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Comment

Comment on "Domain wall motion in thin ferromagnetic nanotubes: Analytic results" by Goussev Arseni et al.

M. C. Depassier

Instituto de Física, Pontificia Universidad Católica de Chile - Casilla 306, Santiago 22, Chile

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The propagation of a domain wall (DW) in a thin ferromagnetic nanotube when an external field is applied along the axis of the cylinder is studied in the recent letter [1]. It is shown in [1] that the time evolution of the magnetization distribution is governed by a modified Landau-Lifshitz-Gilbert equation which, in the limit of strong penalization, reduces to a reaction diffusion equation. The exact formula for the speed of the DW is obtained and its nonlinear dependence on the nanotube radius analyzed.

In this Comment we complement the results of [1] showing that above a critical external applied magnetic field $H_a^{crit}(R)$ there is a transition to a different regime and the speed is no longer given by the formula derived in [1]. The front becomes of Kolmogorov-Petrovskii-Piscounov type (KPP) and the speed is given by a different expression, which we give below. In this regime the speed of the DW has a different dependence on the nanotube radius and on the applied field which yields lower values than those predicted in [1]. In a similar way, there is a critical radius $R_{crit}(H_a)$ above which the KPP regime holds. The functional dependence of the speed on the external field and on the nanotube radius exhibits a sudden change in slope which is reminiscent of that reported in [2].

The dynamics of the magnetization found in [1] in the case of strong penalization (weak anisotropy) is given by the reaction diffusion equation, eq. (12) in [1],

$$\frac{\partial\theta}{\partial\tau}=\left(\frac{\alpha}{\gamma}+\frac{\gamma}{\alpha}\right)(\Delta_S^2\theta-(1+\kappa)\sin\theta\cos\theta-\mathcal{H}_a\sin\theta). \quad (1)$$

Here θ denotes the angle of the magnetization vector with the longitudinal axis $\hat{x}$, that is, $\vec{m}=\cos\theta\hat{e}_x+\sin\theta\hat{e}_\psi$ and $\hat{e}_\psi$ is the unit axis tangential to the surface of the cylinder. The dimensionless parameters κ and $\mathcal{H}_a$ are defined in terms of the physical constants as $\kappa=KR^2/A$ and $\mathcal{H}_a=\mu_0 M_S R^2 H_a/(2A)$, where R is the nanotube radius, A is the exchange constant, K the easy-axis uniaxial anisotropy constant, μ_0 is the magnetic permeability of vacuum, H_a is the applied field and M_s is the saturation magnetization. The constant γ is the electron gyromagnetic ratio and α is a phenomenological damping parameter. The dimensionless time variable is defined by $\tau=[2\gamma A/(\mu_0 M_s R^2)]t$. The subscript S denotes surface derivatives [1].

This equation has an exact solution, a monotonic decaying front $\theta(\xi+c_P\tau)$ joining the unstable equilibrium $\theta=\pi$ to the stable equilibrium $\theta=0$ propagating with speed

$$c_P=\left(\frac{\alpha}{\gamma}+\frac{\gamma}{\alpha}\right)\frac{\mathcal{H}_a}{\sqrt{1+\kappa}}. \quad (2)$$

In terms of the physical parameters it is given by (eq. (19) in [1]),

$$c_{NL}=\left(\frac{\alpha}{\gamma}+\frac{\gamma}{\alpha}\right)\frac{\gamma R H_a}{\sqrt{1+KR^2/A}}. \quad (3)$$

We may scale the time variable τ in (1) to absorb the factor $\alpha/\gamma+\gamma/\alpha$ and consider the reaction diffusion equation

$$\dot{\theta}=\Delta_S^2\theta-(1+\kappa)\sin\theta\cos\theta-\mathcal{H}_a\sin\theta.$$

The reaction term $-(1+\kappa)\sin\theta\cos\theta-\mathcal{H}_a\sin\theta$ may give rise to monostable or bistable equilibria. In the monostable case, in which $\theta=0$ is the stable equilibrium and $\theta=\pi$ is the unstable equilibrium, the speed of the front may be of KPP (or pulled) type [3] or it may be of pushed type [4], depending on the value of the parameters. The solution given in [1] corresponds to the exact solution in the case of the pushed front which also holds in the bistable regime. As we show below there is a regime in which the front becomes of KPP type. In this regime the speed shows a different functional dependence on the applied field and nanotube radius.

