8.17}$$

where K, S, W, N, M are parameters which must be specify for each model and ∇_μ is a covariant derivative:

$$\nabla_\alpha T^{\beta}_{i}{}^{j} = \partial_\alpha T^{\beta}_{i}{}^{j} + \Gamma_{\alpha\gamma}{}^{\beta} T^{\gamma}_{i}{}^{j} + \omega_\alpha{}^k{}_i T^{\beta}_{k}{}^{j} - \omega_\alpha{}^j{}_k T^{\beta}_{i}{}^{k}, \tag{8.18}$$

$$\nabla_\mu \Phi_i = \partial_\mu \Phi_i + \omega_\mu{}^j{}_i \Phi_j, \tag{8.19}$$

here:

$$\Gamma_{\mu\nu}{}^{\alpha} = \frac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}), \tag{8.20}$$

and $\omega_{\mu}{}^{j}{}_{i}$ is a general connection that ensures the object in question to transform in the right way depending on which model we are considering (scalar-Yang Mills, vectorial, tensorial, etc). The computation of the divergent part of Effective Action at one loop is done through a lengthy and cumbersome calculation that consist in the sum of a finite number of one loop divergent Feynman diagrams, the details are given in [23] and the result by equation (30) in the same reference. This last result is too large to show here, but it depends on the parameters involved in $D_i{}^j$ (8.17). So basically, what we need is the quadratic part of the lagrangian of the model to obtain the divergent part of the effective action.

In $\tilde{\delta}$ Gravity, we have:

$$\phi_i \to \vec{h}_{(\alpha\beta)}, \tag{8.21}$$

where $\vec{h}$ is defined in (5.8). As the covariant derivative acting on $\vec{h}$ is given by (5.12) this means $i \to (\alpha\beta)$:

$$\omega_{\mu}{}^{j}{}_{i} \to -\left([\Gamma_{\mu\alpha}{}^{\rho}]\delta^{\nu}_{\beta} + [\Gamma_{\mu\beta}{}^{\rho}]\delta^{\nu}_{\alpha}\right), \tag{8.22}$$

where $[\Gamma_{\mu\alpha}{}^{\rho}]$ is given by equation (5.13). The other relevant parameters in our model are given by:

$L = 2$

$K^{\mu_1\mu_2\ldots\mu_L}{}_{i}^{j}$ given by (5.9).

$S^{\mu_1\mu_2\ldots\mu_{L-1}}{}_{i}^{j} = 0.$

$W^{\mu_1\mu_2\ldots\mu_{L-2}}{}_{i}^{j}$ given by (5.10).

On the other side, to the Faddeev-Popov ghosts we have:

$$\phi_i \to \vec{c}_\alpha, \tag{8.23}$$

where $\vec{c}_\alpha$ is defined by (5.15) and the Covariant Derivative (5.18) says us that:

$$\omega_{\mu\ i}^{\ j} \to -[\Gamma_{\mu\alpha}^{\ \ \rho}], \tag{8.24}$$

Finally, the other parameters are given by:

$L = 2$

$K^{\mu_1\mu_2\cdots\mu_L\ j}_{\quad\quad\quad\ \ i}$ given by (5.16).

$S^{\mu_1\mu_2\cdots\mu_{L-1}\ j}_{\quad\quad\quad\quad\ \ i} = 0$.

$W^{\mu_1\mu_2\cdots\mu_{L-2}\ j}_{\quad\quad\quad\quad\ \ i}$ given by (5.17).

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