

Recent Developments in the Spectral Theory of Orthogonal Polynomials

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Abstract

I will focus on three recent developments in the past year presenting overviews rather than attempting anything comprehensive on any of these three.

Lecture 1: Background. This will first discuss Jacobi and CMV matrices and the spectral-parameter equivalence. Then I'll discuss two fundamental sum rules (Szegő and Killip-Simon) and Szegő asymptotics.

Lecture 2: Sum Rules and Large Deviations. Gamboa, Nagel and Rouault have found a spectacular derivation of sum rules using the theory of large deviations. I'll present their strategy and the application of this strategy by Breuer, Simon and Zeitouni to high order Szegő theorems.

Lecture 3: Killip-Simon Theorems for Finite Gap Sets. After discussing the approach of Damanik-Killip-Simon for periodic spectral sets, I'll give an overview to Yuditskii's recent breakthrough on general finite gap sets.

Lecture 4: Szegő-Widom asymptotics for Chebyshev Polynomials. I'll discuss the Widom surmise for asymptotics of Chebyshev polynomials, and the recent proof by Christensen, Simon and Zinchenko of Widom's 1969 conjecture on the Widom surmise.