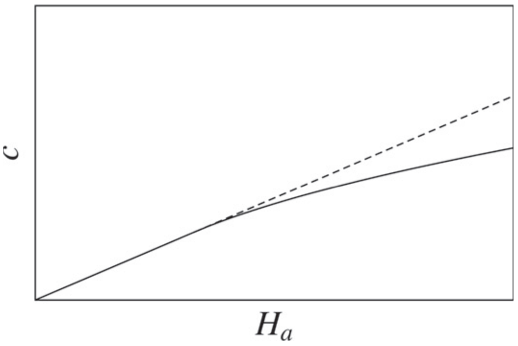

Fig. 1: The solid line represents the speed of the front as a function of applied field. At low field the front is pushed, at higher values of the field the front becomes of KPP type. The dashed line shows the speed of the pushed front which is not the selected speed beyond the critical value of the field H_a^{crit}.

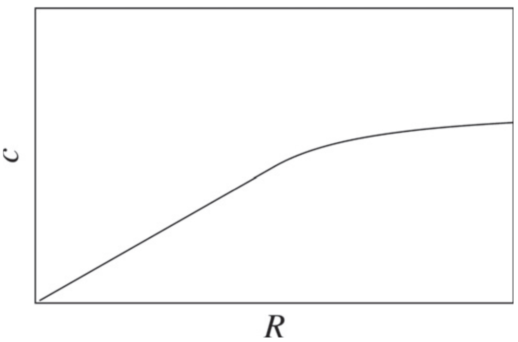

Fig. 2: Speed of the front as a function of nanotube radius. At lower radii the front is pushed, at higher values of the radius the front becomes of KPP-type regime in which the speed increases with the radius but at a lower rate than in the pushed regime.

In order to apply directly the theory of reaction diffusion equations [5,6] it is convenient to make the change of variables $\theta = \pi(1-u)$ to obtain the equation

$$\dot{u} = \Delta_S^2 u + f(u)$$

with

$$f(u) = \frac{\sin(\pi u)}{\pi}(\mathcal{H}_a - (1+\kappa)\cos(\pi u)).$$

In the monostable case, $u = 1$ is the stable equilibrium and $u = 0$ is the unstable equilibrium. In the bistable case both are stable, and the intermediate point becomes unstable; here we shall only consider the transition to the KPP regime in the monostable case. A small perturbation to the unstable state evolves into the front of minimal speed c^* which satisfies [3]

$$2\sqrt{f'(0)} \leq c^* \leq 2\sup\sqrt{f(u)/u}.$$

The KPP value for the speed is given by

$$c_{KPP} = 2\sqrt{f'(0)} = 2\sqrt{\mathcal{H}_a - (1+\kappa)}.$$

In terms of the original physical quantities, the speed is

$$c_{KPP} = \frac{4\gamma A}{\mu_0 M_s R}\left(\frac{\alpha}{\gamma} + \frac{\gamma}{\alpha}\right)\sqrt{\frac{\mu_0 M_s R^2}{2A}H_a - \left(1 + \frac{KR^2}{A}\right)},$$

which has a different functional dependence on the radius and applied field than the speed in the pushed regime, eq. (3).

The front will be with certainty of KPP type when f is concave, that is when $\mathcal{H}_a \geq 4(1+\kappa)$; however the KPP regime may hold for lower values of the applied field [4]. In this problem, for which the solution in the pushed regime is known, we know that the transition occurs at the point where $c_{NL} = c_{KPP}$. It is straightforward to verify that for fixed radius the KPP regime holds when

$$H_a \geq H_a^{crit}(R) = \frac{4A}{\mu_0 M_s R^2}\left(1 + \frac{KR^2}{A}\right).$$

For a given applied magnetic field H_a, the KPP regime holds when

$$R \geq R_{crit}(H_a) = \sqrt{\frac{4A}{\mu_0 M_s H_a - 4K}}.$$

In fig. 1 we show the dependence of the speed on the applied magnetic field for fixed radius. In fig. 2 we show the speed as a function of the nanotube radius for a fixed applied field, large enough so that the KPP regime exists. At the transition value H_a^{crit} in fig. 1 or R_{crit} in fig. 2, the rate of increase of the speed shows a marked decrease.

Similar results hold in the weak penalization regime.

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REFERENCES

[1] Goussev A., Robbins J. M. and Slastikov V., *EPL*, **105** (2014) 67006.
[2] Yan M., Andreas C., Kákay A., García-Sánchez F. and Hertel R., *Appl. Phys. Lett.*, **99** (2011) 122505.
[3] Kolmogorov A., Petrovskii I. G. and Piscounov N., *Bull. Univ. Moscow, Math. Mech.*, **1** (1937) 1.
[4] Benguria R. D. and Depassier M. C., *Phys. Rev. Lett.*, **77** (1996) 1171.
[5] Vol'pert A. I., Vol'pert V. A. and Vol'pert V. A., *Traveling Wave Solutions of Parabolic Systems*, *Translations of Mathematical Monographs*, Vol. **140** (American Mathematical Society, Providence, RI) 1994.
[6] van Saarloos W., *Phys. Rep.*, **386** (2003) 29.